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TM 11-684

DEPARTMENT OF THE ARMY TECHNICAL MANUAL

UNIVERSITY OF VIRGINIA

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ALDERMAN-GOV'T DOCUMENTS

PRINCIPLES AND APPLICATIONS OF MATHEMATICS FOR COMMUNICATIONS-ELECTRONICS

This copy is a reprint which includes current
pages from Change 1.

*HEADQUARTERS, DEPARTMENT OF THE ARMY
OCTOBER 1961*

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CHANGE }
No. 1 }

HEADQUARTERS
DEPARTMENT OF THE ARMY
WASHINGTON, D. C., 24 July 1967

PRINCIPLES AND APPLICATIONS
OF MATHEMATICS FOR
COMMUNICATIONS-ELECTRONICS

TM 11-684, 6 October 1961, is changed as follows:

TAGS 100A

Page	Paragraph	Line	Manual now reads --	Change to read --
4	5, <u>Example 1</u>	4	6.25	.625
6	12g (10)		$1\frac{7}{8}$	$\frac{7}{8}$
12	24g	9	from 1 to 100	from 1 to 1,000
17	31	2	arithmetic (ch. 5).	arithmetic.
18	Caption of figure 6		(-1 and -5).	(-2 and -3).
21	42g, <u>Example 2</u>	4	$-(x^2 - 2x^2 - x + 1)$	$-(x^2 + 2x^2 + x - 1)$
22	45, <u>Example 1</u>	2	$(5^2)^4 = 5^{2 \cdot 4} = 5^8$	$(5^2)^4 = 5^2 \cdot 4 = 5^{10}$
22	45, <u>Example 2</u>	3	or $2^1 \cdot 3^1 \cdot 4^1 \cdot 5^1 \cdot 6^1 = 2^8 3^4 5^2$	or $2^1 \cdot 3^1 \cdot 4^1 \cdot 5^1 \cdot 6^1 = 2^8 3^4 5^2$
22	45, <u>Example 3</u>	2	$(6x^2)^3 = 6^3 \cdot 2^3 \cdot x^6 = 6^3 2^3 x^6$	$(6x^2)^3 = 6^3 \cdot 2^3 \cdot x^6 = 6^3 2^3 x^6$
22	45, <u>Example 4</u>	2	$[(x^2)^3]^4 = [x^2 \cdot 3]^4 = [x^6]^4 = x^{6 \cdot 4} = x^{24}$	$[(x^2)^3]^4 = [x^2 \cdot 3]^4 = [x^6]^4 = x^{6 \cdot 4} = x^{24}$

Page	Paragraph	Lines	Manual now reads —	Change to read —
22	45, <u>Example 5</u>	2	$\left(\frac{2}{x^2}\right)^5 = \frac{2^{1 \cdot 5}}{x^{2 \cdot 5}} = \frac{2^5}{x^{10}} = \frac{32}{x^{10}}$	$\left(\frac{2}{x^2}\right)^5 = \frac{2^{1 \cdot 5}}{x^{2 \cdot 5}} = \frac{2^5}{x^{10}} = \frac{32}{x^{10}}$
24	49b, <u>Example 3</u>	1	Divide 6a	Divide 6a ³
25	50f (5)		$(3x^3 - 2xy - 5y^2)(3x^2 + 2xy - 5y^2)$	$(3x^3 - 2xy + 5y^2)(3x^2 + 2xy - 5y^2)$
26	56g, <u>Example 3</u>	2	$2ax^2 - 4bx^2 + 6cx^2 = 2x^2(a - 2b + 3c)$	$2ax^2 - 4bx^2 + 6cx^2 = 2x^2(a - 2b + 3c)$
30	59g	6	$xb + xb + ab$	$xb + ab$
32	61c (8)		$\sqrt[3]{-x^3}$	$\sqrt[3]{-x^3}$
32	61d (9)		$\pi r \frac{2}{1} + \pi \frac{2}{2}$	$\pi r^2 + \pi r^2$
33	63c	3	$-\frac{(x-y)(x-2y)}{x+y} = \frac{(x+y)(x-2y)(y-x)(x-2y)}{x+y} = \frac{(x-2y)(y-x)(x-2y)}{x+y}$	$-\frac{(x-y)(x-2y)}{x+y} = \frac{(x-2y)(y-x)(x-2y)}{x+y}$
36	68g, <u>Example 1</u>	4	$\frac{6ab}{7x} \cdot \frac{24x^2y}{21ab} = \frac{3xy}{4}$	$\frac{6ab}{7x} \cdot \frac{24x^2y}{21ab} = \frac{3xy}{4}$

Page	Paragraph	Line	Manual now reads —	Change to read —
37	69b (7)	2	denominator is 2PR.	denominator is $6\pi^2 c$.
38	72a	8	$\sqrt{2 \cdot 5^4} = 5^2 \sqrt{2} = 25\sqrt{2}$	$\sqrt{2 \cdot 5^4} = 5^2 \sqrt{2} = 25\sqrt{2}$
38	72a, <u>Example 1</u>	2 & 3	$\sqrt{50} = \sqrt{25 \cdot 2}$ $= \sqrt{25} \cdot \sqrt{2}$	$\sqrt{50} = \sqrt{25 \cdot 2}$ $= \sqrt{25} \cdot \sqrt{2}$
40	73, <u>Example 3</u>	3	$= (4r)^{\frac{2}{3}} - 4(4r)^{\frac{1}{3}} + (4r)^{\frac{5}{6}}$	$= (4r)^{\frac{2}{3}} - r(4r)^{\frac{1}{3}} + (4r)^{\frac{5}{6}}$
40	73, <u>Example 4</u>		$2\sqrt{6} + 9\sqrt{\frac{2}{3}} - \sqrt[4]{36}$	$2\sqrt{6} + 9\sqrt{\frac{2}{3}} - \sqrt[4]{36}$
			$= 2\sqrt{6} + 9\sqrt{\frac{2 \cdot 3}{3 \cdot 3}} - \sqrt[4]{6 \cdot 6}$	$= 2\sqrt{6} + 9\sqrt{\frac{2}{3}} - \sqrt[4]{6 \cdot 6}$
			$= 2\sqrt{6} + \frac{9}{3} \sqrt{6} - \sqrt[4]{6^2}$	$= 2\sqrt{6} + \frac{9}{3} \sqrt{6} - \sqrt[4]{6^2}$
			$= 2\sqrt{6} + 3\sqrt{6} - \sqrt{6}$	$= 2\sqrt{6} + 3\sqrt{6} - \sqrt{6}$
			$= 4\sqrt{6}$	$= 4\sqrt{6}$
41	76a, <u>Example 1</u>	2	$\frac{\sqrt{16}}{\sqrt{5}} = \sqrt{\frac{16}{5}} = \sqrt{3}$	$\frac{\sqrt{16}}{\sqrt{5}} = \sqrt{\frac{16}{5}} = \sqrt{3}$

Page	Paragraph	Line	Manual now reads —	Change to read —
43	76 ₁ , <u>Example 2</u>	2	$\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}}$ $\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}}$ $\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}}$ $\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}}$ $\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}}$	$\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}} =$ $\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}}$ $\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}}$ $\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}}$ $\frac{\sqrt{1+z} - \sqrt{1-z}}{\sqrt{1+z} + \sqrt{1-z}}$
43	76 ₂ (9)		$(2r_1 + 2r_2)^{\frac{1}{2}}$	$(2r_1 + 2r_2)^{\frac{1}{2}}$
43	76 ₂ (10)		$5 \frac{i+3}{\sqrt{(i+3)^2}}$	$5 \frac{i+3}{\sqrt{(i+3)^2}}$
43	76 ₂ (4)		$\frac{2xy}{\sqrt{2x}}$	$\frac{2xy}{\sqrt{2y}}$
43	76 ₂ (5)		$\frac{2}{\sqrt{6-2}}$	$\frac{2}{\sqrt{6-2}}$
43	76 ₂ (9)		$\frac{\sqrt{e^2 + f^2 + f}}{\sqrt{e^2 + f^2 - f}}$	$\frac{\sqrt{e^2 + f^2 + f}}{\sqrt{e^2 + f^2 - f}}$

Page	Paragraph	Line	Manual now reads —	Change to read —
44	77b	7	$j^3 = j \cdot j \cdot j \cdot j \cdot j = j^4 \cdot j = j^1 = \sqrt{-1}$	$j^3 = j \cdot j \cdot j \cdot j \cdot j = j^4 \cdot j = j^1 = \sqrt{-1}$
44	77c <u>Example 2</u>	2-5	$6\sqrt{-2} + 5\sqrt{-8} + 8\sqrt{-18}$ $= j^3\sqrt{2} + j^3\sqrt{8} + j^3\sqrt{18}$ $= j^3\sqrt{2} + j(5 \cdot 2)\sqrt{2} + j(8 \cdot 3)\sqrt{2}$ $= (j^3 + j^{10} + j^{24})\sqrt{2}$ $= j^{10}\sqrt{2}$	$6\sqrt{-2} + 5\sqrt{-8} + 8\sqrt{-18}$ $= j^6\sqrt{2} + j^5\sqrt{8} + j^8\sqrt{18}$ $= j^6\sqrt{2} + j(5 \cdot 2)\sqrt{2} + j(8 \cdot 3)\sqrt{2}$ $= (j^6 + j^{10} + j^{24})\sqrt{2}$ $= j^{10}\sqrt{2}$
44	77c. <u>Example 3</u>	2	$\sqrt{-36} - \sqrt{-64} = j^3 - j^3 = -j^2$	$\sqrt{-36} - \sqrt{-64} = j^6 - j^8 = -j^2$
44	77c. <u>Example 4</u>	3 & 4	$= (j^{10} - j^3)\sqrt{2}$ $= j^{10}\sqrt{2}$	$= (j^{18} - j^8)\sqrt{2}$ $= j^{10}\sqrt{2}$
45	77d. <u>Example 1</u>	2	$\sqrt{-16} \cdot \sqrt{-4} = j^4 \cdot j^2 = j^8 = (-1)8 = -8$	$\sqrt{-16} \cdot \sqrt{-4} = j^4 \cdot j^2 = j^8 = (-1)8 = -8$
45	77d. <u>Example 2</u>	2	$\sqrt{-81} \cdot \sqrt{-25} \cdot \sqrt{-49}$ $= j^3 \cdot j^3 \cdot j^7$ $= j^{315}$ $= (-j)^{315}$ $= -j^{315}$	$\sqrt{-81} \cdot \sqrt{-25} \cdot \sqrt{-49}$ $= j^9 \cdot j^5 \cdot j^7$ $= j^{315}$ $= (-j)^{315}$ $= -j^{315}$

Page	Paragraph	Line	Manual now reads—	Change to read—
45	77b. <u>Example 4</u>	2	$\frac{6}{j} = \frac{6}{j} \cdot \frac{j}{j} = \frac{j6}{j^2} =$ $\frac{j6}{-1} = -j6$	$\frac{6}{j} = \frac{6}{j} \cdot \frac{j}{j} = \frac{j6}{j^2} =$ $\frac{j6}{-1} = -j6$
47	78b. <u>Example 1</u>	4	$= \frac{8 + j - (1 - 1)4}{1 - (-1)}$	$= \frac{8 + j - (-1)4}{1 - (-1)}$
62	90d (9)	2	$\frac{3}{z} + \frac{1}{y} = 1\frac{3}{5}$	$\frac{3}{z} - \frac{1}{y} = 1\frac{3}{5}$
62	100g (4)	1	$f(z) = z^2 - 6z + 12$	$f(z) = z^2 - 6z + 12$
70	118. <u>Example 2</u> <u>Step 3</u>	5	.0021	.00121
71	119. <u>Example 2</u>	1	6.7166 - 10	6.8166 - 10
71	119. <u>Example 2</u> <u>Step 6</u>	4	6.7166 - 10	6.8166 - 10
84	140b. <u>Example 2</u>	10	$D = \sqrt[3]{\frac{A}{v}}$	$D = 2\sqrt{\frac{A}{v}}$

Page	Paragraph	Line	Manual now reads —	Change to read —
84	140g. <u>Example 2</u>	12 & 13	$D = \sqrt[2]{\frac{78.54}{3.1416}}$ $D = \sqrt[2]{25}$	$D = 2\sqrt{\frac{78.54}{3.1416}}$ $D = 2\sqrt{25}$
89	147. <u>Example 1</u>	1	Given the right triangle ABC (fig. 23):	Given the right triangle ABC (fig. 24):
90	147. <u>Example 2</u>	1	Given the right triangle ABC (fig. 23): side A is $\sqrt{5}$;	Given the right triangle ABC (fig. 24): side a is $\sqrt{5}$;
91	148. <u>Example 1</u>	15	$\cos A = \frac{\sqrt{6}}{30} = \frac{1}{5}$	$\cos A = \frac{6}{30} = \frac{1}{5}$
95	152d. <u>Example 1</u>	3	$3\sqrt{71^\circ 22' 21''}$	$3\sqrt{71^\circ 22' 21''}$
95	152d. <u>Example 2</u>	3	$6\sqrt{165^\circ 17' 36''}$	$6\sqrt{165^\circ 17' 36''}$
106	164g (4)	1	Side B	Side b
107	165	1	An oblique triangle is one in which one of	An oblique triangle is one in which none of
122	172g	4	$40^\circ = \frac{1}{4} \pi$ radians	$45^\circ = \frac{1}{4} \pi$ radians

Page	Paragraph	Line	Material now reads—	Change to read—
185	182d	4	cuit c	cuit A
189	204f			Delete the first sentence and insert the following: A 300-volt, 60-cycle ac generator is connected in series with a 6-ohm resistance, and 8-ohm inductive reactance, and a 16-ohm capacitive reactance.
179	224	25	(par. 207).	(par. 223).
202	21f	2	2,567 pounds	2,567 pounds
202	42g (3)		$\frac{3}{35}$	$-\frac{3}{35}$
202	42f (5)		$-\frac{2}{3}$ ampere	$\frac{2}{3}$ ampere
202	50g (1)		$4x^4 - 4x^3 + 4x^2$	$6x^4 - 4x^3 + 4x^2$
202	50f (2)		$\frac{1}{x^2}$	$\frac{1}{x^2}$
202	50f (6)		$\frac{a - e}{es}$	$\frac{a - e}{es}$

Page	Paragraph	Line	Manual now reads —	Change to read —
223	61 ₄ (2)		- 8	8
223	61 ₄ (5)		- 10ab ³	± 10ab ³
223	61 ₄ (7)		14 ⁴ xy	14xy ³
223	60 ₂ (4)		$z = -1 \frac{1}{12}$	$z = -1 \frac{13}{16}$
223	60 ₂ (7)		$\frac{y}{C^2 - c^2} \cdot \frac{z(c+d)}{C^2 - c^2}$	$\frac{y}{c^2 - c^2} \cdot \frac{z(c+d)}{c^2 - c^2}$
223	60 ₂ (4)		$\frac{6(x^2 - 2)}{x^4 - 5x^2 + 14}$	$\frac{6(x^2 - 2)}{x^4 - 5x^2 + 4}$
222	60 ₂ (5)		$\frac{9c + 2ad + 12d}{12c^2c^2}$	$\frac{9c^2 + 2ad - 12c^2}{12c^2c^2}$
223	60 ₂ (10)		$\frac{3c + 4y}{12c^2c^2}$	$\frac{3c + 4y}{12c^2c^2}$
223	76 ₂ (4)		$2\sqrt{2}$	$2\sqrt{2}$
223	76 ₂ (5)		$5\sqrt{2}$	$5\sqrt{2}$

Page	Paragraph	Line	Manual now reads —	Change to read —
223	762 (6)		$\sqrt[4]{a^4b^4}$	$\sqrt[4]{a^4b^4}$
223	762 (8)		$2b\sqrt{c^3}$	$2b\sqrt{c^3}$
223	762 (3)		$\frac{\sqrt{3a}}{3}$	$\frac{2\sqrt{3a}}{3}$
224	762 (9)		$\frac{a^2 + b^2 + 2b\sqrt{a^2 + b^2}}{c^2}$	$\frac{a^2 + 2b^2 + 2b\sqrt{a^2 + b^2}}{c^2}$
224	762 (7)		$\sqrt{5} + 3\sqrt{2}$	$\sqrt{5} + 3\sqrt{2}$
224	762 (4)		$4 + j10$	$44 + j10$
224	762 (7)		$p + jio - e^p$	$p + 2jio - e^p$
224	762 (5)		$\frac{x^2 + jxy - y^2}{x^2y^2}$	$\frac{x^2 + 2jxy - y^2}{x^2 + y^2}$
224	762 (10)		$\frac{p + j2/E - E^2}{p^2}$	$\frac{p + j2/E - E^2}{p^2 + E^2}$

Page	Paragraph	Line	Manual now reads —	Change to read —
234	86 _g (5)		28 — 9	24 9
235	94 _g (4)		3, 4 — 2 3	3, 4 — 2 3
235	127 _f (4)		83. 28	33. 37
235	153 _h (1)		$\tan A = \frac{4}{\sqrt{33}}$	$\tan A = \frac{4}{33} \sqrt{33}$
236	164 _g (11)		32. 9	30.9
236	164 _g (12)		side opposite 60° / 5. 196 inches, side opposite 30° / 3 inches.	side opposite / 60° 5. 196 inches, side opposite / 30° 3 inches.

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For explanation of abbreviations used, see AR 390-50.

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HEADQUARTERS,
DEPARTMENT OF THE ARMY
WASHINGTON 25, D.C., 6 October 1961

PRINCIPLES AND APPLICATIONS OF MATHEMATICS FOR COMMUNICATIONS-ELECTRONICS

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PART I

MATHEMATICAL PRINCIPLES ESSENTIAL TO COMMUNICATIONS-ELECTRONICS

CHAPTER 1

INTRODUCTION

1. Purpose and Scope

a. Purpose. This manual provides the basic mathematics required by communications-electronics personnel.

b. Scope. This manual covers those principles and applications of arithmetic, algebra, logarithms, geometry, and trigonometry that are required for a practical understanding of electricity and electronics. The manual is divided into two parts:

- (1) Part I is a review of the mathematical principles essential to communications-electronics.
- (2) Part II covers the application of the

mathematical principles to common communications-electronics problems.

2. Mathematics and Electronics

Skill in the use of mathematics, particularly arithmetic, algebra, and trigonometry, is essential in the fields of electricity and electronics. Most of our basic ideas of electrical phenomena are based on mathematical reasoning and are stated in mathematical terms. Therefore, a thorough knowledge of mathematics and of the specific applications of mathematics to the field of electricity will serve as a foundation for the technical knowledge needed by communications-electronics personnel.

CHAPTER 2

PERCENTAGE

3. General

a. *Definition.* Percentage is the process of computation in which the basis of comparison is a *hundred*. The term *percent*—from *per*, by, and *centum*, hundred—means *by* or *on the hundred*. Thus, 2 percent of a quantity means two parts of every hundred parts of the quantity.

b. *Symbol.* The symbol of percentage is %. Percent may also be indicated by a fraction or a decimal. Thus, $5\% = \frac{5}{100} = .05$. Figure 1 shows the relationship between fractions, decimals, and percentage.

c. *Base, Rate, and Percentage.*

- (1) The *base* is the number on which the percentage is computed.
- (2) The *rate* is the amount (in hundredths) of the base to be estimated.
- (3) The *percentage* is a part or proportion of a whole expressed as so many per hundred. Percentage is the portion of the base determined by the rate.

4. Conversion of Decimal to Percent

To change a decimal to percent, move the decimal point two places to the right and add the percent symbol.

Example: Change .375 to percent.

Move decimal point two places to right: 37.5

Add percent symbol: 37.5%

5. Conversion of Fraction to Percent

To convert a fraction to percent, divide the numerator by the denominator and convert to a decimal. Then, convert the decimal to percent (par. 4).

Example: Change fraction $\frac{5}{8}$ to percent.

Divide numerator by denominator: $5 \div 8 = .625$

Convert decimal to percent: $6.25 = 62.5\%$

Thus, $\frac{5}{8} = 62.5\%$.

6. Conversion of Percent to Decimal

To change a percent to a decimal, omit the percent symbol and move the decimal point two places to the left.

Example 1: Change 15% to a decimal.

Omit percent symbol: 15% becomes 15

Move decimal point two places to the left: 15 becomes .15

Thus, $15\% = .15$.

Example 2: Change 110% to a decimal.

Omit percent symbol: 110% becomes 110

Move the decimal point two places to the left: 110 becomes 1.10.

Thus, $110\% = 1.10$.

7. Conversion of Percent to Fraction

To change a percent to a fraction, first change the percent to a decimal (par. 6) and then to a fraction. Reduce the fraction to its lowest terms.

Example 1: Change 25% to a fraction.

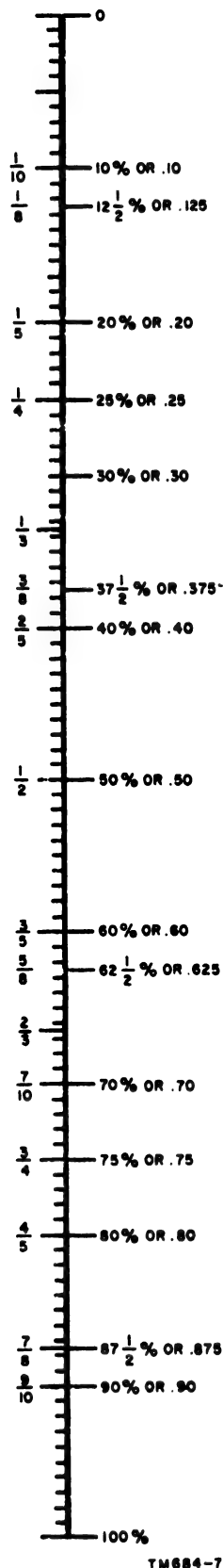
Change to a decimal: $25\% = .25$

Change to a fraction: $.25 = \frac{25}{100}$

Reduce fraction to lowest terms:

$$\frac{25}{100} = \frac{1}{4}$$

Thus, $25\% = \frac{1}{4}$.



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Figure 1. Relationship between fractions, decimals, and percentage.

Example 2: Change 37.5% to a fraction.

Change to a decimal: $37.5\% = .375$

Change to a fraction:

$$.375 = \frac{375}{1000}$$

Reduce fraction to lowest terms:

$$\frac{375}{1000} = \frac{3}{8}$$

Thus, $37.5\% = \frac{3}{8}$.

8. Finding Percentage

a. General. To find the percent of a number, write the percent as a decimal and multiply the number by this decimal. In this case, the *base* and *rate* are given. The problem is to find the *percentage*.

Example 1: Find 5% of 140 (140 is the base, 5% is the rate, and the product is the percentage).

$$5\% \text{ of } 140 = .05 \times 140 = 7$$

Example 2: Find 5.2% of 140.

$$5.2\% \text{ of } 140 = .052 \times 140 = 7.28$$

Example 3: Find 150% of 36.

$$150\% \text{ of } 36 = 1.50 \times 36 = 54$$

Example 4: Find $\frac{1}{2}\%$ of 840.

$$\frac{1}{2}\% = .5\%$$

$$.5\% \text{ of } 840 = .005 \times 840 = 4.20$$

Thus, $\frac{1}{2}\%$ of 840 = 4.20.

b. Application of Percentage. In communications-electronics, typical applications of percentage computation are used in determining tolerance values of resistors (par. 206) or in determining the efficiencies of motors and generators (par. 209).

9. Finding Rate

To find the percent one number is of another, write the problem as a fraction, change the fraction to a decimal, and write the decimal as a percent. In this case, the *percentage* and *base* are given. The problem is to find the *rate*.

Example 1: 3 is what percent of 8? (3 is the percentage, 8 is the base, and the quotient is the rate.)

$$\frac{3}{8} = .375$$

$$.375 = 37.5\% = 37\frac{1}{2}\%$$

Therefore, 3 is $37\frac{1}{2}\%$ of 8.

Example 2: What percent of 542 is 234?

$$\frac{234}{542} = .4317 + (\text{round off})$$

$$.432 = 43.2\%$$

Therefore, 234 is 43.2% of 542.

Example 3: 125 is what percent of 50?

$$\frac{125}{50} = 2.50$$

$$2.50 = 250\%$$

Therefore, 125 is 250% of 50.

10. Finding Base Numbers

To find a number when a percent of the number is known, first find 1% of the number, and then find 100% of the number. In this case, the *percentage* of the number and the *rate* are given. The problem is to find the *base*.

Example 1: 42 is 12% of what number?

$$12\% (\text{base number}) = 42$$

$$1\% (\text{base number}) =$$

$$\frac{42}{12} = 3.50$$

$$100\% (\text{base number}) =$$

$$100 \times 3.50 = 350$$

Therefore, the base number is 350.

Example 2: 45 is 150% of what number?

$$150\% (\text{base number}) = 45$$

$$1\% (\text{base number}) = \frac{45}{150} = .3$$

$$100\% (\text{base number}) =$$

$$100 \times .3 = 30$$

Therefore, the base number is 30.

11. Expressing Accuracy of Measurements in Percent

a. *Relative error* is the accuracy of a measurement expressed in percent of the total measurement. In determining the relative error, it is first necessary to establish the *limit of error*.

b. The *limit of error* is the difference between the *true value* and the *measured value*. Assume that the reading on a scale, to the nearest tenth of an inch, is 2.2 inches. If the true value is 2.15 inches, the limit of error is the difference between 2.15 and 2.20, or .05 inch.

c. Relative error is computed by solving the ratio $\frac{\text{LIMIT OF ERROR}}{\text{MEASURED VALUE}}$, and expressing the result as a percent. In the scale reading above, the relative error = $\frac{.05}{2.2} = 2.27\%$, or 2.3%.

12. Review Problems—Percentage

a. Show each of the following in three forms—as a fraction or mixed number, as a decimal, and as a percent:

(1) $\frac{3}{5}$

(2) 50%

(3) .375

(4) $\frac{1}{4}$

(5) $62\frac{1}{2}\%$

(6) .6

(7) $\frac{3}{10}$

(8) 70%

(9) 2.25

(10) $1\frac{7}{8}$

(11) .08

(12) $\frac{3}{50}$

(13) .18

(14) $\frac{1}{4}\%$

(15) .025

(16) .05

(17) $8\frac{1}{3}\%$

(18) $37\frac{1}{2}\%$

(19) 105%

(20) 4%

b. Evaluate the following:

(1) 250% of 60

(2) 125% of 40

(3) 200% of 2

(4) 225% of 400

c. What percent of a number is—

(1) 1.5 times the number?

(2) $2\frac{3}{4}$ times the number?

(3) $\frac{3}{2}$ times the number?

(4) $5\frac{1}{2}$ times the number?

d. Find the following:

(1) $\frac{2}{5}\%$ of 410

(2) $\frac{3}{5}\%$ of 416,000

(3) $\frac{2}{5}\%$ of 85

(4) 5.2% of 85

e. Solve the following problems:

(1) Find the relative error for a limit of error of .05 inch in measuring 24.2 inches.

(2) Find the relative error for a limit of error of 2 inches in measuring 200 yards.

f. Find the number when—

(1) 12% of the number is 52

(2) 15% of the number is 375

(3) 32% of the number is 166.4

(4) 8% of the number is 16

(5) 84% of the number is 168

CHAPTER 3

RATIO AND PROPORTION

Section I. RATIO

13. Understanding Ratio

It is often desirable, for the purpose of comparison, to express one quantity in terms of another quantity of the same kind. One way to express this relationship is by means of a *ratio*. For example, if one resistor has a resistance of 800 ohms and another has a resistance of 100 ohms, the first resistor has 8 times as much resistance as the second. In other words, the ratio between the resistors is 8 to 1.

14. Expressing Ratio

Ratio can be expressed in four different ways. For example, the ratio of 12 to 3 can be expressed as follows: 12 to 3, 12:3, $12 \div 3$, or $\frac{12}{3}$. The numbers 12 and 3, which are the terms of the ratio, are called the *antecedent* and the *consequent*, respectively. The antecedent is the dividend or the numerator; the consequent is the divisor or denominator.

15. Obtaining Value of Ratio

Both terms of any ratio may be multiplied and divided by the same number without changing the value of the expression. In the ratio

$\frac{12}{3}$, for example, the 12 is divided by 3, giving the value of 4. This means that the ratio 12:3 is equal to the ratio 4:1.

Example 1: What is the ratio of 6:2?

$$\frac{6}{2} = 3, \text{ or } 3:1$$

Example 2: What is the ratio of 7:3?

$$\frac{7}{3} = 2\frac{1}{3} \text{ or } 2\frac{1}{3}:1$$

Example 3: Find the ratio of the areas (par. 26) of two squares the sides of which are 6 and 8 inches, respectively. The areas of similar figures are in the same ratios as the squares of their like dimensions.

$$8^2:6^2 = 64:36$$

$$\frac{64}{36} = 1\frac{28}{36} = 1\frac{7}{9} \text{ or } 1\frac{7}{9}:1$$

Thus, the second square (8 inches on a side) is $1\frac{7}{9}$ times as large as the first square (6 inches on a side).

Section II. PROPORTIONS

16. Understanding Proportion

A proportion is a statement of equality between two ratios. If the value of one ratio is equal to the value of another ratio, they are said to be in proportion. For example, the ratio 3:6 is equal to the ratio 4:8. Therefore, this can be written $3:6 :: 4:8$ or $3:6 = 4:8$. In any proportion, the first and last terms are called the *extremes*; the second and third terms are called the *means* (fig. 2).

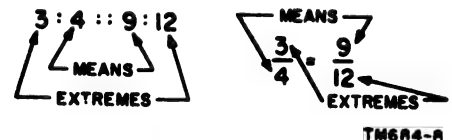


Figure 2. Terms of proportion.

17. Rules of Proportion

There are three rules of proportion that are used in determining an unknown quantity.

They also can be used to prove that the proportion is true.

a. In any proportion, *the product of the means equals the product of the extremes.*

Example 1: $3:4 :: 9:12$.

$$3 \times 12 = 36 \text{ (product of extremes)}$$

$$4 \times 9 = 36 \text{ (product of means)}$$

Example 2: $\frac{3}{4} = \frac{9}{12}$.

Note. When the proportion is expressed in fractional form, the numerator of one fraction is multiplied by the denominator of the other fraction. This process is called *cross-multiplication*.

$$3 \times 12 = 36 \text{ (product of extremes)}$$

$$4 \times 9 = 36 \text{ (product of means)}$$

b. In any proportion, *the product of the means divided by either extreme gives the other extreme.*

Example: $6:8 :: 18:24$.

$$8 \times 18 = 144 \text{ (product of means)}$$

$$144 \div 6 = 24 \text{ (one extreme)}$$

$$144 \div 24 = 6 \text{ (other extreme)}$$

c. In any proportion, *the product of the extremes divided by either mean gives the other mean.*

Example: $5:7 :: 15:21$

$$5 \times 21 = 105 \text{ (product of extremes)}$$

$$105 \div 7 = 15 \text{ (one mean)}$$

$$105 \div 15 = 7 \text{ (other mean)}$$

18. Solving for Unknown Term

As demonstrated in paragraph 49, the unknown term of a proportion can be determined if the other three terms are known.

Example 1: In the proportion $\frac{5}{10} = \frac{10}{y}$, solve

for y (the unknown quantity).

Find the product of the means:

$$10 \times 10 = 100$$

Find the product of the ex-

$$\text{tremes: } 5 \times y = 5y$$

The products of the means and extremes are equal: $5y = 100$

Divide both sides by 5:

$$\frac{5y}{5} = \frac{100}{5}$$

$$y = 20$$

$$\text{Therefore, } \frac{5}{10} = \frac{10}{20}$$

Example 2: In the proportion $6:12 :: 24:y$, solve for y .

Write the proportion in fractional form:

$$\frac{6}{12} = \frac{24}{y}$$

Cross-multiply.

$$6y = 288$$

Divide both sides by 6.

$$\frac{6y}{6} = \frac{288}{6}$$

$$y = 48$$

$$\text{Therefore, } 6:12 :: 24:48.$$

Example 3: In the proportion $\frac{z}{20} =$

$$\frac{5}{10}, \text{ solve for } z.$$

Cross-multiply.

$$10z = 100$$

Divide both sides by 10:

$$\frac{10z}{10} = \frac{100}{10}$$

$$z = 10$$

$$\text{Therefore, } \frac{10}{20} = \frac{5}{10}.$$

19. Stating Ratios for Problems in Proportion

When setting up a proportion problem, be sure to state the ratios correctly. Analyze each problem carefully to determine whether the unknown quantity will be greater or lesser than the known term of the ratio in which it occurs. Arrange the terms of the ratio as shown below, and solve for the unknown quantity as explained in paragraph 18.

$$\frac{\text{LESSER}}{\text{GREATER}} = \frac{\text{LESSER}}{\text{GREATER}}, \text{ or LESSER : GREATER :: LESSER : GREATER}$$

Example: The weight of 15 feet of iron pipe is 8 pounds.

What is the weight of 255 feet of the same pipe? Let the unknown quantity be represented by the letter y . Since ratios must express a relation between quantities of the same kind, one ratio must be between feet and feet and the other between pounds and pounds.

Study the problems; 255 feet of pipe will weigh more than 15 feet of pipe. Arrange the first ratio in the order LESSER to GREATER—15 feet:

255 feet, or $\frac{15}{255}$.

Arrange the second ratio in the same order—LESSER to GREATER—8 pounds: y

pounds, or $\frac{8}{y}$.

Write the proportion and solve.

$$15:255 = 8:y, \text{ or}$$

$$\frac{15}{255} = \frac{8}{y}$$

$$15y = 255 \times 8$$

$$15y = 2040$$

$$y = \frac{2040}{15}$$

$$y = 136 \text{ pounds}$$

20. Inverse Proportion

a. The ratio 2:3 is the inverse of the ratio 3:2. In proportion, when a second ratio is equal to the inverse of the first ratio, the elements are said to be *inversely proportional*.

b. Two numbers are inversely proportional when one increases as the other decreases. In this case, their product is always the same. In problems dealing with pulleys, the speeds of different size pulleys connected by belts are inversely proportional to their diameters. A smaller pulley rotates faster than a larger pulley.

Example 1: A pulley 30 inches in diameter is turning at a speed of 300 revolutions per minute. If this pulley is belted to a pulley 15 inches in diameter (fig. 8), determine the speed at which the smaller pulley is turning.

Let the speed of the smaller pulley be represented by y . Study the problem; the first ratio will be between inches and the second will be between revolutions per minute (rpm). Also note that the second pulley is smaller than the first and must make more revolutions than the first. Therefore, the answer will be a number larger than 300.

Arrange the ratios in the order LESSER to GREATER.

First ratio:

$$15:30, \text{ or } \frac{15}{30}$$

Second ratio:

$$300:y, \text{ or } \frac{300}{y}$$

The proportion:

$$15:30 = 300:y, \text{ or } \frac{15}{30} = \frac{300}{y}$$

Solve the proportion:

$$\frac{15}{30} = \frac{300}{y}$$

$$15y = 300 \times 30$$

$$15y = 9000$$

$$y = \frac{9000}{15}$$

$$y = 600 \text{ rpm}$$

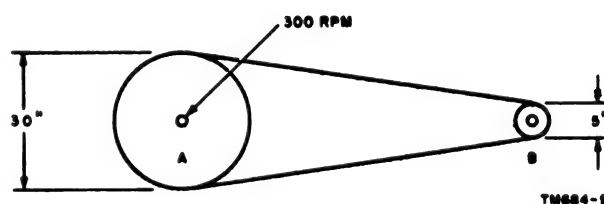


Figure 8. Pulleys and inverse ratio.

Example 2: A 24-inch pulley is fixed to a drive shaft that is turning at the rate of 400 rpm. This pulley is belted to a 6-inch pulley. Determine the speed of the smaller pulley in revolutions per minute. Driving pulley (400 rpm, 24 inches in diameter).

Driven pulley (y rpm, 6 inches in diameter).

$$\frac{6}{24} = \frac{400}{y}$$

$$6y = 400 \times 24 = 9,600$$

$$y = 1,600 \text{ rpm}$$

21. Problems Using Proportion

a. A steel plate $\frac{1}{2}$ inch thick, 12 inches wide, and 9 feet long weighs 183.6 pounds. What is the weight of a piece of steel plate of the same thickness and width if it is 16 feet 6 inches long?

b. If three men complete a certain job in 8 days, how many days would it take seven men

to complete the same job, considering that they will work at the same speed?

c. If 3 resistors cost 25 cents, find the cost of 60 resistors at the same rate?

d. If the upkeep on 62 trucks for a year is \$3,100, what would be the upkeep on 28 such trucks for 1 year at the same rate?

e. At a given temperature, the resistance of a wire increases with its length. If the resistance of a wire per 1,000 feet at 68°F is .248 ohm, what is the resistance of 1,500 feet; of 1,200 feet; of 1,850 feet; of 3,600 feet?

f. If 21-gage wire weighs 2.452 pounds per 1,000 feet, what is the weight of 1,150 feet; 1,540 feet; 1,680 feet; 349 yards?

g. The speeds of gears running together are inversely proportional to the number of teeth in the gears. A driving gear with 48 teeth meshes with a driven gear with 16 teeth. If the driving gear turns at the rate of 100 rpm, how many rpm are made by the driven gear?

h. A 36-tooth gear running at a speed of 280 rpm drives another gear with 64 teeth. What is the speed of the other gear?

CHAPTER 4

POWERS AND ROOTS

22. Powers

There are many times in mathematics when a number must be multiplied by itself a number of times, such as $4 \times 4 \times 4 \times 4 \times 4$. This is written as 4^5 and is described as 4 raised to the fifth power. A number multiplied by itself once is said to be raised to the second power (squared). Thus, 5×5 is written 5^2 . The number 2, written to the right and above the number 5, is the *exponent*; the number 5 is the *base*. The base number is a *factor* of a number written in exponential form because the product is evenly divisible by the base.

23. Roots

The root of a number is that number which, when multiplied by itself a given number of times, will equal the given number. The square root of 25 is 5, since 5×5 or 5^2 equals 25. The third root (cube root) of 216 is 6, since $6 \times 6 \times 6$ or 6^3 equals 216. The fourth root of 81 is 3, since $3 \times 3 \times 3 \times 3$ or 3^4 equals 81. Extraction of a root is generally indicated by placing, in front of the number, a *radical sign* ($\sqrt{\quad}$). A small figure is placed in the angle at the front of the sign to indicate the root to be taken. If the small figure is omitted, it is understood that the operation required is square root.

Thus,

$$\sqrt{25} = 5$$

$$\sqrt[3]{216} = 6$$

$$\sqrt[4]{81} = 3$$

24. Finding Square Root of a Number

a. Finding Square Root by Mental Calculation. In some instances, the square root can be determined mentally from a knowledge of common multiplication. For example, $\sqrt{25}$ is 5, since 5×5 or $5^2 = 25$. Similarly, $\sqrt{144}$ is 12, since 12×12 or $12^2 = 144$.

b. Finding Square Root by Arithmetical Process. In most cases, the square root of a number must be determined by a mathematical process. If the number is a perfect square, the square root will be an integral number; if the number is not a perfect square, the square root will be a continued decimal. To save time in calculation, a table of square roots of numbers from 1 to 100 is given in appendix III.

Example 1: Evaluate $\sqrt{3398.89}$.

Step 1. Starting at the decimal point mark off the digits in pairs in both directions.

$$\sqrt{33 \ 98.89}$$

Step 2. Place the decimal point for the answer directly above the decimal point that appears under the radical sign.

$$\sqrt{33 \ 98.89}$$

Step 3. Determine by inspection the largest number that can be squared without exceeding the first pair of digits—33. The answer is 5, since the square of any number larger than 5 will be greater than 33. Place the 5 above the first pair of digits.

$$\begin{array}{r} 5 \\ \sqrt{33 \ 98.89} \end{array}$$

Step 4. Square 5 to obtain 25, and place it under 33. Subtract 25 from 33 and obtain 8. Bring down the next pair of digits—98.

$$\begin{array}{r} 5 \\ \sqrt{33 \ 98.89} \\ 25 \\ \hline 898 \end{array}$$

Step 5. Double the answer, 5, to obtain a trial divisor of 10. Divide the trial divisor into all but the last

digit of the modified remainder. It will go into 89 eight times. Place the 8 above the second pair of digits, and also place the 8 to the right of the trial divisor. Thus, the true divisor is 108. Multiply 108 by 8 and obtain 864. Subtract 864 from 898 to obtain 34. Bring down the next pair of digits—89.

$$\begin{array}{r}
 5 \quad \boxed{8} \\
 \sqrt{33 \ 98.89} \\
 \underline{25} \\
 2 \times 5 = 10 \boxed{8} \\
 \boxed{8} \times 108 = \underline{864} \\
 3489
 \end{array}$$

Note. With each new successive digit in the answer:

1. Place the digit in the answer above the pair of digits involved.

2. Place the same digit to the right of the trial divisor to obtain the true divisor.

3. Multiply the digit by the true divisor. (Do not use the square boxes in actual problems.)

Step 6.

Double the answer, 58, to obtain a trial divisor of 116. Divide the trial divisor into all but the last digit of the remainder. It will go into 348 three times. Place the 3 above the third pair of digits, and also place the 3 to the right of the trial divisor. Thus, the true divisor is 1163. Multiply 1163 by 3 to obtain 3489. Subtract 3489 from 3489. There is no remainder. Therefore 3398.89 is a perfect square and its square root is 58.3.

$$\begin{array}{r}
 5 \quad 8. \quad \boxed{3} \\
 \sqrt{33 \ 98.89} \\
 \underline{25} \\
 898 \\
 \underline{864} \\
 3489 \\
 \boxed{3} \times 1163 = \underline{3489}
 \end{array}$$

Step 7.

Check the answer by squaring $58.3 - 58.3^2 = 3398.89$.

The complete calculation is shown below:

$$\begin{array}{r}
 5 \quad 8. \quad 3 \\
 \sqrt{33 \ 98.89} \\
 \underline{25} \\
 898 \\
 \underline{864} \\
 3489 \\
 \boxed{3} \times 1163 = \underline{3489}
 \end{array}$$

Example 2: Evaluate $\sqrt{786.808}$

Step 1. Starting at the decimal point, mark off the digits in pairs in both directions.

$$\sqrt{07 \ 86.80 \ 80}$$

Note. The extreme left-hand group may have only one digit. However, there must be an even number of digits to the right of the decimal point. If necessary, add a zero.

Step 2.

Place the decimal point for the answer directly above the decimal point that appears under the radical sign.

$$\sqrt{07 \ 86.80 \ 80}$$

Step 3.

Determine the largest number that can be squared without exceeding the first digit—7. The answer is 2, since the square of any whole number larger than 2 will be greater than 7. Place the 2 above the 7.

$$\begin{array}{r}
 2 \\
 \sqrt{07 \ 86.80 \ 80}
 \end{array}$$

Step 4.

Square 2 to obtain 4 and place it under 7. Subtract 4 from 7 to obtain 3. Bring down the next pair of digits—86.

$$\begin{array}{r}
 2 \\
 \sqrt{07 \ 86.80 \ 80} \\
 \underline{4} \\
 386
 \end{array}$$

Step 5.

Double the answer, 2, to obtain a trial divisor of 4. Divide the trial divisor into all but the last digit of the modified remainder. It will go into 38 nine times. Place the 9 above the second pair of digits, and also place the 9 to the right of the trial divisor. The true divisor is 49. Multiply 49 by 9 to obtain 441. However,

441 cannot be subtracted from 386, so the next lower digit must be tried. Substitute 8 for 9 in both the answer and the divisor and multiply 48 by 8 to obtain 384. Subtract 384 from 386 to obtain a remainder of 2. Bring down the next pair of digits—80.

$$\begin{array}{r}
 2 \ 9. \\
 \sqrt{07 \ 86.80 \ 80} \\
 \underline{4} \\
 386 \\
 \underline{441} \\
 2 \ 8. \\
 \sqrt{07 \ 86.80 \ 80} \\
 \underline{4} \\
 386 \\
 \underline{384} \\
 280
 \end{array}$$

$$\begin{array}{l}
 2 \times 2 = 4 \ \boxed{9} \\
 \boxed{9} \times 49 =
 \end{array}$$

$$\begin{array}{l}
 4 \ \boxed{8} \\
 \boxed{8} \times 48 =
 \end{array}$$

Step 6.

Double the answer, 28, to obtain a trial divisor of 56. Divide the trial divisor into all but the last digit of the remainder. Since it is not possible to divide 56 into 28, place a zero above the third pair of digits and bring down the next pair of digits—80.

$$\begin{array}{r}
 2 \ 8. \ \boxed{0} \\
 \sqrt{07 \ 86.80 \ 80} \\
 \underline{4} \\
 386 \\
 \underline{384} \\
 280
 \end{array}$$

$$2 \times 28 = 56$$

Step 7.

Multiply 280 by 2 to obtain a trial divisor of 560. Divide the trial divisor into all but the last digit of the remainder. It will go 5 times. Place the 5 above the fourth pair of digits, and also place the 5 to the right of the trial divisor. Thus, the true divisor is 5605. Multiply 5605 by 5 to obtain 28025. Subtract 28025 from 28080. There is a remainder of 55. Thus, the square root of 786.808 is 28.05, with a remainder of 55. A more exact answer can be obtained by

adding pairs of zeros and continuing the square root process.

$$\begin{array}{r}
 2 \ 8.0 \ 5 \\
 \sqrt{07 \ 86.80 \ 80} \\
 \underline{4} \\
 386 \\
 \underline{384} \\
 28080 \\
 \underline{28025} \\
 55
 \end{array}$$

$$\begin{array}{l}
 2 \times 280 = 560 \ \boxed{5} \\
 \boxed{5} \times 5605 =
 \end{array}$$

Check the answer by squaring 28.05 and adding the remainder (28.05² + .0055). Place the extreme right digit of the remainder under the extreme right digit of the squared number. The complete calculation is shown below:

$$\begin{array}{r}
 2 \ 8.0 \ 5 \\
 \sqrt{07 \ 86.80 \ 80} \\
 \underline{4} \\
 386 \\
 \underline{384} \\
 28080 \\
 \underline{26025} \\
 55
 \end{array}$$

$$\begin{array}{l}
 2 \times 2 = 4 \ \boxed{8} \\
 \boxed{8} \times 48 = \\
 2 \times 28 = 56 \\
 2 \times 280 = 560 \ \boxed{5} \\
 \boxed{5} \times 5605 =
 \end{array}$$

25. Review Problems—Square Root

a. Solve the following:

- (1) $\sqrt{441}$
- (2) $\sqrt{1089}$
- (3) $\sqrt{2500}$
- (4) $\sqrt{8.40}$
- (5) $\sqrt{2510.01}$
- (6) $\sqrt{4901.4001}$
- (7) $\sqrt{7482.25}$
- (8) $\sqrt{5759.2921}$

b. Solve the following to nearest thousandth.

- (1) $\sqrt{5}$
- (2) $\sqrt{7}$
- (3) $\sqrt{11}$
- (4) $\sqrt{13}$
- (5) $\sqrt{15}$
- (6) $\sqrt{17}$

c. The current (in amperes) flowing through a resistor can be determined by taking the square root of the quotient obtained by dividing the value of power supplied to the resistor (in watts) by the value of the resistance (in

ohms). Thus, if a resistance of 300 ohms is absorbing 60 watts of power, it is drawing a current of $\sqrt{\frac{60}{300}}$ amperes. This equals about .447 ampere. In the same manner, find the value of current for each of the following values of power and resistance:

<i>Power (watts)</i>	<i>Resistance (ohms)</i>	<i>Current (amperes)</i>
(1) 25	1,000	?
(2) 50	7,000	?
(3) 40	500	?
(4) 75	60	?

CHAPTER 5

ALGEBRA

Section I. INTRODUCTION

26. General

a. Algebra is an extension of arithmetic. All of the four basic operations of arithmetic—addition, subtraction, multiplication and division—apply also to algebra. Arithmetic deals only with particular numbers; algebra may also employ letters or symbols to represent numbers.

b. Algebra is often referred to as the short-hand language of mathematicians. The simplest example of the algebraic language is the formula, in which letters are used to represent words or numbers. For example, the area (A) of a rectangle can be determined by multiplying the length (l) by the width (w). Algebraically, this is stated as $A = lw$.

27. Algebraic Expressions and Terms

a. An *algebraic expression* is the representation of any quantity in algebraic signs and symbols; for example, $2x - 7$. A *numerical algebraic expression* consists entirely of numerals and signs, such as $8 - (6 \times 2)$. A *literal algebraic expression* contains only letters and symbols, such as $ax - ay$.

b. Each algebraic expression contains two or more terms, separated by one of the signs of operation ($+$, $-$, $\div$, $\times$). The expression $3x - 4xy - 2y$, for example, contains three terms: $3x$, $4xy$, and $2y$. If the terms have the same letters and exponents, such as $3a^2x$, $9a^2x$, and $12a^2x$, they are called *similar terms*. Terms that do not contain the same letters and exponents, such as $3ab^2$, $3a^2b$, and $3x^2y$, are *dissimilar terms*.

c. If an algebraic expression contains one term, such as $3abc$ or $5a^4x^2$, it is called a *monomial*; if it contains two terms, such as $x - y$,

it is called a *binomial*; and if it contains three terms, such as $5x^2 - 3xy - 2y^2$, it is called a *trinomial*. A more general rule of algebraic expressions states that any expression containing more than one term is called a *polynomial*.

28. Signs of Operation

In algebra, the conventional signs of operation ($+$, $-$, $\times$ and $\div$) retain the same meaning as in arithmetic. In algebra, however, certain other signs may be used.

a. *Multiplication* may be indicated as follows:

Arithmetic	Algebra
$a \times b$	ab
$a \times b$	$a . b$
$a \times b$	$(a)(b)$

b. *Division* may be indicated as follows:

Arithmetic	Algebra
$x \div y$	$\frac{x}{y}$
$(a + b) \div (a - b)$	$\frac{a + b}{a - b}$

c. The arithmetical signs for both *addition* and *subtraction* are retained in algebra.

Arithmetic	Algebra
$4 + 5$	$4 + 5$
$a - b$	$a - b$

29. Coefficients

Any factor of a product is known as a coefficient of the remaining factors. In the term $2\pi f$, 2 is the numerical coefficient of πf , f is the coefficient of 2π , and π is the coefficient of $2f$. However, it is common practice to speak of the numerical part of the term as the coefficient. If a term contains no numerical coefficient, the number 1 is understood. Thus, abc is 1 abc , and xyz is 1 xyz .

30. Subscripts

In expression such as $R = R_1 + R_2 + R_3$, the small numbers or letters written to the right and below the literal terms are called subscripts. Subscripts are used to designate different values of a variable quantity. They are read: R sub 1, R sub 2, etc.

Section II. POSITIVE AND NEGATIVE NUMBERS

32. Signed Numbers

Only positive numbers are used in arithmetical operations, but both *positive* and *negative* numbers may appear in algebraic expressions. The plus sign (+) is used to indicate a positive number and the minus sign (—) to indicate a negative number. If the sign is omitted, the number is understood to be positive. Positive and negative numbers are called *signed numbers*.

33. Need for Negative Numbers

The need for negative numbers may be seen from the succession of subtraction below:

$$\begin{array}{cccccccccc} 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ -0 & -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 \\ \hline 6 & 5 & 4 & 3 & 2 & 1 & 0 & -1 & -2 & -3 \end{array}$$

When the subtrahend is greater than the minuend, the difference becomes less than zero and the negative sign is placed before the difference. Thus, a negative number may be defined as a number less than zero.

34. Application of Positive and Negative Numbers

In technical work, many scales are calibrated above and below (or to the right and left of)

31. The Radical Sign

The radical sign ($\sqrt{\quad}$) has the same meaning in algebra as in arithmetic (ch. 5). Thus, the expression $z = 2\sqrt{R^2 + x^2}$ states that z is equal to 2 times the square root of $R^2 + x^2$.

a center point designated 0 (zero). For example, the degrees of temperature indicated on a thermometer scale are measurements of distance taken on a scale in opposite directions from some point chosen to represent a reference or zero point. Temperature is always so many degrees above or below zero. In mathematics, it is convenient to indicate that a temperature is so many degrees above or below zero by prefixing the reading with a positive or negative sign. Thus, 45° above zero is $+45^\circ$ and 15° below zero is -15° . Similarly, in electronic and electrical measuring instruments, scales are often calibrated to read positive numbers on one side of a zero and negative numbers on the other.

35. Graphical Representation of Positive and Negative Numbers

a. *Principle.* Positive and negative numbers may be represented graphically as shown in figure 4. The zero is the reference point. This graph can be used to illustrate both addition and subtraction.

b. *Addition.* To add numbers graphically, start at the zero reference point and mark off the first number, going to the right if the number is positive, or to the left if the number is

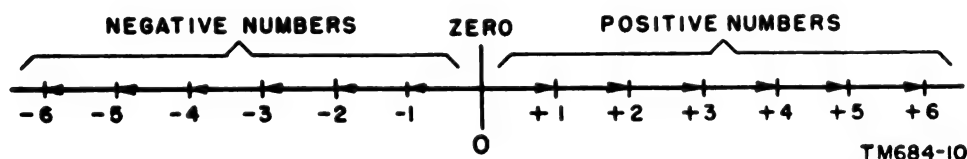


Figure 4. Graphical representation of positive and negative numbers.

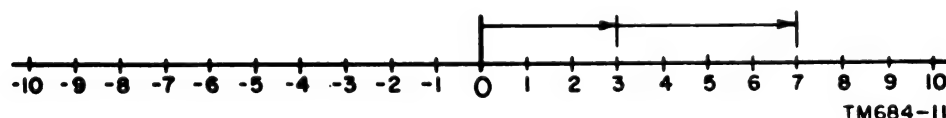


Figure 5. Graphical representation of addition of positive numbers.

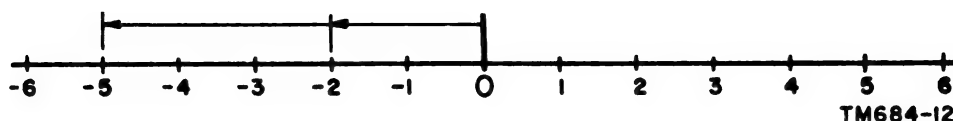


Figure 6. Graphical representation of addition of negative numbers (-1 and -5).

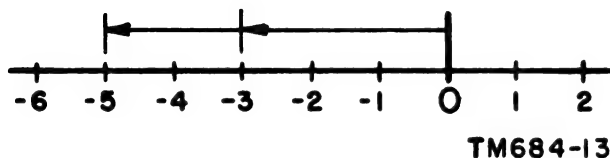


Figure 7. Graphical representation of addition of negative numbers (-3 and -2).

negative. From this new point, mark off the second number, again going to the right if the number is positive, or to the left if it is negative. The number of units between zero and the final point is the sum of the two numbers. This procedure can be continued for more than two numbers. Figure 5 shows graphical addition of positive numbers; figures 6 and 7 show graphical addition of negative numbers; and figure 8 shows the addition of a combination of a positive and a negative number. Figures 6 and 7 show that the order in which the negative numbers are taken does not affect the answer.

c. *Subtraction.* To subtract numbers graphically, change the sign of the subtrahend (number to be subtracted) and proceed as for addition. Figure 9 shows the subtraction of $+3$ from $+5$ to obtain the difference of $+2$.

36. Absolute Value of a Number

The numerical value of a number, without regard to its sign, is called the *absolute value*

of the number. Thus, the absolute value of -3 or $+3$ is 3. This is written $|3|$.

37. Addition of Positive and Negative Numbers

a. *Positive Numbers.* To add two or more positive numbers, find the sum of their absolute values and prefix the sum with a plus sign. When there is no possibility of misunderstanding, the plus sign is usually omitted.

Example: Add $+4$, $+5$, and $+6$
 $+4 + (+5) + (+6) = +15$ or
 15

b. *Negative Numbers.* To add two or more negative numbers, find the sum of their absolute values and prefix the sum with a minus sign.

Example: Add -4 , -5 , and -6
 $-4 + (-5) + (-6) = -15$

c. *Positive and Negative Numbers.* To add a positive and a negative number, find the difference between their absolute values and prefix the sum with the sign of the number that has the greater absolute value. This is called *algebraic addition*. When three or more positive and negative numbers are to be added, first find the sum of all positive numbers, and then the sum of all negative numbers. Add these sums algebraically as above.

Example 1: Add $+6$ and -9 .
 $+6 + (-9) = -3$

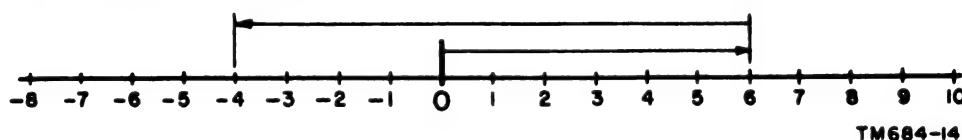


Figure 8. Graphical representation of addition of positive and negative numbers.

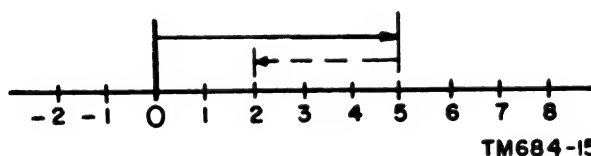


Figure 9. Graphical representation of subtraction of positive numbers.

Example 2: Add +5, —8, +12, and —6.

$$+5 + (+12) = +17$$

$$-8 + (-6) = -14$$

$$(+17) + (-14) = +3$$

38. Subtraction of Positive and Negative Numbers

To subtract positive and negative numbers, change the sign of the subtrahend and proceed as in addition (par. 37).

a. Positive Numbers.

Example 1: Subtract +2 from +5.

$$+5 - (+2) = +5 - 2 = +3 \text{ or } 3$$

Example 2: Subtract $+5a^2$ from $+6a^2$.

$$+6a^2 - (+5a^2) = +6a^2 - 5a^2 = +1a^2 = a^2$$

b. Negative Numbers.

Example 1: Subtract —3 from —5.

$$-5 - (-3) = -5 + 3 = -2$$

Example 2: Subtract $-4a$ from $-2a$.

$$-2a - (-4a) = -2a + 4a = +2a \text{ or } 2a$$

c. Positive and Negative Numbers.

Example 1: Subtract —2 from +5.

$$+5 - (-2) = +5 + 2 = +7 \text{ or } 7.$$

Example 2: Subtract $-3x^2$ from $+5x^2$.

$$+5x^2 - (-3x^2) = +5x^2 + 3x^2 = +8x^2 \text{ or } 8x^2$$

39. Multiplication of Positive and Negative Numbers

a. Numbers Having Like Signs. If the two numbers to be multiplied have the same signs, the product is positive.

Example 1: Multiply +5 by +3.

$$(+5)(+3) = +15 \text{ or } 15$$

Example 2: Multiply —5 by —3.

$$(-5)(-3) = +15 \text{ or } 15$$

b. Numbers Having Unlike Signs. If the two numbers to be multiplied have unlike signs, the product is negative.

Example 1: Multiply —5 by +3.

$$(-5)(+3) = -15$$

Example 2: Multiply +5 by —3.

$$(+5)(-3) = -15$$

c. Several Positive and Negative Numbers. To multiply several positive and negative numbers, multiply the numbers in groups of two in the order in which they appear.

Example 1:

$$\begin{aligned} &\text{Multiply } (-5)(+3)(+7)(-2)(-4). \\ &\quad (-5)(+3) \quad (+7)(-2) \quad (-4) \\ &= (-15) \quad (-14) \quad (-4) \\ &= \quad (+210) \quad (-4) \\ &= \quad \quad \quad -840 \end{aligned}$$

Example 2:

$$\begin{aligned} &\text{Multiply } (+7)(+2)(-5)(-3)(-1)(-4). \\ &= (+7)(+2) \quad (-5)(-3) \quad (-1)(-4) \\ &= (+14) \quad (+15) \quad (+4) \\ &= \quad (+210) \quad (+4) \\ &= \quad \quad \quad 840 \end{aligned}$$

40. Division of Positive and Negative Numbers

a. Numbers Having Like Signs. The quotient of two numbers that have the same signs is positive.

Example 1: Divide —15 by —5.

$$-15 \div -5 = +3 \text{ or } 3$$

Example 2: Divide +24 by +6.

$$+24 \div +6 = +4 \text{ or } 4$$

b. Numbers Having Unlike Signs. The quotient of two numbers that have opposite signs is negative.

Example 1: Divide 35 by —7.

$$+35 \div -7 = -5$$

Example 2: Divide —8,988 by 28.

$$-8988 \div 28 = -321$$

41. Order of Signs

When only addition and subtraction signs appear in a series of terms, addition and subtraction procedures may be performed in any order. However, when multiplication and division signs appear in the same series with addition and subtraction signs, the multiplication and division must be performed first, and then the addition and subtraction.

Example 1: Evaluate $15 + 5 - 3 + 4 - 8$.

Step 1. Add the + terms:

$$15 + 5 + 4 = 24$$

Step 2. Add the — terms:

$$(-3) + (-8) = -11$$

Step 3. Add the + terms and — terms algebraically:

$$24 - 11 = 13.$$

Example 2: Evaluate $9 \times 4 + 6 - 3 + 5 \times 2$.

Step 1. Perform the multiplication first:

$$(9 \times 4) + 6 - 3 + (5 \times 2) = (36) + 6 - 3 + (10)$$

Step 2. Add the + terms:

$$36 + 6 + 10 = 52$$

Step 3. Add the + terms and the — terms algebraically:

$$52 - 3 = 49$$

Example 3: Evaluate $81 \div 9 - 3 + 6 - 15 + 4 \times 5$.

Step 1. Perform the division:

$$(81 \div 9) - 3 + 6 - 15 + (4 \times 5) = (9) - 3 + 6 - 15 + (4 \times 5)$$

Step 2. Perform the multiplication:

$$9 - 3 + 6 - 15 + (4 \times 5) = 9 - 3 + 6 - 15 + (20)$$

Step 3. Add the + terms:

$$9 + 6 + 20 = 35$$

Step 4. Add the — terms:

$$(-3) + (-15) = -18$$

Step 5. Add the $\div$ terms and the — terms algebraically:

$$35 - 18 = 17.$$

42. Review Problems—Positive and Negative Numbers

a. Add the following:

(1) 23 and -6

(2) 21 and 37

(3) -54 and 33

(4) -43° and -96°

(5) 682 volts and -934 volts

b. Subtract the following:

(1) -104 amperes from 147 amperes

(2) -37 volts from -45 volts

(3) $.64cy$ from $.0025cy$

(4) $21.36ax^2$ from $-10.63ax^2$

(5) $-.986x^2y$ from $.824x^2y$

c. Find the product of the following:

(1) -6.4 and 2.8

(2) 3, -6 , and 4

(3) $-\frac{2}{3}$, $-\frac{6}{7}$, and $-\frac{2}{5}$

(4) 3.01, $-.02$, and -1.26

(5) $-.0025$, 150, $-.10$, and $.075$

(6) -2 , 5, 3, -1 , and 4

d. Divide:

(1) 36 by 4

(2) $-\frac{5}{7}$ by $\frac{3}{4}$

(3) -5.6 by $-.008$

(4) -750 by -3

(5) $\frac{1}{3}$ ampere by $\frac{1}{2}$ ampere

(6) $-.3750$ by 150

e. Evaluate the following:

(1) $2 + 3 - 9$

(2) $3 + 4 + 2 \times 5 - 8$

(3) $2 - 3 \times 9$

(4) $3 \times 4 + 2 \times 5 - 3$

(5) $5 + 3 \times 7 - 2 \times 11 + 7$

(6) $28 \div 14 - 8 + 16 + 3 \times 2$

(7) $46 - 18 + 3 \times 4 - 8 + 12$

(8) $5 - 3 + 6 \times 4 + 40$

(9) $8 - 16 + 4 \times 3 - 10 \times 5$

(10) $15 \div 5 - 3 + 2 \times 10 - 2$

Section III. FUNDAMENTAL OPERATIONS

43. Addition and Subtraction of Algebraic Expressions

a. General. Only similar algebraic terms—those that are exactly alike in all respects other than numerical coefficients—may be added or subtracted. For example, the sum of $3x^2y$ and $5x^2y$ is $8x^2y$. Dissimilar terms cannot be added or subtracted directly, but the processes of addition or subtraction can be indicated by the use of plus or minus signs. For example, the sum of $4x^2y$ and $2xy^2$ is $4x^2y + 2xy^2$.

b. Procedure. To add or subtract algebraic expressions, arrange the terms so that like terms are in the same vertical column, and preferably in descending order of powers. Add or subtract the terms according to the rules of signed numbers (pars. 37 and 38).

Example 1: Add $x^3 - 3x^2 + 1$, $x^3 + x - 3$, and $x^2 + x + 1$.

$$\begin{array}{r} x^3 - 3x^2 \qquad \qquad + 1 \\ x^3 \qquad \qquad + x - 3 \\ \qquad \qquad x^2 + x + 1 \\ \hline 2x^3 - 2x^2 + 2x - 1 \end{array}$$

Example 2: Subtract $x^3 + 3x^2 + x - 1$ from $x^4 + x^3 - x + 2$.

$$\begin{array}{r} x^4 + x^3 \qquad \qquad - x + 2 \\ -(x^3 - 3x^2 - x + 1) \\ \hline \text{Remove parentheses and change} \\ \text{signs.} \\ x^4 + x^3 \qquad \qquad - x + 2 \\ -x^3 - 3x^2 - x + 1 \\ \hline x^4 \qquad - 3x^2 - 2x + 3 \end{array}$$

44. Multiplication and Division of Monomials

a. Multiplication. In multiplying monomials, multiply the numerical coefficients and write this result as the coefficient of the product. After the coefficient, write each literal factor with an exponent equal to the sum of all the exponents of that letter in the original factors.

For example, $3a^n \cdot 2a^m = 6a^{n+m}$.

Example 1: Multiply x^2 by x^3 .

$$x^2 \cdot x^3 = x^{2+3} = x^5$$

Example 2: Multiply x , x^3 , and x^{10} .

$$x^1 \cdot x^3 \cdot x^{10} = x^{1+3+10} = x^{14}$$

Example 3: Multiply x^3y^6 by $3xy^2$.

Step 1. Multiply the coefficients:
 $1 \cdot 3 = 3$

Step 2. Multiply the two factors having the base x :

$$x^3 \cdot x = x^{3+1} = x^4$$

Step 3. Multiply the two factors having the base y :

$$y^6 \cdot y^2 = y^{6+2} = y^8$$

Step 4. The product is:

$$x^4y^8 \cdot 3xy^2 = 3x^4y^8$$

Example 4: Multiply x^2y^4z and wx^3yz^5 .

$$x^2y^4z \cdot wx^3yz^5 = wx^{2+3}y^{4+1}z^{1+5}$$

$$x^{2+3} = x^5$$

$$y^{4+1} = y^5$$

$$z^{1+5} = z^6$$

$$\text{Therefore, } x^2y^4z \cdot wx^3yz^5 = wx^5y^5z^6$$

b. Division. In dividing a monomial by a monomial, divide the numerical coefficient of the dividend by the coefficient of the divisor and write the result as the coefficient of the quotient. After the coefficient, write each literal factor with an exponent equal to its exponent in the dividend minus its exponent in the divisor. Thus, to divide $6a^n$ by $3a^m$ (n greater than m), $\frac{6a^n}{3a^m} = 2a^{n-m}$.

Example 1: Divide x^3 by x^2 .

$$\frac{x^3}{x^2} = x^{3-2} = x^1 = x$$

Example 2: Divide $5x^4yz^3$ by $6x^3z^2$.

$$\begin{aligned} \frac{5x^4yz^3}{6x^3z^2} &= \frac{5}{6}x^{4-3}yz^{3-2} \\ &= \frac{5}{6}x^1yz \text{ or } \frac{5x^1yz}{6} \end{aligned}$$

c. Removal of Parentheses and Brackets.

- (1) In multiplying a quantity in parentheses by a given factor, multiply each term inside the parentheses by that factor and drop the parentheses. If the factor is a negative quantity, the sign of every term inside the parentheses is changed. For example, $-5(a - b + c) = -5a + 5b - 5c$.
- (2) When an algebraic expression, such as $5x - 4[x - 2(x - 3)]$, has more than one grouping symbol (parentheses and brackets), remove the inside grouping symbol first and then successively remove the outer grouping symbols.

Example 1: Simplify $5x - 4 [x - 2(x - 3)]$.

$$\begin{aligned} 5x - 4 [x - 2(x - 3)] &= 5x - 4 [x - 2x + 6] \\ &= 5x - 4x + 8x - 24 \\ &= 9x - 24 \\ &= 3(3x - 8) \end{aligned}$$

Example 2: Simplify $4a - \{6a - 2b + 2 [2a - b + 42] - (c + 2b)\}$.

$$\begin{aligned} 4a - \{6a - 2b + 2 [2a - b + 42] - (c + 2b)\} \\ &= 4a - \{6a - 2b + 4a - 2b + 84 - c - 2b\} \\ &= 4a - 6a + 2b - 4a + 2b - 84 + c + 2b \\ &= -6a + 6b + c - 84 \end{aligned}$$

Example 3: Simplify $-(-1 [- (x - y - z) + 29] - 39 + 2y - z)$.

$$\begin{aligned} &-(-1 [- (x - y - z) + 29] - 39 + 2y - z) \\ &= -(-1 [-x + y + z + 29] - 39 + 2y - z) \\ &= -(+x - y - z - 29 - 39 + 2y - z) \\ &= -x + y + z + 29 + 39 - 2y + z \\ &= -x - y + 2z + 68 \end{aligned}$$

45. Raising Algebraic Functions to Powers

To raise an algebraic function to a power, multiply the exponents. Thus, $(a^m)^n = a^{mn}$.

Example 1: Simplify $(5^3)^4$.

$$(5^3)^4 = 5^{3 \cdot 4} = 5^{12}$$

Example 2: Simplify $(2ab)^3$.

$$\begin{aligned} (2ab)^3 &= 2ab \cdot 2ab \cdot 2ab = 8a^3 b^3 \\ \text{or } 2^{1 \cdot 3} a^{1 \cdot 3} b^{1 \cdot 3} &= 8a^3 b^3 \end{aligned}$$

Example 3: Simplify $(ax^2)^3$.

$$(ax^2)^3 = a^{1 \cdot 3} x^{2 \cdot 3} = a^3 x^6$$

Example 4: Simplify $[(x^3)^4]^5$.

$$[(x^3)^4]^5 = [x^{3 \cdot 4}]^5 = [x^{12}]^5 = x^{12 \cdot 5} = x^{60}$$

Example 5: Simplify $\left(\frac{2}{x^2}\right)^5$

$$\left(\frac{2}{x^2}\right)^5 = \frac{2^{1 \cdot 5}}{x^{2 \cdot 5}} = \frac{2^5}{x^{10}} = \frac{32}{x^{10}}$$

46. Negative Exponents

The rule for dividing monomials (par. 44b) also holds when the exponents of the denominator is greater than the exponent of the numerator. For example, $a^3 \div a^5 = a^{3-5} = a^{-2}$; however, a quantity such as a^{-2} may be written as $\frac{1}{a^2}$.

Example: Multiply x^2 , x^{-1} , and $\frac{1}{x^{-3}}$.

Step 1. Write down the factors of the multiplication:

$$x^2 \cdot x^{-1} \cdot \frac{1}{x^{-3}}$$

Step 2. Place all factors in the numerator:

$$x^2 \cdot x^{-1} \cdot x^3$$

Step 3. Multiply the factors (add their exponents):

$$x^{2-1+3} = x^4$$

47. Zero Exponents

The zero power of any quantity is equal to 1. For example $x^2 \cdot x^{-2} = x^0$ when the exponents are added. However, x^{-2} can also be written $\frac{1}{x^2}$; in this case, $x^2 \cdot x^{-2} = \frac{x^2}{x^2} = 1$.

Therefore, $x^0 = 1$. Any number (except zero) raised to the zero power is equal to 1.

Example: Solve $\frac{x^2y^2}{z} \cdot \frac{z^4}{xy} + \frac{x^2y^2}{z^3}$.

$$\begin{aligned}\frac{x^2y^2}{z} \cdot \frac{z^4}{xy} + \frac{x^2y^2}{z^3} &= \frac{x^2y^2z^4}{xyz} + \frac{x^2y^2}{z^3} = \frac{x^2y^2z^4}{xyz} \cdot \frac{z^3}{z^3} \\ &= \frac{x^2y^2z^7}{x^2y^3z} = x^{2-2}y^{2-3}z^{7-1} \\ &= x^{-1}y^{-1}z^6 = x^{-1} \cdot 1 \cdot z^6 = \frac{z^6}{x}\end{aligned}$$

48. Multiplication of Polynomials

a. By a Monomial. To multiply a polynomial by a monomial, multiply each term in the polynomial separately by the monomial and add the products. Observe the rules for the multiplication of signed numbers (par. 39) and exponents (par. 44a).

Example 1: Multiply $3a + 2ab + 5c$ by $2b$.

$$\begin{array}{r}3a + 2ab + 5c \\ 2b \\ \hline 6ab + 4ab^2 + 10bc\end{array}$$

Example 2: Multiply $ad - ae + af$ by $3a^2$.

$$\begin{array}{r}ad - ae + af \\ 3a^2 \\ \hline 3a^2d - 3a^2e + 3a^2f\end{array}$$

Example 3: Multiply $3x^2y^2 - 2xy^3 + 5x^4y$

$$\begin{array}{r}4x^3y \\ 3x^2y^2 - 2xy^3 + 5x^4y \\ 4x^3y \\ \hline 12x^5y^3 - 8x^4y^4 + 20x^7y^2\end{array}$$

b. By a Polynomial. To multiply a polynomial by another polynomial, multiply each term of one polynomial by each term of the other and add the products.

Example 1: Multiply $(a + b)$ by $(a + b)$.

$$\begin{array}{r}a + b \\ a + b \\ \hline a^2 + ab \\ ab + b^2 \\ \hline a^2 + 2ab + b^2\end{array}$$

Example 2: Multiply $2x + 3y$ by $2x + 3z$.

$$\begin{array}{r}2x + 3y \\ 2x + 3z \\ \hline 4x^2 + 6xy \\ + 6xz + 9yz \\ \hline 4x^2 + 6xy + 6xz + 9yz\end{array}$$

Example 3: Multiply $5x^2 - 6xy + 3y^2$ by $x + y$.

$$\begin{array}{r}5x^2 - 6xy + 3y^2 \\ x + y \\ \hline 5x^3 - 6x^2y + 3xy^2 \\ + 5x^2y - 6xy^2 + 3y^3 \\ \hline 5x^3 - x^2y - 3xy^2 + 3y^3\end{array}$$

49. Division of Polynomials

a. By a Monomial. To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example 1: Divide $3a^2 + 4ab + 5ac$ by a .

$$\frac{3a^2 + 4ab + 5ac}{a} = 3a + 4b + 5c$$

Example 2: Divide $7x^2 + 14xy - 21ax^2$ by $7x$.

$$\frac{7x^2 + 14xy - 21ax^2}{7x} = x + 2y - 3ax$$

Example 3: Divide $4r(s + t) - r^2(s + t)^2 + qr^2(s + t)^3$ by $r^2(s + t)$.

$$\begin{aligned}\frac{4r(s + t) - r^2(s + t)^2 + qr^2(s + t)^3}{r^2(s + t)} \\ = \frac{4r(s + t)}{r^2(s + t)} - \frac{r^2(s + t)^2}{r^2(s + t)} + \frac{qr^2(s + t)^3}{r^2(s + t)} \\ = \frac{4}{r} - r(s + t) + q(s + t)^2\end{aligned}$$

b. By a Polynomial. To divide a polynomial by a polynomial, just arrange the dividend and the divisor according to descending powers of one variable, starting with the highest powers at the left. Then proceed as shown in the examples below. If there is a remainder, write it as the numerator of a fraction the denominator of which is the divisor.

Example 1: Divide $ab + ac + db + dc$ by $a + d$.

Step 1. Divide the first term of the divisor, a , into the first term of the dividend, ab . The quantity a is contained in the first term, ab , b times. Write b as the first term of the quotient.

Step 2. Multiply both terms of the divisor by b :

Step 3. Subtract the result from the original dividend:

Step 4. Divide the first term of the divisor into the first term of the

remainder. It is contained in the first term, ac , c times. Write c as the second term of the quotient.

$$\begin{array}{r} b + c \\ a + d \overline{) ab + ac + db + dc} \\ \underline{ab \quad \quad + db} \\ ac \quad \quad + dc \end{array}$$

Step 5. Multiply both terms of the divisor by c and subtract. There is no remainder:

$$\begin{array}{r} b + c \\ a + d \overline{) ab + ac + db + dc} \\ \underline{ab \quad \quad + db} \\ ac \quad \quad + dc \\ \underline{ac \quad \quad + dc} \end{array}$$

Step 6. Therefore, $\frac{ab + ac + db + dc}{a + d} = b + c$.

Example 2: Divide $x^2 + 2xy + y^2$ by $x + y$.

$$\frac{x^2 + 2xy + y^2}{x + y} =$$

$$\begin{array}{r} x + y \\ x + y \overline{) x^2 + 2xy + y^2} \\ \underline{x^2 + xy} \\ xy + y^2 \\ \underline{xy + y^2} \end{array}$$

Therefore, $\frac{x^2 + 2xy + y^2}{x + y} = x + y$.

Example 3: Divide $6a^2 - ab - 27ac - 15b^2 + 7bc + 30c^2$ by $3a - 5b - 6c$.

$$\begin{array}{r} 2a + 3b - 5c \\ 3a - 5b - 6c \overline{) 6a^2 - ab - 27ac - 15b^2 + 7bc + 30c^2} \\ \underline{6a^2 - 10ab - 12ac} \\ 9ab - 15ac - 15b^2 + 7bc + 30c^2 \\ \underline{9ab - 15b^2 - 18bc} \\ -15ac + 25bc + 30c^2 \\ \underline{-15ac + 25bc + 30c^2} \end{array}$$

50. Review Problems—Fundamental Operations

a. Add the following algebraic expressions:

(1) $2a^4 + 3a^2b^2 + 5b^4$, $a^4 - 5a^2b^2 - 2b^4$, and $3a^4 - 2a^2b^2 + b^4$.

(2) $3E - 2R - 15Z$, $6R + 24Z$, and $-2E - R + 11Z$.

(3) $10w - 4x + 3y + 6z$, $2x - 5w + y$, $3z - 2x - y$, and $6y - 4w - z + 5x$.

b. Subtract the following algebraic expressions:

- (1) $-7ax - 2by + cz$ from $12ax + 15by - 8cz$.
- (2) $10w - 3y - 4z + 6x$ from $3x + 5y - 2z - 15w$.
- (3) $8a^2 + 10ab - 4b^2$ from $12a^2 - 24ab + 2b^2$.

c. Simplify:

- (1) $7a^0$
- (2) $(5x + 9)^0$
- (3) $(3x^2 + 7x + 1)^0$

d. Perform the indicated operations:

- (1) $f^3 \cdot f^4$
- (2) $y^a \cdot y^b$
- (3) $v^{x+1} \cdot v^{x-1}$
- (4) $\frac{r^{10}}{r^5}$
- (5) $(R^3)^m$
- (6) $\frac{r^{m+5}}{r^4}$

e. Express with positive exponents:

- (1) $4x^{-4}$
- (2) $r^{-3}x^{-4}$
- (3) $(6a)^{-2b}$
- (4) $I^{-2}R^{-1}$
- (5) $2^{-3}a^2b^{-3}$
- (6) $\frac{3EI^{-2}R^{-1}}{4}$

f. Perform the indicated operations:

- (1) $(5ab)(2a^2 - 3ab + 7b^2)$
- (2) $4a(a^2 + 3a + 1)$
- (3) $(i^2 + 3i + 9)(i - 3)$
- (4) $(2x^2 + 3xy - y^2)(x^2 + xy + y^2)$
- (5) $(3x^2 - 2xy - 5y^2)(3x^2 + 2xy - 5y^2)$
- (6) $[(x - 1)a - (x - 1)c] \div [(x - 1)ac]$
- (7) $(3rL - rR^2) \div rR$
- (8) $(5a^4b - 10a^3b^2 + 15a^2b^3) \div 5a^4b$
- (9) $(1 + 2z^4 + 4z^2 - z^3 + 7z) \div (3 + z^2 - z)$
- (10) $(100b^3 - 13b^2 - 3b) \div (3 + 25b)$

Section IV. FACTORING

51. Understanding Factoring

Factoring is the breaking up of an expression into the *factors* or *individual parts* of which it is composed. In other words, to factor an algebraic expression means to find two or more expressions which, when multiplied together, will result in the original expression. For example, since $3 \cdot 5 = 15$, 3 and 5 are the factors of 15; since $4 \cdot a \cdot b = 4ab$, 4, a , and b , are the factors of $4ab$; since $a(x + y) = ax + ay$, a and $(x + y)$ are the factors of $ax + ay$.

52. Factors of Positive Integers

It is often difficult to determine at a glance the factors of which a number is composed. For example, consider the numerical expression 36. There are many different combinations of numbers that would result in an answer of 36; for example, the desired factors for 36 in a certain problem might $36 \cdot 1$, $18 \cdot 2$, $12 \cdot 3$, $9 \cdot 4$, $6 \cdot 6$, $2 \cdot 2 \cdot 9$, $4 \cdot 3 \cdot 3$, $2 \cdot 3 \cdot 6$, and so on.

53. Factors of a Monomial

Because the factors of a monomial are evident, usually a monomial is not separated into its prime factors. The factors of a^4b^2c are $a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot c$, and the factors of $15a^2b^3$ are $3 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot b$.

54. Square Root of a Monomial

The square root of an algebraic expression is one of its two equal factors. Thus, the square root of 49 is 7, the square root of 81 is 9, the square root of a^2 is a , and the square root of x^2y^2 is xy . As discussed in paragraph 31, the radical sign is used to indicate the square root of a number. Actually, every number has two square roots, one positive and one negative. If no sign precedes the radical, the positive or *principal root* is understood. For example, $\sqrt{9} = +3$. If a negative sign *precedes the radical*, however, the negative root is intended. Thus, $-\sqrt{9} = -3$. When dealing with literal terms, the values of the various factors often

are unknown. Therefore, when extracting the square root of a monomial, extract the square root of the numerical coefficient, divide the exponents of the literal terms by 2, and prefix the square root with the plus or minus ($\pm$) sign, which denotes that either the positive or negative root may be the correct one.

Example 1: $\sqrt{x^{16}y^4} = \pm x^8y^2$.

Example 2: $\sqrt{49a^4b^2} = \pm 7a^2b$.

55. Cube Root of a Monomial

The cube root of a monomial is one of its three equal factors. The index 3 in the angle of the radical sign ($\sqrt[3]{}$) indicates cube root (par. 31). To extract the cube root of a monomial, extract the cube root of the numerical coefficient, divide the exponents of the literal terms by 3, and prefix the cube with the same sign as that of the monomial.

Example 1: $\sqrt[3]{a^6y^3} = a^2y$.

Example 2: $\sqrt[3]{27x^{12}y^6z^9} = 3x^4y^2z^3$.

Example 3: $\sqrt[3]{-64r^{12}s^3} = -4r^4s$.

56. Factors of a Polynomial

a. *Common Monomial Factor.* In an algebraic expression, the type of factor which can be recognized most easily is the monomial factor (single letter or number) which is common to each term in the expression. For instance, in the expression $xa + xb + xc$, the x is a factor common to each of the terms. Thus, the expression $xa + xb + xc$ can be written $x(a + b + c)$. This relationship is shown pictorially in figure 10. Since the area of a rectangle is equal to its base multiplied by its altitude (par. 136b), the area of the uppermost rectangle in figure 10 is x times a , or xa . The areas of the center and lower rectangles are xb and xc , respectively. The area of the large rectangle formed by the three small rectangles is equal to its base x times its altitude $(a + b + c)$, or $x(a + b + c)$. Since the area of the large

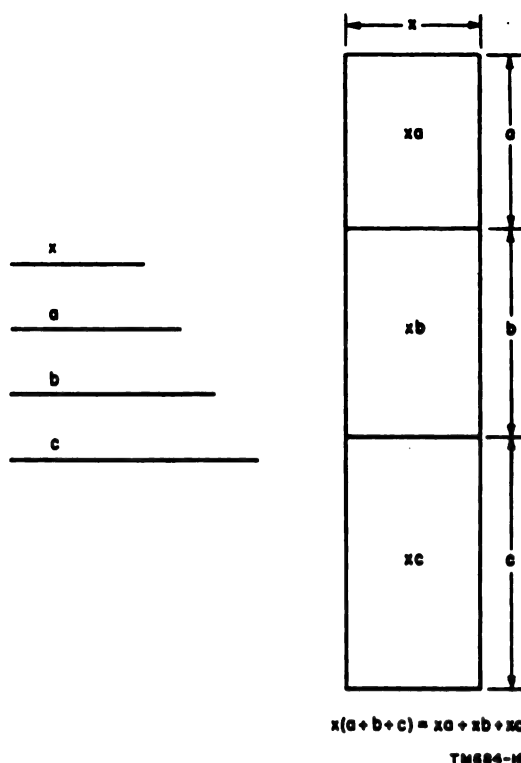


Figure 10. Common monomial factors.

rectangle is equal to the sum of the areas of the three smaller rectangles, then $x(a + b + c)$ is equal to $xa + xb + xc$. This shows that the factor x can be removed from $xa + xb + xc$ and the expression written $x(a + b + c)$. Accuracy of factoring can be checked by multiplying the two factors together—the product should be the original expression. Thus, $x(a + b + c) = xa + xb + xc$. To factor a polynomial the terms of which have a common monomial factor, determine the largest factor common to all of the terms, divide the polynomial by this factor, and write the quotient in parentheses preceded by the monomial factor. The first factor contains all that is common to all of the terms; it may consist of more than one literal number and may be to a power higher than the first.

Example 1: Factor $x^3 - 7x^2 + 4x$.

$$x^3 - 7x^2 + 4x = x(x^2 - 7x + 4)$$

Example 2: Factor $abx + aby - abz$.

$$abx + aby - abz = ab(x + y - z)$$

Example 3: Factor $2az^2 - 4bz^2 + 6cz^2$.

$$2az^2 - 4bz^2 + 6cz^2 = 2z^2(a - 2b + 3c)$$

b. Binomial Factors. Sometimes binomial factors are not immediately apparent, and an algebraic term may appear to have no common factors. For example, the expression $am + bm + an + bn$ may seem to have no factors in common. However, the first pair, $am + bm$, has a common factor, m , and the second pair, $an + bn$, has a common factor, n . Factoring out the common factors, the expression becomes $m(a + b) + n(a + b)$. Since there are two terms containing a common factor $(a + b)$, this factor can be removed to make the expression $(a + b)(m + n)$. Thus, the factors are $(a + b)$ and $(m + n)$. This relationship is shown pictorially in figure 11. Starting with

the upper left-hand rectangle and going clockwise, the areas of the four rectangles are an , am , bm , and bn . The area of the large rectangle formed by the four smaller rectangles is its base $(m + n)$ times its altitude $(a + b)$, or $(m + n)(a + b)$. Since the area of the large rectangle is equal to the sum of the areas of the four smaller rectangles, then $(m + n)(a + b)$ is equal to $an + am + bm + bn$. This shows that the expression $am + bm + an + bn$ can be factored into $(m + n)$ and $(a + b)$. To check the factoring, multiply $(a + b)$ by $(m + n)$; the product is $am + an + bm + bn$. Since the addition of terms can be expressed in any order, the factoring is correct.

Example 1: Factor $py - pz - qy + qz$.

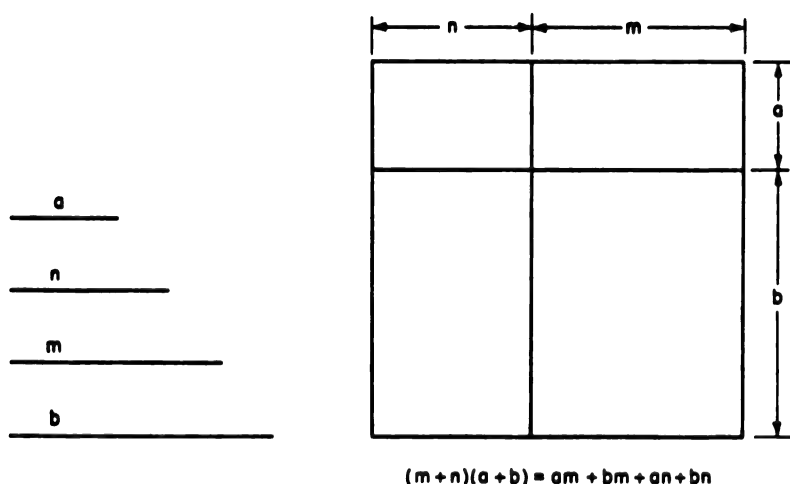
$$\begin{aligned} py - pz - qy + qz &= p(y - z) - q(y - z) \\ &= (p - q)(y - z) \end{aligned}$$

Example 2: Factor $4xa - 8zb - 6ya - 4xb + 8za + 6yb$.

$$\begin{aligned} 4xa - 8zb - 6ya - 4xb + 8za + 6yb &= 4xa - 6ya + 8za - 4xb + 6yb - 8zb \\ &= 2a(2x - 3y + 4z) - 2b(2x - 3y + 4z) \\ &= (2a - 2b)(2x - 3y + 4z) \\ &= 2(a - b)(2x - 3y + 4z) \end{aligned}$$

Example 3: Factor $da + db - dc - ea - eb + ec + fa + fb - fc$.

$$\begin{aligned} da + db - dc - ea - eb + ec + fa + fb - fc &= d(a + b - c) - e(a + b - c) + f(a + b - c) \\ &= (d - e + f)(a + b - c) \end{aligned}$$



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Figure 11. Binomial factors.

57. Factors of the Square of a Binomial

a. Square of Sum of Two Numbers. The square of the sum of two numbers is a special product that should be readily recognized to aid in factoring algebraic expressions. *The square of the sum of two numbers equals the square of the first, plus twice the product of the first and second, plus the square of the second.* To illustrate, $(a + b)^2 = a^2 + 2ab + b^2$. Conversely, the factors of $a^2 + 2ab + b^2$ are $(a + b)(a + b)$ or $(a + b)^2$. This relationship is shown in figure 12. The areas of the four rectangles, as shown on the figure, are a^2 , ab , ab , and b^2 . The area of the large rectangle formed by the four smaller rectangles is equal to its base $(a + b)$ times its altitude $(a + b)$, or $(a + b)^2$. Since the area of the large rectangle is equal to the sum of the areas of the four smaller rectangles, then $(a + b)^2$ is equal to $a^2 + ab + ab + b^2$, or $a^2 + 2ab + b^2$. This shows that the expression $a^2 + 2ab + b^2$ can be factored into $(a + b)(a + b)$, or $(a + b)^2$. Figure 13 shows a similar relationship in which nine small rectangles form one large rectangle.

In this case, the area of the large rectangle is $(a + 2b)^2$ and the sum of the areas of the nine smaller rectangles is $a^2 + 4ab + 4b^2$; consequently, $(a + 2b)$ and $(a + 2b)$ are factors of $a^2 + 4ab + 4b^2$. Thus, the factors of the square of one number, plus twice the product of the first and second number, plus the square of the second number are the square of the sum of the two numbers.

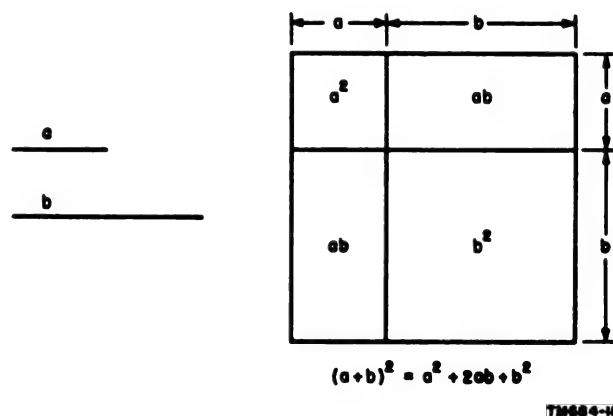


Figure 12. Square of sum of two numbers.

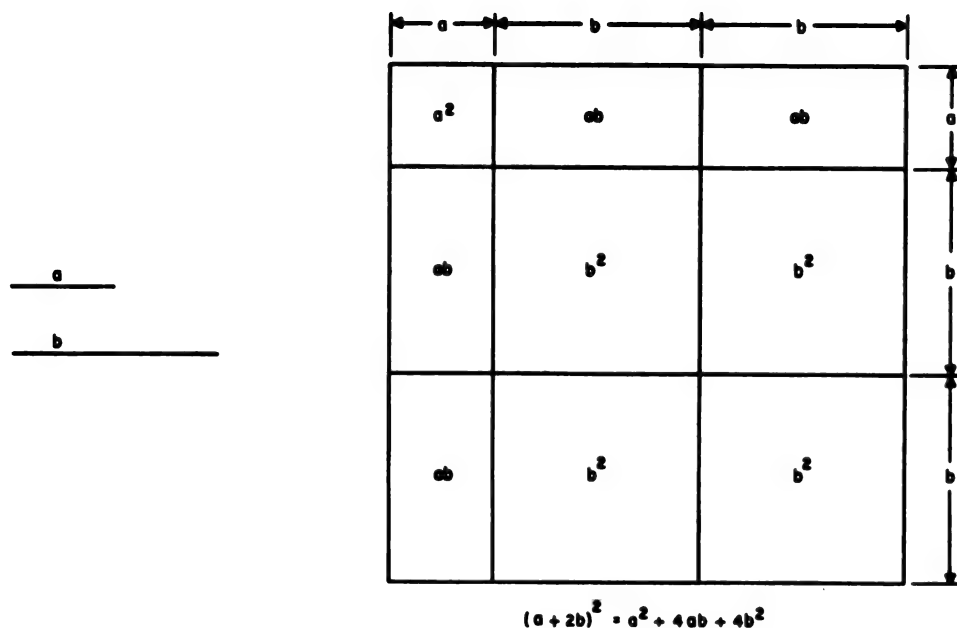


Figure 13. Factors of square of positive binomial.

Example: Factor $4b^2 + 16db + 16d^2$.

$$\begin{aligned} 4b^2 + 16db + 16d^2 &= (2b + 4d)(2b + 4d) \\ &= (2b + 4d)^2 \\ &= [2(b + 2d)]^2 \\ &= 2^2 (b + 2d)^2 \end{aligned}$$

To prove the factoring:

$$\begin{aligned} (2b + 4d)^2 &= (2b)^2 + 2(2b)(4d) + (4d)^2 \\ &= 4b^2 + 16db + 16d^2 \end{aligned}$$

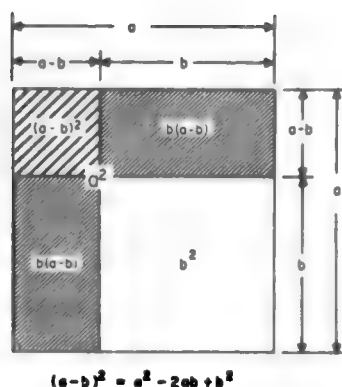
Note that 4 (that is, 2^2) may be removed before factoring the rest of the expression—this often simplifies computation.

$$4(b^2 + 4bd + 4d^2) = 4(b + 2d)^2$$

b. Square of Difference of Two Numbers. The square of the difference of two numbers equals the square of the first, minus twice the product of the first and second, plus the square of the second. For example, $(a - b)^2 = a^2 - 2ab + b^2$. The factors of $a^2 - 2ab + b^2$ are $(a - b)(a - b)$ or $(a - b)^2$. This relationship is shown pictorially in figure 14. The area of the large rectangle formed by the four small rectangles is a^2 . The areas of the four smaller rectangles are shown on the illustration. The area of the upper left-hand rectangle is $(a - b)^2$. It is also equal to the area of the large rectangle minus the areas of the other three rectangles, or $a^2 - b(a - b) - b(a - b) - b^2$.

This can be further simplified as follows:

$$\begin{aligned} a^2 - b(a - b) - b(a - b) - b^2 \\ a^2 - 2b(a - b) - b^2 \\ a^2 - 2ab + 2b^2 - b^2 \\ a^2 - 2ab + b^2 \end{aligned}$$



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Figure 14. Square of difference of two numbers.

Therefore, $(a - b)^2 = a^2 - 2ab + b^2$, and $(a - b)$ and $(a - b)$ are factors of $a^2 - 2ab + b^2$. Thus, the factors of the square of one number, minus twice the product of the first and the second, plus the square of the second are the square of the difference of the two numbers.

Example:

Factor $9b^2 - 12bd + 4d^2$.

$$\begin{aligned} 9b^2 - 12bd + 4d^2 &= (3b - 2d)(3b - 2d) \\ &= (3b - 2d)^2 \end{aligned}$$

To prove the factoring:

$$\begin{aligned} (3b - 2d)^2 &= (3b)^2 - 2(3b)(2d) + (2d)^2 \\ &= 9b^2 - 12bd + 4d^2 \end{aligned}$$

58. Factors of Difference of Two Squares

The product of the sum and difference of two numbers is equal to the difference of their squares. Thus, $(a + b)(a - b) = a^2 - b^2$. To factor the difference of two squares, extract the square roots, then write the sum of the roots as one factor and the difference of the roots as the other factor. Thus, the factors of $a^2 - b^2$ are $(a + b)(a - b)$.

Example:

Factor $4x^2 - 9y^2$.

$$4x^2 - 9y^2 = (2x + 3y)(2x - 3y)$$

To prove the factoring:

$$\begin{aligned} (2x + 3y)(2x - 3y) \\ &= (2x)^2 + (2x)(3y) - (2x)(3y) - (3y)^2 \\ &= 4x^2 - 9y^2 \end{aligned}$$

59. Factors of Trinomials

a. Trinomials Such as $x^2 + x(a + b) + ab$. The factors of a trinomial consisting of the square of the common term, the product of the common term and the algebraic sum of the unlike terms, and the product of the unlike terms are two binomials that have one term in common and the other term unlike. Thus, the factors of $x^2 + x(a + b) + ab$ are $(x + a)(x + b)$ where x is the common term, and a and b are the unlike terms. As proof, the product of $(x + a)(x + b)$ is $x^2 + xa + xb + ab$. By factoring the two terms which have a common factor, x , the original trinomial $x^2 + x(a + b) + ab$ is obtained.

Example: Factor $9r^2 + 6r(s + t) + 4st$.

$$9r^2 + 6r(s + t) + 4st = (3r + 2s)(3r + 2t)$$

To prove the factoring:

$$\begin{aligned}(3r + 2s)(3r + 2t) &= (3r)^2 + (3r)(2s) + (3r)(2t) + (2s)(2t) \\ &= 9r^2 + 6rs + 6rt + 4st \\ &= 9r^2 + 6r(s + t) + 4st\end{aligned}$$

b. Trinomials Such as $x^2 + 6x + 8$. To factor a trinomial of the form $x^2 + 6x + 8$, $x^2 - 6x + 8$, $x^2 + 6x - 8$, or $x^2 - 6x - 8$, much of the work is done by trial and error. The problem is to find two factors of the final term which, when added together, will give the coefficient of the middle term. Taking the first of the trinomials above, the factors of 8 are $8 \cdot 1$ and $4 \cdot 2$. Since $4 + 2 = 6$ and $8 + 1 = 9$, the factors that will be used are 4 and 2. With regards to signs, if the sign of the final term is positive, the signs of the two factors are alike and will be the same as the sign of the middle term. Thus, the factors of $x^2 + 6x + 8$ are $(x + 4)$ and $(x + 2)$, and the factors of $x^2 - 6x + 8$ are $(x - 4)$ and $(x - 2)$. If the sign of the final term is negative, however, the signs containing the two terms of each binomial factor are unlike; the larger factor will take the sign of the middle term. For example, the factors of $x^2 + 2x - 8$ are $(x + 4)$ and $(x - 2)$, and the factors of $x^2 - 2x - 8$ are $(x - 4)$ and $(x + 2)$.

Example 1: Factor $y^2 + 12y + 32$.

$$y^2 + 12y + 32 = (y + 8)(y + 4)$$

Example 2: Factor $z^2 - 11z + 30$.

$$z^2 - 11z + 30 = (z - 6)(z - 5)$$

Example 3: Factor $r^2 + 4r - 12$.

$$r^2 + 4r - 12 = (r + 6)(r - 2)$$

Example 4: Factor $s^2 - s - 20$.

$$s^2 - s - 20 = (s - 5)(s + 4)$$

c. Trinomials Such as $6a^2 - 11a - 10$. The procedure used to factor trinomials of this type

is an extension of the procedure described in *b* above and as shown in the example below.

Example: Factor $6a^2 - 11a - 10$.

Step 1. Find two numbers that, when multiplied together, form the left-hand term, $6a^2$.

$$(6a)(a) = 6a^2$$

$$(2a)(3a) = 6a^2$$

Step 2 Find two numbers that, when multiplied together, form the right-hand term, -10 .

$$(10)(-1) = -10$$

$$(5)(-2) = -10$$

$$(-10)(1) = -10$$

$$(-5)(2) = -10$$

Step 3. By trial and error, set up two binomial expressions containing factors from step 1 in the left-hand term and factors from step 2 in the right-hand term. The proper selection of factors should give the middle term of the trinomial when the binomials are multiplied.

$$(2a + 5)(3a - 2) \text{ (first trial)}$$

$$6a^2 + 15a - 4a - 10 = 6a^2 + 11a - 10 \text{ (multiplying out)}$$

The middle term obtained does not match the middle term of the given trinomial. The numerical value is correct, but the sign is wrong. Make a second

trial with the signs in the binomials changed.

$$(2a - 5)(3a + 2) \\ 6a^2 - 15a + 4a - 10 = 6a^2 - 11a - 10$$

Step 4. Since the second trial results in the correct trinomial, the factors of $6a^2 - 11a - 10$ are $(2a - 5)$ and $(3a + 2)$.

Note. The method of trial and error used above may not work in every case. Other arrangements of factors and signs must be tried until the correct results are obtained.

60. Factors of Two Cubes

a. Sum of Two Cubes. The factors of the sum of two cubes, such as $x^3 + y^3$, are $(x + y)$ and $(x^2 - xy + y^2)$. In this case, the binomial is an expression of the sum of the primes times the sum of the squares of the primes minus the product of the primes. This is seen readily by dividing $x^3 + y^3$ by $x + y$.

Thus,

$$\begin{array}{r} x + y \overline{) x^3 - xy^2 + y^3} \\ \underline{x^3 + x^2y} \\ -x^2y \\ \underline{-x^2y - xy^2} \\ xy^2 + y^3 \\ \underline{xy^2 + y^3} \\ 0 \end{array}$$

Example 1: Factor $z^3 + 8$.

$$z^3 + 8 = (z + 2)(z^2 - 2z + 4)$$

To prove the factoring:

$$\begin{array}{r} z + 2 \overline{) z^3 - 2z^2 + 4z + 8} \\ \underline{z^3 + 2z^2} \\ -2z^2 - 4z \\ \underline{-2z^2 - 4z} \\ 4z + 8 \\ \underline{4z + 8} \\ 0 \end{array}$$

Example 2:

Factor $r^3 + 125x^3$.

$$r^3 + 125x^3 = (r + 5x)(r^2 - 5rx + 25x^2)$$

To prove the factoring:

$$\begin{array}{r} r + 5x \overline{) r^3 - 5r^2x + 25rx^2 + 125x^3} \\ \underline{r^3 + 5r^2x} \\ -5r^2x \\ \underline{-5r^2x - 25rx^2} \\ 25rx^2 + 125x^3 \\ \underline{25rx^2 + 125x^3} \\ 0 \end{array}$$

b. Difference of Two Cubes. The factors of the difference of two cubes, such as $x^3 - y^3$, are $(x - y)(x^2 + xy + y^2)$. These factors are an expression of the difference of the primes times the sum of the squares plus the product of the primes. As in the sum of two cubes, factoring can be proved by dividing the product by the binomial factor.

Example 1: Factor $a^3 - b^3$.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

To prove the factoring:

$$\begin{array}{r} a - b \overline{) a^3 + a^2b + ab^2 + b^3} \\ \underline{a^3 - a^2b} \\ a^2b \\ \underline{a^2b - ab^2} \\ ab^2 + b^3 \\ \underline{ab^2 + b^3} \\ 0 \end{array}$$

Example 2: Factor $z^3 - 27$.

$$z^3 - 27 = (z - 3)(z^2 + 3z + 9)$$

To prove the factoring:

$$\begin{array}{r} z - 3 \overline{) z^3 + 3z^2 + 9z - 27} \\ \underline{z^3 - 3z^2} \\ 3z^2 \\ \underline{3z^2 - 9z} \\ 9z - 27 \\ \underline{9z - 27} \\ 0 \end{array}$$

Example 3: Factor $64s^3 - 216t^3$.

$$64s^3 - 216t^3 = (4s - 6t)(16s^2 + 24st + 36t^2)$$

To prove the factoring:

$$\begin{array}{r} 16s^2 \qquad + 24st \qquad + 36t^2 \\ 4s - 6t \overline{) 64s^3 \qquad \qquad \qquad - 216t^3} \\ \underline{64s^3 \qquad - 96s^2t} \qquad \qquad \qquad \\ 96s^2t \qquad \qquad \qquad \\ \underline{96s^2t \qquad - 144st^2} \qquad \qquad \qquad \\ 144st^2 \qquad - 216t^3 \\ \underline{144st^2 \qquad - 216t^3} \end{array}$$

61. Review Problems—Factoring

a. Factor:

- (1) $25 + 5 - 30$
- (2) $8 + 4 - 32$
- (3) $9 - 18 + 21$
- (4) $7r - 21r + 35r$
- (5) $10x + 8y + 6z$

b. Find the values of the indicated powers:

- (1) $(7xy^3)^2$
- (2) $(-2w^3)^2$
- (3) $(8a^2b^4)^2$
- (4) $(9a^3x)^3$
- (5) $(-3bx^4)^3$

c. Find the value of each of the following:

- (1) $\sqrt{5^3}$
- (2) $\sqrt{4^3}$
- (3) $\sqrt{a^3b^4}$
- (4) $\sqrt{36y^2z^4}$

$$(5) \sqrt{100a^3b^{10}}$$

$$(6) \sqrt{16a^3 \cdot 5^3}$$

$$(7) \sqrt[3]{-27}$$

$$(8) \sqrt[3]{-x^9}$$

$$(9) \sqrt[3]{(-8)^3}$$

$$(10) \sqrt[3]{125x^{15}y^{18}z^3}$$

d. Factor:

- (1) $3x + 6$
- (2) $5a^3 + 15a$
- (3) $10x^3 - 14x^2 - 2x$
- (4) $6azy + 9bzx - 12cz$
- (5) $m^3 + m^2 - 5mx$
- (6) $3a^5 - 6a^4b - 3a^3b^2$
- (7) $7ry^3 - 14ry^2 + 21ry$
- (8) $12x^2am + 14xa^2m + 16xam^3$
- (9) $\pi r \frac{2}{1} + \pi r \frac{2}{2}$
- (10) $\frac{1}{4}c^3d - \frac{1}{8}c^2d^2 + \frac{1}{16}cd^3$

Section V. ALGEBRAIC FRACTIONS

62. General

Algebraic fractions play an important part in equations for electrical and electronic circuits. These fractions can be added, subtracted, multiplied, and divided in the same manner as arithmetical fractions.

63. Changing Signs of Fractions

a. The sign preceding a fraction is the sign of the fraction. It refers to the fraction as a whole and not to either the numerator or the denominator. In addition, the numerator and denominator each has a sign. For example, in the fraction $-\frac{3a}{5b}$, the sign of the fraction is

minus, the sign of the numerator is plus, and the sign of the denominator is plus. Any two of the three signs can be changed without changing the value of the fraction.

$$\text{Thus, } -\frac{3a}{5b} = \frac{-3a}{5b} = \frac{3a}{-5b}.$$

Therefore, the sign of the fraction is not changed if the signs of both the numerator and the denominator are changed. Also, the sign of the fraction must be changed if the sign of either the numerator or denominator, but not both, is changed.

b. If the numerator or denominator is a polynomial, the sign of each term should be changed, not just the first sign. For example,

$$-\frac{a-b}{c-d} = +\frac{-(a-b)}{c-d} = \frac{-a+b}{c-d} = \frac{b-a}{c-d}.$$

c. If the numerator or denominator is in factored form, change only the sign of one of the factors, not both. Thus,

$$-\frac{(x-y)(x-2y)}{x+y} = \frac{(x+y)(x-2y)}{x+y} = \frac{(y-x)(x-2y)}{x+y}.$$

64. Changing Form of Algebraic Fractions

In algebra, as in arithmetic, any fraction can be changed to an equivalent fraction by multiplying or dividing both the numerator and denominator by the same term or number except zero. This will not change the value of the fraction. For example, to change the fraction $\frac{3}{5}$ to a fraction with 10 as its denominator, multiply both the numerator and the denominator by 2. Thus,

$$\frac{3}{5} = \frac{3 \cdot 2}{5 \cdot 2} = \frac{6}{10}.$$

Similarly, to change the fraction $\frac{x}{y}$ to a fraction with yz as its denominator, the denominator is changed to yz by multiplying by z ; the numerator also is multiplied by z to become xz . Thus,

$$\frac{x}{y} = \frac{x \cdot z}{y \cdot z} = \frac{xz}{yz}.$$

Example 1: Change $\frac{4}{a-3}$ to a fraction with a^2-9 as its denominator.

$$\begin{aligned} \frac{4}{a-3} &= \frac{4 \cdot (a+3)}{(a-3)(a+3)} \\ &= \frac{4(a+3)}{a^2-9} \end{aligned}$$

Example 2: Change $\frac{4r-3}{6r}$ to a fraction with $18\pi r^2s$ as its denominator.

$$\frac{4r-3}{6r} = \frac{(4r-3) \cdot 3\pi rs}{6r \cdot 3\pi rs} = \frac{3\pi rs(4r-3)}{18\pi r^2s}$$

65. Reducing Fractions to Lowest Terms

As in arithmetic, when the numerator and denominator of a fraction have no common factor other than 1, the fraction is said to be in its lowest terms. The fraction $\frac{3}{8}$, $\frac{a}{b}$, and

$\frac{p+q}{p-q}$, therefore, are in their lowest terms

since the numerator and denominator of each fraction have no other factor except 1. The fractions $\frac{6}{12}$ and $\frac{3a}{9a^2}$ are not in their lowest

terms. The fraction $\frac{6}{12}$ can be reduced to its lowest term by dividing both the numerator and denominator by 6. Similarly, the fraction

$\frac{6y}{15y^2}$ can be reduced to $\frac{1}{3y}$ by dividing the numerator and denominator by $6y$. Thus, to reduce a fraction to its lowest terms, factor the numerator and denominator into prime factors and cancel the factors common to both (since they are equal to $\frac{1}{1}$).

Example 1: Reduce $\frac{6y}{8y^2}$ to lowest terms.

$$\frac{6y}{8y^2} = \frac{2y(3)}{2y(4y)} = \frac{3}{4y}$$

Example 2: Reduce $\frac{xab^2}{xcb}$ to lowest terms.

$$\frac{xab^2}{xcb} = \frac{xb(ab)}{xb(c)} = \frac{ab}{c}$$

Example 3: Reduce $\frac{a^2 - b^2}{4a + 4b}$ to lowest terms.

$$\frac{a^2 - b^2}{4a + 4b} = \frac{(a + b)(a - b)}{4(a + b)} = \frac{a - b}{4}$$

Example 4: Reduce $\frac{2a^2 + 4ab + 2b^2}{2a + 2b}$ to lowest terms.

$$\frac{2a^2 + 4ab + 2b^2}{2a + 2b} = \frac{2(a + b)(a + b)}{2(a + b)} = \frac{a + b}{1} = a + b$$

66. Finding Lowest Common Denominator

The lowest common denominator (LCD) of two or more fractions is the smallest term or number that is divisible by each of the denominators. Inspect to find this term or number, divide the LCD by the denominator of each fraction, and multiply both the numerator and denominator by the quotient. For example, when changing the fractions $\frac{2}{3}$ and $\frac{4}{5}$ to fractions which have an LCD, inspection shows that 15 is the smallest number which is divisible by both 3 and 5. Thus, the fractions $\frac{2}{3}$ and $\frac{4}{5}$ be-

come $\frac{10}{15}$ and $\frac{12}{15}$. Similarly, the LCD of $\frac{4xy}{3a^2}$ and $\frac{6z}{4ab}$ is $12a^2b$ because this is the smallest term that is divisible by both $3a^2$ and $4ab$. Thus, the fraction $\frac{4xy}{3a^2}$ and $\frac{6z}{4ab}$ become $\frac{16xyb}{12a^2b}$ and $\frac{18za}{12a^2b}$, respectively. When fractions have factors with exponents in the denominators, the highest power of each distinct factor is used to form the LCD. For example, consider the problem of finding the LCD of fractions having the following denominators: x^2y^2z , $x^2y^3z^2$, y^4z^3 , x^2y^4 . The LCD is $x^2y^4z^3$ because x^2 , y^4 , and z^3 are the highest powers of x , y , and z in any one denominator.

Example: Change $\frac{3a}{a^2 - b^2}$ and $\frac{4b}{a^2 - ab - 2b^2}$ to equivalent fractions having an LCD.

Step 1. Factor each denominator into its prime factors:

$$\frac{3a}{a^2 - b^2} = \frac{3a}{(a + b)(a - b)}$$

$$\frac{4b}{a^2 - ab - 2b^2} = \frac{4b}{(a + b)(a - 2b)}$$

Step 2. The lowest common multiple of the denominators is the LCD:

$$(a + b)(a - b)(a - 2b)$$

Step 3. Divide the LCD by the denominators:

$$(a + b)(a - b)(a - 2b) \div (a + b)(a - b) = a - 2b$$

$$(a + b)(a - b)(a - 2b) \div (a + b)(a - 2b) = a - b$$

Step 4. Change $\frac{3a}{(a + b)(a - b)}$ into a fraction having $(a + b)$

$(a - b)(a - 2b)$ as its denominator:

$$\frac{3a}{(a + b)(a - b)} = \frac{3a(a - 2b)}{(a + b)(a - b)(a - 2b)}$$

Step 5. Change $\frac{4b}{(a+b)(a-2b)}$ into a fraction having $(a+b)(a-b)(a-2b)$ as its denominator.

$$\frac{4b}{(a+b)(a-2b)} = \frac{4b(a-b)}{(a+b)(a-b)(a-2b)}$$

Step 6. Therefore, $\frac{3a}{a^2-b^2} = \frac{3a(a-b)}{(a+b)(a-b)(a-2b)}$

$$\text{and } \frac{4b}{a^2-ab-b^2} = \frac{4b(a-b)}{(a+b)(a-b)(a-2b)}$$

67. Addition and Subtraction of Algebraic Fractions

a. Addition. The addition of algebraic fractions is similar to the corresponding operation in arithmetic. To add two or more fractions having a common denominator, add the numerators and place the result over the common denominator. If the fractions have different denominators, convert them to fractions with an LCD. The sum of the fractions is equal to the algebraic sum of the numerators divided by the LCD. Simplify the numerator and reduce the result to its lowest terms. If possible, factor or combine for further simplification.

Example: Find the sum of $\frac{2x}{x+y}$ and $\frac{2y}{x-y}$.

The LCD is $(x+y)(x-y)$. Therefore,

$$\begin{aligned} \frac{2x}{x+y} + \frac{2y}{x-y} &= \frac{2x(x-y)}{(x+y)(x-y)} + \frac{2y(x+y)}{(x+y)(x-y)} \\ &= \frac{2x(x-y) + 2y(x+y)}{(x+y)(x-y)} \\ &= \frac{2x^2 - 2xy + 2xy + 2y^2}{(x+y)(x-y)} \\ &= \frac{2x^2 + 2y^2}{(x+y)(x-y)} \\ &= \frac{2(x^2 + y^2)}{x^2 - y^2} \end{aligned}$$

b. Subtraction. To subtract two fractions having a common denominator, subtract the numerator of the subtrahend from the numerator of the minuend and place the result over the common denominator. If the denominators are different, find the LCD and subtract, as shown below.

Example: Subtract $\frac{8}{x^2+6x-16}$ from $\frac{9}{x^2+7x-18}$.

The LCD is $(x-2)(x+8)(x+9)$. Therefore,

$$\begin{aligned} \frac{9}{x^2+7x-18} - \frac{8}{x^2+6x-16} &= \frac{9(x+8)}{(x-2)(x+8)(x+9)} - \frac{8(x+9)}{(x-2)(x+8)(x+9)} \\ &= \frac{9(x+8) - 8(x+9)}{(x-2)(x+8)(x+9)} \\ &= \frac{9x + 72 - 8x - 72}{(x-2)(x+8)(x+9)} \\ &= \frac{x}{(x-2)(x+8)(x+9)} \end{aligned}$$

68. Multiplication and Division of Algebraic Fractions

a. Multiplication. The process of multiplication of algebraic fractions is the same as in arithmetic. The product of two or more fractions is the product of the numerators divided by the product of the denominators. The operation may be simplified by dividing common factors in the numerator and denominator by the same factor.

Example 1: Multiply $\frac{6a^2b}{7x}$ by $\frac{21x^2y}{24a^3b}$.

The first numerator and the second denominator are divisible by $6a^2b$; the first denominator and the second numerator are divisible by $7x$. Therefore:

$$\frac{\overset{1}{\cancel{6a^2b}}}{\underset{1}{\cancel{7x}}} \cdot \frac{\overset{3xy}{\cancel{21x^2y}}}{\underset{4}{\cancel{24a^3b}}} = \frac{3xy}{4}$$

Example 2: Multiply $\frac{a^2 + 2ab + b^2}{a - b}$ by $\frac{a^2 - 2ab + b^2}{a + b}$

$$\begin{aligned} \frac{a^2 + 2ab + b^2}{a - b} \cdot \frac{a^2 - 2ab + b^2}{a + b} &= \frac{(a + b)(a + b)}{a - b} \cdot \frac{(a - b)(a - b)}{a + b} \\ &= \frac{\overset{1}{\cancel{(a + b)}}(a + b)\overset{1}{\cancel{(a - b)}}(a - b)}{\underset{1}{\cancel{(a - b)}}(\underset{1}{\cancel{a + b}})} \\ &= (a + b)(a - b) \\ &= a^2 - b^2 \end{aligned}$$

b. Division. To divide algebraic fractions, multiply the dividend by the reciprocal of the divisor. Thus, to divide by x , multiply by the reciprocal of x , that is $\frac{1}{x}$. In other words, invert the divisor and proceed as in multiplication.

Example 1: Divide $\frac{2a + 2b}{a - 3}$ by $\frac{a^2 - b^2}{2a - 6}$.

$$\begin{aligned} \frac{2a + 2b}{a - 3} \div \frac{a^2 - b^2}{2a - 6} &= \frac{2a + 2b}{a - 3} \cdot \frac{2a - 6}{a^2 - b^2} \\ &= \frac{\overset{1}{\cancel{2(a + b)}}}{\underset{1}{\cancel{a - 3}}} \cdot \frac{\overset{1}{\cancel{2(a - 3)}}}{\underset{1}{\cancel{(a + b)(a - b)}}} \\ &= \frac{2 \cdot 2}{a - b} \\ &= \frac{4}{a - b} \end{aligned}$$

Example 2: Divide $\frac{z^2 - z - 6}{z^2 - 25}$ by $\frac{z^2 + z - 12}{z^2 - z - 20}$.

$$\begin{aligned} \frac{z^2 - z - 6}{z^2 - 25} \div \frac{z^2 + z - 12}{z^2 - z - 20} &= \frac{z^2 - z - 6}{z^2 - 25} \cdot \frac{z^2 - z - 20}{z^2 + z - 12} \\ &= \frac{\overset{1}{\cancel{(z - 3)}}(z + 2)}{\underset{1}{\cancel{(z - 5)}}(z + 5)} \cdot \frac{\overset{1}{\cancel{(z - 4)}}(z + 5)}{\underset{1}{\cancel{(z + 4)}}(\underset{1}{\cancel{z - 3}})} \\ &= \frac{z + 2}{z + 5} \end{aligned}$$

69. Review Problems—Algebraic Fractions

a. *Changing Signs of Fractions.* Solve for the unknown.

- (1) $\frac{4x+3}{6} - \frac{x-9}{4} = 5$
- (2) $\frac{x-2}{4} = \frac{1}{2}$
- (3) $\frac{r+4}{3} - \frac{r-2}{5} = 2$
- (4) $\frac{4x-3}{6x} - \frac{4x+5}{8x} = 2$
- (5) $\frac{7t+2}{3} = 3$
- (6) $\frac{x-4}{3} + \frac{2x-5}{6} = 3$
- (7) $\frac{2r+3}{2} - \frac{3r+2}{4} = 2$
- (8) $\frac{7x-4}{3} + \frac{x-5}{5} = \frac{1}{5}$

b. *Equivalent Fractions.* Supply missing terms.

- (1) $\frac{4}{8} = \frac{?}{16}$
- (2) $\frac{1}{c} = \frac{?}{cx}$
- (3) $\frac{3}{r-s} = \frac{?}{r^2-s^2}$
- (4) $\frac{a-s}{1} = \frac{?}{3}$
- (5) $\frac{l-6}{l-3} = \frac{?}{(l-3)(l-9)}$
- (6) Change $\frac{4E^2}{R}$ into an equivalent fraction of which the denominator is $2I^2 R$.
- (7) Change $\frac{1}{3\pi/c}$ into an equivalent fraction of which the denominator is $2I^2 R$.

c. *Lowest Common Denominator.* Reduce to equivalent fractions having an LCD.

- (1) $\frac{1}{R}, \frac{1}{R^2}, \frac{1}{r}$
- (2) $\frac{1}{a+1}, \frac{x}{a-1}$
- (3) $\frac{b}{2x}, \frac{c}{3x}$
- (4) $\frac{y}{2}, \frac{y}{2y+6}$
- (5) $\frac{2}{c}, \frac{3}{c+1}$

$$(6) \frac{i}{e-5}, \frac{i}{2e-10}$$

$$(7) \frac{y}{c^2-d^2}, \frac{z}{c-d}$$

d. *Addition and Subtraction of Fractions*
Perform the indicated operations.

$$(1) \frac{1}{a} + \frac{4}{a} + \frac{7}{a}$$

$$(2) \frac{s}{t} + \frac{s+4}{2t} + \frac{s+3}{4t}$$

$$(3) \frac{3a}{4x^2y} + \frac{5b}{6xy^2}$$

$$(4) \frac{2}{z^2-1} + \frac{4}{z^2-4}$$

$$(5) \frac{3c-2d}{4cd^2} + \frac{2c-3d}{3c^2d}$$

$$(6) \frac{(r+1)(r-3)}{r^2+2r-15} + \frac{(r-2)(r+5)}{r^2+2r-15}$$

$$(7) 3y - \frac{1}{4}$$

$$(8) \frac{a+b}{a-b} - \frac{a-b}{a+b}$$

$$(9) \frac{32}{25q^2} - \frac{16}{5q}$$

$$(10) \frac{3t-2t}{4tv^2} - \frac{2t-3t}{3t^2v}$$

e. *Multiplication and Division of Fractions*
Perform the indicated operations.

$$(1) \frac{9y^2}{16} \cdot \frac{2}{3}$$

$$(2) \frac{a^3}{b^4} \cdot \frac{a^6}{b^2}$$

$$(3) \frac{3x^2}{49y^2z} \cdot \frac{7yz^2}{9xm}$$

$$(4) \left(\frac{1}{r} - \frac{1}{s}\right) \left(r - \frac{r^2}{s}\right)$$

$$(5) \frac{2x^2-5xy-3y^2}{x^2-9y^2} \cdot \frac{3x+9y}{10x^2+5xy}$$

$$(6) \frac{a-b}{a^2+2ab+b^2} \cdot \frac{a+b}{a^2-2ab+b^2}$$

$$(7) 3z \div \frac{1}{5}$$

$$(8) \frac{5ba^3}{6cd} \div 5b$$

$$(9) \frac{12s^2t}{20uv} \div \frac{3st}{4u^2v}$$

$$(10) \left(e + 2 - \frac{3}{e}\right) \div \left(e + 1 - \frac{2}{e}\right)$$

Section VI. EXPONENTS AND RADICALS

70. General

Chapter 4 presents exponents and roots consisting only of whole numbers. However, to use exponents and radicals to solve many equations and formulas, a knowledge of additional operations is required.

71. Fractional Exponents

a. General. A fractional exponent is merely another way of expressing the root of a number. For example, the cube root of x usually is written $\sqrt[3]{x}$; however, it also can be written $x^{\frac{1}{3}}$. Similarly, $\sqrt{2}$ also can be written $2^{\frac{1}{2}}$.

b. Application. Fractional exponents have a practical value in simplifying algebraic problems. They follow the same rules as exponents

that consist of integers, and can be added, subtracted, multiplied, or divided in the same way; thus

$$a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} = a^{\frac{1}{2} + \frac{1}{2}} = a^1 = a, \text{ and } a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} = a^{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = a^{\frac{3}{2}} = a \cdot a^{\frac{1}{2}} = a \sqrt{a}.$$

In other words, $a^{\frac{1}{2}}$ is one of two equal factors of a or the square root of a , and $a^{\frac{1}{3}}$ is one of three equal factors of a or the square cube root of a ; therefore, $a^{\frac{1}{2}} = \sqrt{a}$ and $a^{\frac{1}{3}} = \sqrt[3]{a}$.

c. Changing from Radical Form to Exponential Form. To change a radical expression to exponential form, remove the radical sign and annex a fractional exponent to the radicand (number under the radical sign). The numerator of the fractional exponent is the power of the radicand, and the denominator is the index of the root.

Example 1: Change $\sqrt[4]{a^2}$ to exponential form and simplify.

$$\sqrt[4]{a^2} = (a^2)^{\frac{1}{4}}$$

Multiplying exponents and simplifying:

$$(a^2)^{\frac{1}{4}} = a^{2 \cdot \frac{1}{4}} = a^{\frac{2}{4}} = a^{\frac{1}{2}} = \sqrt{a}$$

$$\text{Therefore, } \sqrt[4]{a^2} = \sqrt{a}$$

Example 2: Change $\sqrt[3]{8a^2b^3}$ to exponential form and simplify.

$$\begin{aligned} \sqrt[3]{8a^2b^3} &= \sqrt[3]{2^3a^2b^3} = (2^3a^2b^3)^{\frac{1}{3}} = 2^{\frac{3}{3}} \cdot a^{\frac{2}{3}} \cdot b^{\frac{3}{3}} \\ &= 2^1 a^{\frac{2}{3}} b^1 = 2b (a^2)^{\frac{1}{3}} = 2b \sqrt[3]{a^2} \end{aligned}$$

d. Changing from Exponential Form to Radical Form. To change an expression with a fraction exponent to a radical form, make the base of the fractional exponent the radicand, the numerator of the exponent the power of the radicand, and the denominator of the exponent the index of the root.

Example 1: Change $4^{\frac{1}{2}}$ to radical form.

$$4^{\frac{1}{2}} = \sqrt{4}$$

Example 2: Change $3^{\frac{1}{3}}$ to radical form.

$$3^{\frac{1}{3}} = \sqrt[3]{3^1} = \sqrt[3]{3}$$

Example 3: Change $(5a^2b)^{\frac{1}{3}}$ to radical form.

$$\begin{aligned} (5a^2b)^{\frac{1}{3}} &= \sqrt[3]{(5a^2b)^1} \\ &= \sqrt[3]{5a^2b^1} \end{aligned}$$

72. Simplification of Radicals

a. Removing a Factor from the Radicand. The form in which a radical expression is writ-

ten may be changed without altering its numerical value. Sometimes there is a question as to what actually is the simplest form for an expression. For instance, consider the simplification of an expression such as $\sqrt{1250}$: $\sqrt{1250} = \sqrt{2 \cdot 5^4} = 5^2 \sqrt{2} = 25\sqrt{2}$. The expression $25\sqrt{2}$ usually is accepted as being simpler than $\sqrt{1250}$. As a general rule, the fewer the factors under the radical sign, the simpler the expression. Thus, a radicand may be separated into two factors, one of which is the greater power whose root can be taken. The root of this factor may then be written as the coefficient of a radical of which the other factor is the radicand.

Example 1: Simplify $\sqrt{50}$.

$$\begin{aligned} \sqrt{50} &= \sqrt{25 \cdot 2} \\ &= \sqrt{25} \cdot \sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

Example 2: Simplify $\sqrt[4]{32a^7b^3}$.

$$\begin{aligned}\sqrt[4]{32a^7b^3} &= (2^5a^7b^3)^{\frac{1}{4}} \\ &= 2^{\frac{5}{4}}a^{\frac{7}{4}}b^{\frac{3}{4}} \\ &= 2^{\frac{4}{4}}2^{\frac{1}{4}}a^{\frac{4}{4}}a^{\frac{3}{4}}b^{\frac{3}{4}} \\ &= 2a\sqrt[4]{2a^3b^3}\end{aligned}$$

b. Rationalizing Denominator. Rationalizing a denominator containing a radical means to eliminate the radical in the denominator. For example, to rationalize the expression $\frac{1}{\sqrt[3]{2}}$, first change the denominator into an expression having a fractional exponent; thus, $\frac{1}{\sqrt[3]{2}} = \frac{1}{2^{\frac{1}{3}}}$; then multiply the denominator by a number that will make its exponent equal to 1. This operation eliminates the radical sign below the line. In this case, $2^{\frac{2}{3}}$ is such a factor; thus $2^{\frac{1}{3}} \cdot 2^{\frac{2}{3}} = 2^1 = 2$. Such multiplication can be performed without changing the value of the fraction if the numerator also is multiplied by the same number; thus $\frac{1}{2^{\frac{1}{3}}} \cdot \frac{2^{\frac{2}{3}}}{2^{\frac{2}{3}}} = \frac{2^{\frac{2}{3}}}{2^{\frac{1}{3}+\frac{2}{3}}} = \frac{2^{\frac{2}{3}}}{2}$.

Finally, changing the numerator into radical form, $\frac{\sqrt[3]{2^2}}{2} = \frac{\sqrt[3]{4}}{2}$. Therefore, to rationalize a denominator, multiply both the numerator and the denominator by a number that will make the exponent in the denominator equal to 1; then simplify the radicand in the numerator. The examples below illustrate the method of rationalizing a few different types of denominators.

Example 1: Rationalize $\frac{1}{3^{\frac{2}{7}}}$.

$$\frac{1}{3^{\frac{2}{7}}} = \frac{1}{3^{\frac{2}{7}}} \cdot \frac{3^{\frac{5}{7}}}{3^{\frac{5}{7}}} = \frac{3^{\frac{3}{7}}}{3} = \frac{\sqrt[7]{3^3}}{3}$$

Example 2: Rationalize $\frac{1}{\sqrt{8}}$.

First simplify $\sqrt{8}$.

$$\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt[2]{2} = 2 \cdot 2^{\frac{1}{2}}$$

$$\frac{1}{\sqrt{8}} = \frac{1}{2 \cdot 2^{\frac{1}{2}}} = \frac{2^{\frac{1}{2}}}{2 \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}} = \frac{\sqrt{2}}{4}$$

Example 3: Rationalize $\frac{1}{\sqrt{7}}$.

Here the square root in the denominator is being multiplied by itself, making the number a perfect square.

$$\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{\sqrt{7}\sqrt{7}} = \frac{\sqrt{7}}{7}$$

c. Practical Application. The processes of the simplification of radicals and rationalization of denominators are useful when computing decimals. It is necessary to know, however, that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, etc. For example, consider the problem of evaluating $\frac{1}{\sqrt{2}}$. One way of evaluating this problem is to divide 1 by 1.414. This evaluation is a long-division problem of some length, however. A much more simple way is to rationalize—thus $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, and dividing 1.414 by 2 gives the result, 0.707.

73. Addition and Subtraction of Radicals

As discussed in paragraph 27b, terms that are alike in all respects, except for their coefficients, are called *similar terms*. Similarly, radicals that have the same index and the same radicand and differ only in their coefficients are called *similar radicals*. For example, $-5\sqrt{3}$, $2\sqrt{3}$, and $\sqrt{3}$ are similar radicals. Similar radicals may be added or subtracted in the same way that similar terms are added and subtracted. However, if the radicands are not alike and cannot be reduced to a common radicand, they are dissimilar and addition and subtraction can only be indicated; thus to add or subtract radicals, reduce them to their simplest form, then combine similar radicals, and indicate the addition or subtraction of dissimilar radicals.

Example 1: Perform the indicated operations.

$$4\sqrt{6} - 5\sqrt{6} - \sqrt{6} + 10\sqrt{6} = 8\sqrt{6}$$

Example 2: Add.

$$\begin{aligned}\sqrt{48a} + \sqrt{\frac{a}{3}} + \sqrt{3a} &= 4\sqrt{3a} + \frac{1}{3}\sqrt{3a} + \sqrt{3a} \\ &= \frac{16}{3}\sqrt{3a}\end{aligned}$$

Example 3: Perform the indicated operations.

$$\begin{aligned}\sqrt[3]{16r^3} - r\sqrt[3]{4r} + \sqrt[3]{64r^3} &= \sqrt[3]{(4r)^3} - r\sqrt[3]{4r} + \sqrt[3]{(4r)^3} \\ &= (4r)^{\frac{3}{3}} - 4(4r)^{\frac{1}{3}} + (4r)^{\frac{3}{3}} \\ &= \sqrt[3]{4r} - r\sqrt[3]{4r} + \sqrt[3]{4r} \\ &= \sqrt[3]{4r} (2 - r)\end{aligned}$$

Example 4: Perform the indicated operations.

$$\begin{aligned}2\sqrt{6} + \sqrt[3]{\frac{2}{3}} - \sqrt[3]{36} &= 2\sqrt{6} + \sqrt[3]{\frac{2}{3} \cdot \frac{8}{8}} - \sqrt[3]{6 \cdot 6} \\ &= 2\sqrt{6} + \frac{2}{3}\sqrt{6} - \sqrt{6} \\ &= 2\sqrt{6} + \frac{2}{3}\sqrt{6} - \sqrt{6} \\ &= 4\sqrt{6}\end{aligned}$$

74. Multiplication of Radicals

a. Radicals With Same Indexes. Radicals can be multiplied and combined under the same radical sign even though they differ in value, provided the index of the radicals are the same. To multiply a radical expression when radicals are of the same order, first multiply the coefficients, then multiply the radicands, and then simplify, if possible. For example, $2\sqrt{3} \cdot 3\sqrt{5} = 6\sqrt{15}$. If the radicand is a perfect square, simplify the result by extracting the square root. Remember that there are two square roots, one positive and one negative; thus, $6\sqrt{3} \cdot 4\sqrt{3} = 24\sqrt{9} = 24(\pm 3) = \pm 72$. When polynomial expressions, either or both of which involve radicals, are to be multiplied, proceed in the same manner as with literal polynomial expressions (par. 48). For example,

$$(\sqrt{3} + 2\sqrt{5}) \times (\sqrt{3} - 2\sqrt{5}) =$$

$$\begin{array}{r} \sqrt{3} + 2\sqrt{5} \\ \sqrt{3} - 2\sqrt{5} \\ \hline \sqrt{9} + 2\sqrt{15} \\ - 2\sqrt{15} - 4\sqrt{25} \\ \hline \sqrt{9} \qquad - 4\sqrt{25} = \pm 3 - 4(\pm 5) \\ \qquad \qquad \qquad = \pm 3 \pm 20 \\ \qquad \qquad \qquad = 3 \pm 20 \text{ or } -3 \pm 20 \\ \qquad \qquad \qquad = \pm 17 \text{ or } \pm 23 \end{array}$$

Example 1: Multiply $2\sqrt[3]{3a}$, $5\sqrt[3]{4a}$, and $3\sqrt[3]{18a}$.

$$\begin{aligned}2\sqrt[3]{3a} \cdot 5\sqrt[3]{4a} \cdot 3\sqrt[3]{18a} &= 2 \cdot 5 \cdot 3 \cdot \sqrt[3]{3a} \cdot \sqrt[3]{4a} \cdot \sqrt[3]{18a} \\ &= 30\sqrt[3]{216a^3} \\ &= 30 \cdot 6a \\ &= 180a\end{aligned}$$

Example 2: Multiply $\sqrt[3]{8t^3}$ and $\sqrt[3]{4t^3s}$.

$$\begin{aligned}\sqrt[3]{8t^3} \cdot \sqrt[3]{4t^3s} &= \sqrt[3]{32t^6s} \\ &= \sqrt[3]{2^4 \cdot 2 \cdot t^4 \cdot t \cdot s} \\ &= 2t\sqrt[3]{2ts}\end{aligned}$$

b. Radicals With Different Indexes. To multiply radicals when the indexes are different, first express them as radicals with a common index (or common fractional exponent) and proceed as in *a* above. The common index is the lowest common multiple of the indexes of the original radicals.

Example 1: Multiply $\sqrt{2} \cdot \sqrt[3]{4}$.

$$\begin{aligned}\sqrt{2} \cdot \sqrt[3]{4} &= \sqrt{2} \cdot \sqrt[3]{2^2} \\ &= 2^{\frac{1}{2}} \cdot 2^{\frac{2}{3}} \\ &= 2^{\frac{3}{6}} \cdot 2^{\frac{4}{6}} \\ &= 2^{\frac{7}{6}} \\ &= 2^{\frac{6}{6}} \cdot 2^{\frac{1}{6}} \\ &= 2 \cdot 2^{\frac{1}{6}} \text{ or } 2\sqrt[6]{2}\end{aligned}$$

Example 2: Multiply $\sqrt[3]{4x} \cdot \sqrt[4]{8x^3}$.

$$\begin{aligned}\sqrt[3]{4x} \cdot \sqrt[4]{8x^3} &= \sqrt[12]{(4x)^4} \cdot \sqrt[12]{(8x^3)^3} \\ &= \sqrt[12]{(2^2x)^4} \cdot \sqrt[12]{(2^3x^3)^3} \\ &= \sqrt[12]{2^8x^4} \cdot \sqrt[12]{2^9x^9} \\ &= \sqrt[12]{2^{17}x^{13}} \\ &= \sqrt[12]{2^{12} \cdot 2^5 \cdot x^{12} \cdot x} \\ &= 2x \sqrt[12]{2^5 \cdot x} \\ &= 2x \sqrt[12]{32x}\end{aligned}$$

75. Division of Radicals

a. Monomial Radical Expressions. The division of radicals is essentially the opposite of multiplication. When radicals are of the same order, the division of two radicals may be expressed under one radical sign—for example, $\frac{\sqrt{4}}{\sqrt{2}} = \sqrt{\frac{4}{2}} = \sqrt{2}$. When radicals are of different orders, they must be expressed as radicals having the same index or be changed to fractional exponents.

Example 1: Divide $\sqrt{15}$ by $\sqrt{5}$.

$$\frac{\sqrt{15}}{\sqrt{5}} = \sqrt{\frac{15}{5}} = \sqrt{3}$$

Example 2: Divide $\sqrt[3]{x^3y}$ by $\sqrt[3]{y^7}$.

$$\begin{aligned}\frac{\sqrt[3]{x^3y}}{\sqrt[3]{y^7}} &= \sqrt[3]{\frac{x^3y}{y^7}} \\ &= \sqrt[3]{\frac{x^3}{y^6}} \\ &= \frac{x}{y^2} \sqrt[3]{x^3}\end{aligned}$$

Example 3: Divide $\sqrt{35}$ by $\sqrt{15}$.

$$\begin{aligned}\frac{\sqrt{35}}{\sqrt{15}} &= \sqrt{\frac{35}{15}} \\ &= \sqrt{\frac{7}{3}} \\ &= \frac{1}{3}\sqrt{21}\end{aligned}$$

Example 4: Divide $\sqrt{4ab} \sqrt[3]{2ab}$ by $\sqrt[6]{4a^3b^3}$.

$$\begin{aligned}\frac{\sqrt{4ab} \sqrt[3]{2ab}}{\sqrt[6]{4a^3b^3}} &= \frac{\sqrt{(4ab)^2} \sqrt[3]{(2ab)^3}}{\sqrt[6]{4a^3b^3}} \\ &= \frac{\sqrt{16a^2b^2} \sqrt[3]{8a^3b^3}}{\sqrt[6]{4a^3b^3}} \\ &= \frac{4ab \sqrt[3]{8a^3b^3}}{\sqrt[6]{4a^3b^3}} \\ &= \frac{4ab \cdot 2\sqrt[3]{a^3b^3}}{\sqrt[6]{4a^3b^3}} \\ &= \frac{8ab \sqrt[3]{a^3b^3}}{\sqrt[6]{4a^3b^3}} \\ &= \sqrt[6]{2^6b^3} \text{ or } (2^6b^3)^{\frac{1}{6}} \\ &= 2\sqrt[6]{b^3}\end{aligned}$$

b. Binomial Expressions With Radical in Divisor. When the divisor is a binomial in which one or more of the terms contains a square root, division is performed by first rationalizing the divisor. Multiply the numerator and denominator of the fraction by the denominator with the sign between the terms changed; then simplify the numerator and the denominator.

Example 1: Divide 3 by $4 + \sqrt{6}$.

$$\begin{aligned}\frac{3}{4 + \sqrt{6}} &= \frac{3}{4 + \sqrt{6}} \cdot \frac{4 - \sqrt{6}}{4 - \sqrt{6}} \\ &= \frac{3(4 - \sqrt{6})}{16 - 6} \\ &= \frac{3}{10} (4 - \sqrt{6})\end{aligned}$$

Example 2: Divide $\sqrt{1+x} - \sqrt{1-x}$ by $\sqrt{1+x} + \sqrt{1-x}$.

$$\begin{aligned} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} &= \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \cdot \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}} \\ &= \frac{(1+x) - 2\sqrt{1-x^2} + (1-x)}{(1+x) - (1-x)} \\ &= \frac{2 - 2\sqrt{1-x^2}}{2x} \\ &= \frac{1 - \sqrt{1-x^2}}{x} \end{aligned}$$

76. Review Problems—Exponents and Radicals

a. Simplify.

- (1) $2^{\frac{1}{2}}(2^{\frac{3}{2}})$
- (2) $(8^{\frac{2}{3}})^{\frac{3}{2}}$
- (3) $\sqrt{50}$
- (4) $\sqrt[3]{\frac{1}{16}}$
- (5) $\sqrt{18x-9}$
- (6) $\sqrt[n]{\frac{6x^{3n}}{y^n}}$
- (7) $(x^{10}y^5)^{\frac{1}{5}}$
- (8) $(d^6e^4)^{\frac{3}{4}}$
- (9) $\left(\frac{64r^4}{s^3}\right)^{\frac{1}{3}}$
- (10) $(a^9b^3)^{\frac{2}{3}}$

b. Express with radical signs.

- (1) $4^{\frac{1}{2}}$
- (2) $a^{\frac{3}{2}}b^{\frac{2}{3}}$
- (3) $6^{\frac{2}{3}}$
- (4) $(8f)^{\frac{1}{2}}$
- (5) $5x^{\frac{5}{6}}$
- (6) $a^{\frac{2}{3}}c^{1.5}$
- (7) $6r^{\frac{1}{3}}$
- (8) $(8a^2b^3)^{\frac{1}{3}}$
- (9) $(^2r_1 + ^3r_2)^{\frac{1}{2}}$
- (10) $3(x^{\frac{1}{2}}y^2)^{\frac{1}{2}}$

c. Express with fractional exponents.

- (1) $\sqrt[4]{a^3}$
- (2) $\sqrt[3]{5x}$
- (3) $6x\sqrt[3]{a^2}$
- (4) $\sqrt[5]{z^3}$
- (5) $\sqrt[4]{3a^3b^5}$
- (6) $y^3\sqrt[4]{a^3}$
- (7) $8\sqrt[3]{3e}$
- (8) $9\sqrt[5]{g^4}$
- (9) $3b\sqrt[4]{cd^2}$
- (10) $\sqrt[3]{(x-y)^2}$

d. Simplify by removing suitable factors from radicand.

- (1) $\sqrt{12}$
- (2) $\sqrt{63}$
- (3) $\sqrt{63x^2}$
- (4) $2\sqrt{72a^2b^4}$
- (5) $\sqrt{60b^3d^2}$
- (6) $\sqrt{8I^2R}$
- (7) $3\sqrt{63p^3z^2}$
- (8) $2dr^2\sqrt{108dr^4s^3}$
- (9) $5a\sqrt{81a^2b}$
- (10) $16w^2x\sqrt{98w^4x^3y^2z}$

e. Rationalize denominators.

- (1) $\frac{1}{\sqrt{50}}$
- (2) $\frac{1}{\sqrt{4x}}$
- (3) $\frac{2a}{\sqrt{3a}}$
- (4) $\frac{1}{\sqrt[3]{x}}$
- (5) $\frac{1}{\sqrt[4]{3ax^3}}$

$$(6) \frac{1}{\sqrt[3]{3-2x}}$$

$$(7) \frac{a+b}{\sqrt[3]{a^3}}$$

$$(8) \frac{a}{\sqrt[3]{a^3bc}}$$

$$(9) \frac{1}{\sqrt[3]{(s+1)^3}}$$

$$(10) \frac{i+3}{\sqrt[5]{(i+3)^5}}$$

f. Simplify.

$$(1) 6\sqrt{4} - 3\sqrt{4} + 2\sqrt{4}$$

$$(2) 6\sqrt{45} - 2\sqrt{20}$$

$$(3) x - \sqrt{\frac{3x^2}{4}}$$

$$(4) \frac{a}{2} + \sqrt{\frac{9a^2}{2}}$$

$$(5) r\sqrt{rst} + rt\sqrt{\frac{s}{rt}}$$

$$(6) \sqrt{\frac{x+y}{x-y}} - \sqrt{\frac{x-y}{x+y}}$$

$$(7) \sqrt{5} + 3\sqrt{x} + 5\sqrt{x}$$

$$(8) 7\sqrt{a} - 4\sqrt{b} - 2\sqrt{b}$$

$$(9) 4\sqrt{x-y} + 3\sqrt{x+y} - 8\sqrt{x-y}$$

$$(10) 3\sqrt{125a^3b^3} + b\sqrt{20a^3} - \sqrt{500a^3b^3}$$

g. Find product and simplify.

$$(1) 3\sqrt{5} \cdot 4\sqrt{2}$$

$$(2) 2\sqrt[3]{9} \cdot 3\sqrt[3]{3}$$

$$(3) 4\sqrt[3]{a^3b^3} \cdot 2\sqrt[3]{ab^3}$$

$$(4) \sqrt{4x^2} \cdot x\sqrt{3z^2}$$

$$(5) \sqrt[3]{4x^3y^3} \cdot \sqrt[3]{2x^3y^3} \cdot \sqrt[3]{4xy^3}$$

$$(6) 2\sqrt[3]{2pq^2r} \cdot \sqrt[3]{4pq^3r^2} \cdot 3\sqrt[3]{8pq^2r^3}$$

$$(7) (\sqrt{a} + \sqrt{b} + \sqrt{c})^2$$

$$(8) a\sqrt{x}(a\sqrt{ax} + x\sqrt{ax} + \sqrt{ax})$$

$$(9) \sqrt{9 - \sqrt{17}} \cdot \sqrt{9 + \sqrt{17}}$$

$$(10) \sqrt[3]{x^3y^3} \sqrt[3]{256a^3}$$

h. Divide and simplify.

$$(1) \frac{\sqrt{12}}{\sqrt{8}}$$

$$(2) \frac{\sqrt[3]{625y}}{\sqrt[3]{5y}}$$

$$(3) \frac{\sqrt[3]{16x^3}}{\sqrt[3]{2x}}$$

$$(4) \frac{3zy}{\sqrt{zv}}$$

$$(5) \frac{2}{\sqrt{6-2}}$$

$$(6) \frac{\sqrt{30a} \sqrt[3]{24a^3} \sqrt[3]{72a}}{\sqrt[3]{5a}}$$

$$(7) \frac{\sqrt{2} + \sqrt{c}}{\sqrt{c} + 2\sqrt{2}}$$

$$(8) \frac{4\sqrt{3} - 3\sqrt{2}}{\sqrt{6}} \div \frac{\sqrt{10}}{4\sqrt{3} + 3\sqrt{2}}$$

$$(9) \frac{\sqrt{e^2 + f^2 + f}}{\sqrt{e^2 + f^2 - f}}$$

$$(10) \frac{2b + \sqrt{1-4b^2}}{2b - \sqrt{1-4b^2}}$$

Section VII. IMAGINARY AND COMPLEX NUMBERS

77. Imaginary Numbers

a. Indicated Square Root of Negative Numbers.

- (1) In the study of roots to this point, only the roots of positive numbers have been considered. Sometimes a negative expression will appear under the radical. Such an expression originally was given the designation *imaginary number* to distinguish it from real numbers. In electricity and electronics, however, so-called imaginary numbers are used for real physical calculations—the reactance of a large capaci-

tor or inductor must be calculated by using this type of number.

- (2) In multiplication, when a real number is multiplied by itself the result is always positive. For example, $+5 \cdot +5 = 25$, and $-5 \cdot -5 = 25$. Therefore, any number raised to a power having an even exponent will be positive because like signs are being multiplied. However, this is not true for the interpretation of an expression such as $\sqrt{-9}$. Any negative number can be regarded as the product of a positive number of the same absolute value and -1 , and the square root of a negative

number can be written as the square root of a positive number times $\sqrt{-1}$; thus, $\sqrt{-9} = \sqrt{9} \sqrt{-1} = 3\sqrt{-1}$, with $\sqrt{-1}$ being the imaginary number. Most mathematics texts represent the imaginary number $\sqrt{-1}$ by the letter i . However, the letter I or i means current in electrical formulas; therefore, the letter j , commonly called the *operator j*, is used in electronics.

Example 1: $\sqrt{-36} = \sqrt{(-1)36} = \sqrt{-1} \cdot \sqrt{36} = \sqrt{-1} \cdot 6 = j6$

Example 2: $\sqrt{-Z^2} = \sqrt{(-1)Z^2} = \sqrt{-1} \cdot \sqrt{Z^2} = \sqrt{-1} \cdot Z = jZ$

Example 3: $-\sqrt{-9a^2} = -\sqrt{(-1)9a^2} = -\sqrt{-1} \cdot \sqrt{9a^2} = -\sqrt{-1} \cdot 3a = -j3a$

b. Powers of Operator j . Imaginary numbers follow the fundamental laws of addition, subtraction, multiplication, and division. They also can be raised to a power; thus, $j^3 = j^2 \cdot j = -1(j) = -j$, and $j^4 = j^2 \cdot j^2 = -1(-1) = 1$. The values of the powers of j are obtained as follows:

$$j^2 = j \cdot j = \sqrt{-1} \cdot \sqrt{-1} = -1;$$

$$j^3 = j \cdot j \cdot j = \sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{-1} = -1\sqrt{-1} = -j; \text{ and}$$

$$j^4 = j \cdot j \cdot j \cdot j = \sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{-1} = -1 \cdot -1 = 1; \text{ but}$$

$j^5 = j \cdot j \cdot j \cdot j \cdot j = j^4 \cdot j = j^1 = \sqrt{-1}$, and the whole cycle starts over again. Therefore, j^4 can be eliminated as many times as it is contained in an expression, reducing the quantity to j , j^2 , or j^3 and getting its value from the following:

$$j = j = \sqrt{-1}$$

$$j^2 = -1$$

$$j^3 = -j$$

$$j^4 = 1$$

Example 1: Simplify j^{13} .

$$j^{13} = j^{12} \cdot j = j = \sqrt{-1}$$

Example 2: Simplify j^{27} .

$$j^{27} = j^{24} \cdot j^3 = j^3 = -j = -\sqrt{-1}$$

c. Addition and Subtraction of Imaginary Numbers. These numbers may be added or subtracted in the same manner that any algebraic expression is added or subtracted (par. 44). First change the expression to the j form; then treat the j as any other letter in an algebraic expression.

Example 1: Add $\sqrt{-25}$, $\sqrt{-36}$, and $\sqrt{-9}$.

$$\sqrt{-25} + \sqrt{-36} + \sqrt{-9} = j5 + j6 + j3 = j14$$

Example 2: Add $6\sqrt{-2} + 5\sqrt{-8} + 8\sqrt{-18}$.

$$\begin{aligned} 6\sqrt{-2} + 5\sqrt{-8} + 8\sqrt{-18} &= j^2\sqrt{2} + j^2\sqrt{8} + j^2\sqrt{18} \\ &= j^2\sqrt{2} + j(5 \cdot 2)\sqrt{2} + j(8 \cdot 3)\sqrt{2} \\ &= (j^2 + j^{10} + j^{24})\sqrt{2} \\ &= j^{10}\sqrt{2} \end{aligned}$$

Example 3: Subtract $\sqrt{-64}$ from $\sqrt{-36}$.

$$\sqrt{-36} - \sqrt{-64} = j^2 - j^2 = -j^2$$

Example 4: Subtract $4\sqrt{-8}$ from $6\sqrt{-18}$.

$$\begin{aligned} 6\sqrt{-18} - 4\sqrt{-8} &= j(6 \cdot 3)\sqrt{2} - j(4 \cdot 2)\sqrt{2} \\ &= (j^{13} - j^9)\sqrt{2} \\ &= j^{10}\sqrt{2} \end{aligned}$$

d. *Multiplication of Simple Imaginary Numbers.* When multiplying two imaginary numbers, remember that $j^2 = -1$, $j^3 = -j$, and $j^4 = 1$ (b above); then, proceed as with any problem in multiplication (par. 45).

Example 1: Multiply $\sqrt{-16}$ and $\sqrt{-4}$.

$$\sqrt{-16} \cdot \sqrt{-4} = j^4 \cdot j^2 = j^6 = (-1)8 = -8$$

Example 2: Multiply $\sqrt{-81}$, $\sqrt{-25}$, and $\sqrt{-49}$.

$$\sqrt{-81} \cdot \sqrt{-25} \cdot \sqrt{-49} = j^9 \cdot j^5 \cdot j^7 = j^{21} = (-j)315 = -j315$$

e. *Division of Single Imaginary Numbers.* In the division of two simple imaginary numbers, when both the dividend and divisor contain operator j , divide both by j and proceed as with ordinary integers. If a j remains in the denominator, the denominator must be rationalized because the j represents a radical expression. To rationalize, multiply both the numerator and denominator by the imaginary number.

Example 1: Divide $\sqrt{-100}$ by $\sqrt{-16}$.

$$\frac{\sqrt{-100}}{\sqrt{-16}} = \frac{j \cdot 10}{j \cdot 4} = 2\frac{1}{2}$$

Example 2: Divide 12 by $\sqrt{-6}$.

$$\frac{12}{\sqrt{-6}} = \frac{12}{j\sqrt{6}} = \frac{12 \cdot j\sqrt{6}}{j\sqrt{6} \cdot j\sqrt{6}} = \frac{j12\sqrt{6}}{j^2 6} = \frac{j2\sqrt{6}}{-1} = -j2\sqrt{6}$$

Example 3: Divide $\sqrt{-3}$ by $\sqrt{-4}$.

$$\frac{\sqrt{-3}}{\sqrt{-4}} = \frac{j\sqrt{3}}{j^2} = \frac{\sqrt{3}}{2} \text{ or } \frac{1}{2}\sqrt{3}$$

Example 4: Divide 6 by j .

$$\frac{6}{j} = \frac{6}{j} \cdot \frac{j}{j} = \frac{j6}{j^2} = \frac{j6}{-1} = -j6$$

78. Complex Numbers

a. *Operations With Complex Numbers.* A *complex number* is a real number united to an imaginary number by a plus or minus sign; thus, $10 - j5$, $x + jy$, and $R + jx$ are complex numbers. Complex numbers are of great importance in alternating-current electricity in which many problems would be difficult to solve without their use. A complex number expressed in the form $x + jy$ may be considered a bi-

nomial; thus, the addition, subtraction, multiplication, and division of complex numbers are reduced to the corresponding operations with binomials in which one term is real and the other imaginary.

b. *Addition and Subtraction of Complex Numbers.* To add or subtract complex numbers, first combine the real parts, then combine the imaginary parts, and write the results as a binomial with the appropriate sign separating the real and imaginary terms.

Example 1: Add $3 + j5$ and $5 - j$.

$$(3 + j5) + (5 - j) = 3 + j5 + 5 - j \\ = 8 + j4$$

Example 2: Add $6 + \sqrt{-25}$ and $8\sqrt{-16}$.

$$\begin{aligned}(6 + \sqrt{-25}) + (8\sqrt{-16}) &= 6 + j5 + (8 \cdot j4) \\ &= 6 + j5 + j32 \\ &= 6 + j37\end{aligned}$$

Example 3: Add $8 + \sqrt{-12}$ and $9 + \sqrt{-75}$.

$$\begin{aligned}(8 + \sqrt{-12}) + (9 + \sqrt{-75}) &= 8 + j2\sqrt{3} + 9 + j5\sqrt{3} \\ &= 17 + j7\sqrt{3}\end{aligned}$$

Example 4: Subtract $7 - j6$ from $3 - j2$.

$$\begin{aligned}(3 - j2) - (7 - j6) &= 3 - j2 - 7 + j6 \\ &= -4 + j4\end{aligned}$$

Example 5: Subtract $2 - 3\sqrt{-4}$ from $10 + \sqrt{-4}$.

$$\begin{aligned}(10 + \sqrt{-4}) - (2 - 3\sqrt{-4}) &= (10 + j2) - (2 - j6) \\ &= 10 + j2 - 2 + j6 \\ &= 8 + j8 \text{ or } 8(1 + j)\end{aligned}$$

Example 6: Subtract $3 + 7\sqrt{-24}$ from $5 + 3\sqrt{-6}$.

$$\begin{aligned}(5 + 3\sqrt{-6}) - (3 + 7\sqrt{-24}) &= 5 + j3\sqrt{6} - [3 + j(7 \cdot 2)\sqrt{6}] \\ &= 5 + j3\sqrt{6} - 3 - j14\sqrt{6} \\ &= 2 - j11\sqrt{6}\end{aligned}$$

c. Multiplication of Complex Numbers. As in addition and subtraction, when complex numbers are multiplied they are treated as ordinary binomials. Remember, however, that $j^2 = -1$.

Example 1: Multiply $3 - j6$ by $4 + j2$.

$$\begin{array}{r} 3 - j6 \\ 4 + j2 \\ \hline 12 - j24 \\ + j6 - j^2 12 \\ \hline 12 - j18 - j^2 12 \end{array} = j12 - j18 - (-1)(12)$$

$$\begin{aligned} &= 12 - j18 + 12 \\ &= 24 - j18 \end{aligned}$$

Example 2: Multiply $8 - \sqrt{-5}$ by $-2 + \sqrt{-6}$.

$$\begin{array}{r} 8 - j\sqrt{5} \\ -2 + j\sqrt{6} \\ \hline -16 + j2\sqrt{6} + j8\sqrt{6} - j^2\sqrt{30} \end{array} = -16 + j2\sqrt{6} + j8\sqrt{6} - (-1)\sqrt{30}$$

$$\begin{aligned} &= -16 + j2\sqrt{6} + j8\sqrt{6} + \sqrt{30} \\ &= -16 + \sqrt{30} + j(2\sqrt{6} + 8\sqrt{6}) \end{aligned}$$

d. Division of Complex Numbers. When dividing complex numbers, the denominator of the expression in its fractional form must first be rationalized (par. 74). To obtain a real number as a divisor, multiply both the numerator and denominator by the complex number of the denominator with its sign changed (called the *conjugate* of the complex number). In carrying out the multiplication, the radical expression is eliminated. Since $j^2 = -1$, the sign of the coefficient of j^2 is changed; the complex number thus becomes a real number to combine with the other real number in the denominator.

Example 1: Divide $3 + j4$ by $1 + j$.

$$\begin{aligned}\frac{3 + j4}{1 + j} &= \frac{3 + j4}{1 + j} \cdot \frac{1 - j}{1 - j} \\ &= \frac{3 + j - j^2 4}{1 - j^2} \\ &= \frac{3 + j - (1 - 1) 4}{1 - (-1)} \\ &= \frac{3 + j + 4}{2} \\ &= \frac{7}{2} + j \frac{1}{2}\end{aligned}$$

Example 2: Divide 6 by $3 + \sqrt{-2}$.

$$\begin{aligned}\frac{6}{3 + \sqrt{-2}} &= \frac{6}{3 + j\sqrt{2}} \cdot \frac{3 - j\sqrt{2}}{3 - j\sqrt{2}} \\ &= \frac{6(3 - j\sqrt{2})}{(3 + j\sqrt{2})(3 - j\sqrt{2})} \\ &= \frac{18 - j6\sqrt{2}}{9 - j^2 2} \\ &= \frac{18 - j6\sqrt{2}}{11}\end{aligned}$$

79. Review Problems—Imaginary and Complex Numbers

a. Simplify the radical, using operator j .

- (1) $\sqrt{-75}$
- (2) $\sqrt{-23}$
- (3) $-\sqrt{-64ax^5}$
- (4) $-\sqrt{-100x^5y^4}$
- (5) $\sqrt{-\frac{1}{9}}$
- (6) $\sqrt[3]{-128x^5y^4}$

b. Add.

- (1) $-47 + j17$ and $63 + j92$
- (2) $27 - j11$ and $14 - j11$
- (3) $123 - j114$ and $-62 - j137$
- (4) $44 + j17$ and $-j7$
- (5) $6 + j10$ and $j1$
- (6) $14 + j15$ and $-16 - j62$

c. Subtract.

- (1) $-69 + j432$ from $710 + j61$
- (2) $14 - j121$ from $73 - j7$
- (3) $84 - j62$ from $62 - j47$
- (4) $-74 - j20$ from $81 - j81$
- (5) $-87 - j7$ from $82 + j16$
- (6) $-9 + j$ from $-j7$

d. Multiply.

- (1) $4 + \sqrt{-81}$ by $2 + \sqrt{-49}$
- (2) $2 + 2\sqrt{-2}$ by $3 + 3\sqrt{-3}$
- (3) $2 - j3$ by $2 + j3$
- (4) $(2 - j3)^2$
- (5) $(j^4 + j^2 + j^2 + j^4)^2$
- (6) $4 - j7$ by $8 + j2$
- (7) $f + jg$ by $f + jg$
- (8) $I + jE$ by $I - jE$
- (9) $8 - j13$ by $11 - j12$
- (10) $5 + \sqrt{-16}$ by $7 - \sqrt{-81}$

e. Divide.

- (1) 1 by $3 + j2$
- (2) $6 + j$ by j
- (3) $2 + j3$ by $3 - j4$
- (4) $4 + \sqrt{-9}$ by $2 - \sqrt{-1}$
- (5) $x + jy$ by $x - jy$
- (6) 10 by $1 + j2$
- (7) 3 by $1 - j$
- (8) $3 + \sqrt{-25}$ by $4 - \sqrt{-4}$
- (9) $6 - j2$ by $4 - j7$
- (10) $I + jE$ by $I - jE$

Section VIII. EQUATIONS

80. General

An *equation* is a statement of equality between two expressions. For example, $x + y = 12$, $3x + 5 = 20$, and $3 \cdot 9 = 27$ are equations; therefore, all expressions separated by the equality sign are equations, whether the expressions are algebraic or arithmetical. The expression to the left of the equality sign is called the *left-hand member* of the equation; the expression to the right of the equality sign is called the *right-hand member*. Finding the values of the unknown quantities of an algebraic equation is known as solving the equation, and the answer is called the *solution*. If only one unknown is involved, the solution is also called the *root*.

81. Solving Simple Equations

a. Adding Same Quantity to Both Members of Equation. Equal quantities may be added to both sides of an equation without changing the equality.

Example 1: Solve the equation $x - 4 = 7$ for x .

$$\begin{aligned} x - 4 &= 7 \\ x - 4 + 4 &= 7 + 4 \\ x &= 11 \end{aligned}$$

Example 2: Solve the equation $x - 7 = 14$ for x .

$$\begin{aligned} x - 7 &= 14 \\ x - 7 + 7 &= 14 + 7 \\ x &= 21 \end{aligned}$$

b. Subtracting Same Quantity From Both Members of Equation. Equal quantities may be subtracted from both sides of an equation.

Example 1: Solve the equation $x + 2 = 5$ for x .

$$\begin{aligned} x + 2 &= 5 \\ x + 2 - 2 &= 5 - 2 \\ x &= 3 \end{aligned}$$

Example 2: Solve the equation $x + 5 = 12$ for x .

$$\begin{aligned} x + 5 &= 12 \\ x + 5 - 5 &= 12 - 5 \\ x &= 7 \end{aligned}$$

c. Multiplying Both Members of Equation by Same Quantity. Both sides of an equation can be multiplied by the same quantity.

Example 1: Solve the equation $\frac{x}{3} = 5$ for x .

$$\begin{aligned} \frac{x}{3} &= 5 \\ \frac{x}{3} \cdot \frac{3}{1} &= 5 \cdot 3 \\ x &= 15 \end{aligned}$$

Example 2: Solve the equation $\frac{z}{3} + \frac{z}{9} = 4$ for z .

Multiply both sides of the equation by 9.

$$\begin{aligned} 9 \left(\frac{z}{3} + \frac{z}{9} \right) &= 9 \cdot 4 \\ 3z + z &= 36 \\ 4z &= 36 \\ z &= 9 \end{aligned}$$

d. Dividing Both Members of Equation by Same Quantity. Both sides of an equation may be divided by the same quantity.

Example 1: Solve the equation $3x = 12$ for x .

$$\begin{aligned} 3x &= 12 \\ \frac{3x}{3} &= \frac{12}{3} \\ x &= 4 \end{aligned}$$

Example 2: Solve the equation $PV = RT$ for T .

$$\begin{aligned} PV &= RT \\ \frac{PV}{R} &= \frac{RT}{R} \\ T &= \frac{PV}{R} \end{aligned}$$

82. Solving More Difficult Equations

a. Transposition. The process of adding to or subtracting from both members of an equation (par. 81a and b) can be shortened by shifting a term or terms from one side of the equation to the other and changing the signs. This operation is called *transposition*.

Example 1: Solve the equation $6x + 4 = x - 16$ for x .

$$\begin{aligned} 6x + 4 &= x - 16 \\ 6x - x &= -16 - 4 \\ 5x &= -20 \\ x &= -4 \end{aligned}$$

Example 2: Solve the equation $5a - 7 = 2a + 2$ for a .

$$\begin{aligned} 5a - 7 &= 2a + 2 \\ 5a - 2a &= 2 + 7 \\ 3a &= 9 \\ a &= 3 \end{aligned}$$

b. Equations With Fractions. In solving a fractional equation, first find the LCD and multiply both members of the equation, term by term; then perform the operations in paragraph 81 or *a* above.

Example 1: Solve the equation $\frac{x}{2} + \frac{x}{3} = 10$ for x .

$$\begin{aligned} \frac{x}{2} + \frac{x}{3} &= 10 \\ \frac{3x + 2x}{6} &= 10 \\ \frac{5x}{6} &= \frac{10}{1} \\ 5x &= 60 \\ x &= 12 \end{aligned}$$

Example 2: Solve the equation $\frac{x-1}{2} = 3 + x$ for x .

$$\begin{aligned} \frac{x-1}{2} &= 3 + x \\ \frac{x-1}{2} &= \frac{3+x}{1} \\ 1(x-1) &= 2(3+x) \\ x-1 &= 6+2x \\ x-2x &= 6+1 \\ -x &= 7 \\ x &= -7 \end{aligned}$$

Example 1: In simple problems, an equation may be written by an almost direct translation into algebraic symbols; thus,

Seven times a certain voltage diminished by 3
 $\underbrace{7}_{7} \times \underbrace{\text{certain voltage}}_E - \underbrace{3}_3$
 gives the same result as the voltage increased by 75,
 $\underbrace{\hspace{10em}}_{+} \underbrace{\hspace{10em}}_{75}.$

Solving the equation:

$$\begin{aligned} 7E - 3 &= E + 75 \\ 7E - E &= 75 + 3 \\ 6E &= 78 \\ E &= 13 \end{aligned}$$

Example 3: Solve the equation $\frac{2}{x-2} + \frac{2}{x+4} = \frac{4}{x-3}$ for x .

$$\begin{aligned} \frac{2}{x-2} + \frac{2}{x+4} &= \frac{4}{x-3} \\ \frac{2(x+4) + 2(x-2)}{(x-2)(x+4)} &= \frac{4}{x-3} \\ \frac{2x+8+2x-4}{(x-2)(x+4)} &= \frac{4}{x-3} \\ \frac{4x+4}{(x-2)(x+4)} &= \frac{4}{x-3} \\ (4x+4)(x-3) &= 4(x-2)(x+4) \\ 4x^2 - 8x - 12 &= 4(x^2 + 2x - 8) \\ 4x^2 - 8x - 12 &= 4x^2 + 8x - 32 \\ 4x^2 - 4x^2 - 8x - 8x &= -32 + 12 \\ -16x &= -20 \\ 16x &= 20 \\ x &= \frac{20}{16} = \frac{5}{4} = 1\frac{1}{4} \\ x &= 1\frac{1}{4} \end{aligned}$$

83. Written Equations

Many practical problems are stated in words and must be translated into symbols before the rules of algebra can be applied. There are no specific rules for the translation of a written problem into an equation of numbers, signs, and symbols. The following general suggestions may be helpful in developing equations:

a. From the worded statement of the problem, select the unknown quantity (or one of the unknown quantities) and represent it by a letter, such as x . Write the expression, stating exactly what x represents and the units in which it is measured.

b. If there is more than one unknown quantity in the problem, try to represent each unknown in terms of the first unknown.

$$\begin{aligned}\text{Check: } 7(13) - 3 &= 13 + 75 \\ 91 - 3 &= 13 + 75 \\ 88 &= 88\end{aligned}$$

Example 2: A triangle has a perimeter of 30 inches. The longest side is 7 inches longer than the shortest side, and the third side is 5 inches longer than the shortest side. Find the length of the three sides.

Let x = length of shortest side.

$x + 7$ = length of longest side.

$x + 5$ = length of third side.

$$x + (x + 5) + (x + 7) = 30$$

Solving the equation:

$$x + x + 5 + x + 7 = 30$$

$$3x + 12 = 30$$

$$3x = 30 - 12$$

$$3x = 18$$

$$x = 6 = \text{shortest side.}$$

$$6 + 5 = 11 = \text{third side.}$$

$$6 + 7 = 13 = \text{longest side.}$$

84. Simultaneous Equations

a. Definition. Simultaneous equations are two or more equations satisfied by the same sets of values of the unknown quantities. They are used to solve a problem with two or more unknown quantities.

b. Example. Assume that the sum of two numbers is 17, and that three times the first number less two times the second number is equal to 6. What are the numbers? In setting up equations for this problem, let x equal the first number and y equal the second number. The first equation is $x + y = 17$, and the second equation is $3x - 2y = 6$. This problem can be solved in three ways: by substitution, by addition, or by subtraction. All three methods are explained below.

(1) Substitution.

$$x + y = 17 \text{ or } x = 17 - y$$

Substitute $x = 17 - y$ in the second equation:

$$3x - 2y = 6$$

$$3(17 - y) - 2y = 6$$

Remove the parentheses:

$$51 - 3y - 2y = 6$$

Transpose:

$$-5y = 6 - 51$$

$$-5y = -45$$

$$5y = 45$$

$$y = 9$$

Substitute $y = 9$ in the first equation and solve for x :

$$x + y = 17 \text{ or } x + 9 = 17$$

Transpose:

$$x = 17 - 9$$

$$x = 8$$

(2) Addition.

$$x + y = 17$$

$$3x - 2y = 6$$

Before adding, change the y in the first equation to $2y$ so that the y terms drop out when added; thus, the first equation must be multiplied by 2.

$$2x + 2y = 34$$

$$3x - 2y = 6$$

$$5x = 40$$

$$x = 8$$

Substitute $x = 8$ in the first equation and solve for y :

$$x + y = 17 \text{ or } 8 + y = 17$$

$$y = 17 - 8$$

$$y = 9$$

(3) Subtraction.

Before subtracting, multiply the first equation by 3 so that the x terms drop out when subtracted.

$$3x + 3y = 51$$

$$3x - 2y = 6$$

Subtract the second equation from the first equation:

$$\begin{array}{r}
 3x + 3y = 51 \\
 -3x + 2y = -6 \\
 \hline
 5y = 45 \\
 y = 9
 \end{array}$$

Substitute $y = 9$ in the first equation and solve for x : Refer to (1) and (2) above.

c. *Additional Examples.* If the coefficients of the unknowns differ (for example, $3x$ and x and $2y$ and $4y$), multiply one or both equations to establish equal coefficients for one of the unknowns (x or y).

Example 1: Solve for x and y if $3x + 2y = 7$ and $x + 4y = 9$.

$$\begin{array}{r}
 3x + 2y = 7 \\
 x + 4y = 9
 \end{array}$$

Multiply the first equation by 2 so that $2y$ will become $4y$:

$$\begin{array}{r}
 6x + 4y = 14 \\
 x + 4y = 9
 \end{array}$$

Subtract the second equation from the first equation:

$$\begin{array}{r}
 6x + 4y = 14 \\
 -x - 4y = -9 \\
 \hline
 5x = 5 \\
 x = 1
 \end{array}$$

Solve for y by substituting $x = 1$ in either equation.

Example 2: Solve for x and y if $2x + 3y = 24$ and $3x - 4y = 2$.

$$\begin{array}{r}
 2x + 3y = 24 \\
 3x - 4y = 2
 \end{array}$$

Multiply the first equation by 4 to change $3y$ to $12y$; multiply the second equation by 3 to change $4y$ to $12y$; then add the two equations:

$$\begin{array}{r}
 8x + 12y = 96 \\
 9x - 12y = 6 \\
 \hline
 17x = 102 \\
 x = 6
 \end{array}$$

Solve for y by substituting $x = 6$ in either equation.

85. Solving Formulas

a. *The Formula.* A formula is a rule or law that states a scientific relationship. It can be

expressed in an equation by using letters, symbols, and constant terms. For example, a formula in electricity (par. 184) states that the voltage across any part of a circuit is equal to the product of the current and resistance of that part of the circuit. In formula form, this is expressed as $E = IR$, where E is the *voltage or difference in potential* expressed in *volts*, I is the *current* expressed in *amperes*, and R is the *resistance* expressed in *ohms*.

b. *Solving the Formula.* To solve a formula, perform the same operations on both members of an equation until the desired unknown can be isolated in one member of the equation. If the numerical values for some variables are given, substitute in the formula and solve for the unknown as in any other equation.

Example 1: Solve the formula $T =$

$$\frac{12(D - d)}{l} \text{ for } D.$$

$$T = \frac{12(D - d)}{l}$$

$$T = \frac{12D - 12d}{l}$$

Multiply both sides by l :

$$Tl = 12D - 12d$$

Transpose and change signs:

$$12D = Tl + 12d$$

Divide both sides by 12:

$$\frac{12D}{12} = \frac{Tl}{12} + \frac{12d}{12}$$

$$D = \frac{Tl}{12} + d$$

Example 2: Given the formula for electrical power, $P = I^2R$, find the value of P in watts when $I = 15.4$ amperes and $R = 25.7$ ohms.

$$P = I^2R$$

Substituting the given numerical values for I and R :

$$\begin{aligned}
 P &= (15.4)^2 \times 25.7 \\
 &= 237.16 \times 25.7 \\
 &= 6,095 \text{ watts}
 \end{aligned}$$

Example 3: Given the formula for the total resistance of two resistors in parallel,

$$R_T = \frac{R_1 R_2}{R_1 + R_2}, \text{ solve for } R_x \text{ in ohms when}$$

$$R_1 = 40 \text{ ohms and } R_2 = 60 \text{ ohms.}$$

$$R_r = \frac{R_1 R_2}{R_1 + R_2}$$

Substitute the given numerical values for R_1 and R_2 :

$$\begin{aligned} R_r &= \frac{40 \times 60}{40 + 60} \\ &= \frac{2,400}{100} \\ &= 24 \text{ ohms} \end{aligned}$$

86. Review Problems—Equations

a. Solve for the unknown quantity in each of the following:

$$(1) y + 12 = 15$$

$$(2) \frac{n}{8} = \frac{1}{4}$$

$$(3) 0.63s = 53.55$$

$$(4) 47x - 17 = 235 - 37x$$

$$(5) (10m + 6) - (11 - 15m) = 14m + 6m$$

$$(6) \begin{aligned} x + y &= 3 \\ 3x + 2y &= 1 \end{aligned}$$

$$(7) \begin{aligned} a - 3b &= 0 \\ 5a - 4b &= 11 \end{aligned}$$

$$(8) \begin{aligned} 7x - 5y &= 1 \\ 5x + y &= 19 \end{aligned}$$

$$(9) \begin{aligned} 4m - 2n &= 2 \\ 3m + n &= 14 \end{aligned}$$

$$(10) \begin{aligned} 3r - 9s &= 15 \\ 6r - 7s &= 41 \end{aligned}$$

b. Solve the following formulas for the quantity indicated:

$$(1) Fd = Wh \text{ for } d$$

$$(2) v^2 = v_0^2 + 2gh \text{ for } g$$

$$(3) F = \frac{w}{y} a \text{ for } a$$

$$(4) H = \frac{D^2 N}{2.534} \text{ for } N$$

$$(5) F = \frac{22.5 B l}{10^3} \text{ for } l$$

c. Solve the following linear equations for the unknown quantity:

$$(1) 7(2x - 6) - 8 = 10x + 10$$

$$(2) 10(x - 2) - 10(2 - x) = 4x - 40$$

$$(3) 9.8a - 9.4 = 6.8a + .6$$

$$(4) 2x + 3 + \frac{11x - 11}{3} = 22$$

$$(5) 3R + (2R - 4) = 6R - 10(R - 2)$$

$$(6) \frac{5Z}{4} + 2Z = \frac{3 + Z}{3} - 7Z$$

$$(7) -(5x + 15) = 5x + 21 - \frac{5(2 - x)}{2}$$

$$(8) \frac{11y - 13}{25} + \frac{17y + 4}{21} + \frac{19y + 3}{7} = 28\frac{1}{7} + \frac{5y - 25\frac{1}{2}}{4}$$

$$(9) \frac{4\bar{X}_L}{5} - 6X_L + 2 = \frac{X_L}{4}$$

$$(10) (x - 1)(x + 1) + x(1 - x) = 4x(2x + 1) - 8x(x - 2)$$

d. Solve the following sets of simultaneous linear equations:

$$(1) \begin{aligned} 5x - 2y &= 10 \\ 3x - y &= 7 \end{aligned}$$

$$(2) \begin{aligned} 6a + 15b &= 69 \\ 6a - 6b &= 14 \end{aligned}$$

$$(3) \begin{aligned} x - 3y &= -17 \\ 2x + 6y &= 50 \end{aligned}$$

$$(4) \begin{aligned} 6x - 8y &= 20 \\ 3x + 2y &= -14 \end{aligned}$$

$$(5) \begin{aligned} -4x + y &= 13 \\ 8x - 5y &= -29 \end{aligned}$$

$$(6) \begin{aligned} 2I + \frac{2Z - 22}{3} &= 30 \\ \frac{3I - 15}{4} + 6Z &= 108 \end{aligned}$$

$$(7) \begin{aligned} \frac{2}{x} + y &= 1 \\ \frac{1}{x} + 2y &= 1\frac{1}{2} \end{aligned}$$

$$(8) \begin{aligned} \frac{a}{3} + \frac{b}{4} &= 1 \\ \frac{a}{5} + \frac{b}{2} &= -\frac{1}{2} \end{aligned}$$

$$(9) \begin{aligned} \frac{5}{x} + \frac{2}{y} &= -1 \\ \frac{3}{x} + \frac{1}{y} &= 1\frac{1}{2} \end{aligned}$$

$$(10) \begin{aligned} \text{Solve for } r \text{ and } s: \\ (a - b)r + (a + b)s &= a^2 - b^2 \\ (a + b)r - (a - b)s &= 2ab \end{aligned}$$

c. Solve the following problems:

- (1) Three times a voltage (E) diminished by 2 is equal to that voltage. What is the voltage?
- (2) The sum of two resistances in series is R ohms. One resistance is 20 ohms. Give the algebraic expression for the other.
- (3) If a certain voltage (E) is tripled and the result is diminished by 220 volts, the remainder is equal to the original voltage. What is the voltage?
- (4) When two resistors are connected in series, the total resistance (R) is the sum of the two resistances. If one resistor is 25 ohms and the total resist-

ance is 100 ohms, what is the value of the other resistor?

- (5) The current (I) from a battery is divided among three circuits. The first circuit draws 20 milliamperes more than the second circuit, and the second circuit draws 20 milliamperes more than the third circuit. If the total current drawn is 240 milliamperes, what is the current in each circuit?
- (6) Solving by the formula $I = \frac{E}{R}$, how much current (I) does an electric circuit having a resistance (R) of 20 ohms take if the voltage (E) is 110 volts?

Section IX. QUADRATIC EQUATIONS

87. General

A quadratic equation is one which can be reduced to the form $ax^2 + bx + c = 0$ where a , b , and c are known and x is the unknown quantity. In other words, a quadratic equation contains the square of the unknown quantity, such as x^2 , but no higher power. For example, $3x^2 + 5x - 2 = 0$ and $x^2 - 4x + 3 = 0$ are quadratic equations. The form $ax^2 + bx + c = 0$ is called the *general quadratic equation*.

88. Pure Quadratic Equations

A pure quadratic equation is obtained from the general quadratic equation when b is equal to zero and the middle term (bx) does not appear. The equation then becomes $ax^2 + c = 0$. The pure quadratic equation has two roots that are equal in absolute value but have opposite signs. As discussed in paragraph 49, all numbers have two square roots. The equation $x^2 - 36 = 0$ is a pure quadratic equation since there are two numbers which, when substituted for x , will satisfy the equation. Thus $(+6)^2 - 36 = 0$ since $36 - 36 = 0$; also, $(-6)^2 - 36 = 0$ since $36 - 36 = 0$. Therefore, $x = \pm 6$.

Example: Solve the equation $x^2 - 5 = 20$ for x .

$$\begin{aligned}x^2 - 5 &= 20 \\x^2 &= 25 \\x &= \pm 5\end{aligned}$$

Check:

$$\begin{aligned}(\pm 5)^2 - 5 &= 20 \\25 - 5 &= 20 \\20 &= 20\end{aligned}$$

89. Solution by Factoring

a. Quadratic equations are found in many applications of even the simplest nature. For example, suppose that a sheet of metal is to be cut so that it has an area of 30 square inches, and that the length of the piece will be 1 inch longer than the width. With x representing the unknown width and $x + 1$ the unknown length, $x(x + 1)$ equals the area; therefore, the equation that must be satisfied is $x(x + 1) = 30$. By performing the indicated multiplication and subtracting 30 from each side, the equation now can be written in the form of a quadratic equation, as $x^2 + x - 30 = 0$.

b. To solve this equation, factor the left-hand side into the equivalent equation: $(x - 5)(x + 6) = 0$. The product of two factors is zero if either of the factors is zero (par. 53). Thus, each factor is set equal to zero and solved for the unknown. The equation is satisfied if $x - 5 = 0$ or $x = 5$. Note that the equation also is satisfied if $x + 6 = 0$. This illustrates an important fact concerning quadratic equations: *Every quadratic equation has two solutions.* Only one solution, however, may be appropriate when quadratic equations are used to solve

actual problems. The quadratic equation only gives two *possible* solutions—the *actual* solution must be determined by referring to the facts in the original problem.

Example 1: Solve the equation $x^2 - 2x = 0$ for x .

$$x^2 - 2x = 0$$

Factoring:

$$x(x - 2) = 0$$

$$x = 0$$

or $x - 2 = 0$

$$x = 2$$

Thus, 0 or 2 are the roots of the equation $x^2 - 2x = 0$.

Example 2: Solve the equation $2x^2 - 3x - 5 = 0$ for x .

$$2x^2 - 3x - 5 = 0$$

Factoring:

$$(2x - 5)(x + 1) = 0$$

so $x + 1 = 0$

and $x = -1$

or $2x - 5 = 0$

$$2x = 5$$

and $x = \frac{5}{2}$ or $2\frac{1}{2}$

Thus, -1 and $2\frac{1}{2}$ are the roots of the equation $2x^2 - 3x - 5 = 0$.

90. Solution by Completing the Square

In solving quadratic equations, the method of factoring described in paragraph 89 usually is best if the factors are immediately apparent by inspection. When the values of the unknown are not whole numbers or rational fractions, a quadratic equation can be solved more easily by the method of *completing the square*. This method also is used to derive the quadratic formula (par. 91). For example, to solve the equation $2x^2 - x - 2 = 0$ by completing the square, proceed as follows:

a. Transpose all terms involving x to the left-hand side of the equation and all other terms to the right-hand side. The equation is now in the form $2x^2 - x = 2$, or $x^2 - \frac{1}{2}x = 1$. When using this method, the coefficient of the squared term must be unity (one).

b. Add a number to both sides of the equation so that the left-hand side will be a perfect

trinomial square. To determine this number, divide the coefficient of the middle term ($-\frac{1}{2}$) by 2 and square the resulting number.

$$x^2 - \frac{1}{2}x = 1$$

$$x^2 - \frac{1}{2}x + \frac{1}{16} = 1 + \frac{1}{16}$$

c. Replace the trinomial square on the left-hand side of the equation with the square of a binomial.

$$(x - \frac{1}{4})^2 = \frac{17}{16}$$

d. Extract the square root of both sides of the equation.

$$x - \frac{1}{4} = \frac{\pm\sqrt{17}}{4}$$

Thus, $x = \frac{1 \pm \sqrt{17}}{4}$

91. The General Quadratic Equation

a. *General.* Another method of solving quadratic equations consists of substitution in a formula derived from the general quadratic equation (b below). The general quadratic equation is in the form $ax^2 + bx + c = 0$, and any quadratic equation can be written in this form (par. 87). Thus, in the equation $2x^2 + 5x - 3 = 0$, $a = 2$, $b = 5$, and $c = -3$. Similarly, in the equation $9x^2 - 25 = 0$, $a = 9$, $b = 0$, and $c = -25$.

b. *Deriving Formula for Solving any Quadratic Equation.* Since the general quadratic equation, $ax^2 + bx + c = 0$, represents any quadratic equation, the roots of this equation will represent the roots of any quadratic equation; then, if the general quadratic equation is solved for the unknown values, the roots obtained will serve as a formula for finding the roots of any quadratic equation. The formula is derived from the general form by the method of completing the square; thus, given the general equation $ax^2 + bx + c = 0$, proceed as follows:

(1) Divide through by the coefficient a .

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

(2) Subtract the term $\frac{c}{a}$ from both sides of the equation.

$$x^2 + \frac{bx}{a} = -\frac{c}{a}$$

This operation prepares the equation for the addition of a quantity to both sides of the equation that will make the left-hand side a perfect square. This quantity is obtained by dividing the coefficient of the x term by 2, and squaring the quotient. Since the coefficient of the x term is $\frac{b}{a}$, the quantity to be added to both sides of the equation is $(\frac{b}{2a})^2$, or $\frac{b^2}{4a^2}$.

- (3) Add $\frac{b^2}{4a^2}$ to both sides of the equation.

$$x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

- (4) Factor the left-hand side of the equation, and add the fraction on the right-hand side.

$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

- (5) Take the square root of both sides of the equation.

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

- (6) Subtract $\frac{b}{2a}$ from both sides of the equation.

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

- (7) Collect the terms on the right-hand side of the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This equation is known as the *quadratic formula*. The two roots of any quadratic equation can be obtained by substituting in the formula the particular values of a , b , and c .

92. Solution by the Quadratic Formula

In practical problems, pure quadratic equations (par. 88) are seldom found, and solution

by factoring (par. 89) can be used only occasionally. However, any quadratic equation can be solved by the method of completing the square (par. 90)—the method used to derive the quadratic formula (par. 91). This method is unnecessary, however, when the values for a , b , and c for any quadratic equation can be substituted in the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 1: Solve the equation $2x^2 - 6x + 3 = 0$ by using the quadratic formula.

$$2x^2 - 6x + 3 = 0$$

$$a = 2; b = -6; c = 3$$

Substituting in the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{36 - (4)(2)(3)}}{4}$$

$$= \frac{6 \pm \sqrt{12}}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

$$\text{Thus, } x = \frac{3 + \sqrt{3}}{2} \text{ or } x = \frac{3 - \sqrt{3}}{2}.$$

$$\text{Check: } x = \frac{3 + \sqrt{3}}{2}$$

$$x = \frac{3 + 1.732}{2} = 2.366$$

Substituting in the equation:

$$2(2.366)^2 - 6(2.366) + 3 = 0$$

$$11.20 - 14.20 + 3 = 0$$

$$14.20 - 14.20 = 0$$

$$x = \frac{3 - \sqrt{3}}{2}$$

$$x = \frac{3 - 1.732}{2} = .634$$

Substituting in the equation:

$$2(.634)^2 - 6(.634) + 3 = 0$$

$$2(.40) - 3.80 + 3 = 0$$

$$3.80 - 3.80 = 0$$

Example 2: Solve the equation $3x^2 + 5x - 2 = 0$ by using the quadratic formula.

$$3x^2 + 5x - 2 = 0$$

$$a = 3; b = 5; c = -2$$

Substituting in the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{25 - (4)(3)(-2)}}{(2)(3)}$$

$$= \frac{-5 \pm 7}{6}$$

$$\text{Thus, } x = \frac{1}{3} \text{ or } x = -2.$$

$$\text{Check: } x = \frac{1}{3}$$

Substituting in the equation:

$$3\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) - 2 = 0$$

$$\frac{3}{9} + \frac{5}{3} - 2 = 0$$

$$\frac{1}{3} + \frac{5}{3} - 2 = 0$$

$$\frac{1}{3} + \frac{5}{3} - \frac{6}{3} = 0$$

$$\frac{6}{3} - \frac{6}{3} = 0$$

$$x = -2$$

Substituting in the equation:

$$3(-2)^2 + 5(-2) - 2 = 0$$

$$12 - 10 - 2 = 0$$

$$12 - 12 = 0$$

93. Character of the Roots

a. The values for unknowns that are not

whole numbers or rational fractions are called *irrational roots*. A *rational* number is a number which can be expressed as the ratio of two integers. For example, 9, $\frac{7}{3}$, $\frac{1}{8}$, and $\sqrt{16}$ are rational numbers. Any whole number is rational since it is the quotient of itself and unity; thus, $9 = \frac{9}{1}$. Numbers such as $\frac{7}{3}$ and $\frac{1}{8}$ are often referred to as rational fractions. A radical is rational if it can be expressed as the quotient of two whole numbers. Thus $\sqrt{16}$ is rational since $\sqrt{16} = 4 = \frac{4}{1}$. A number such as $\sqrt{3}$ which cannot be written as the ratio of two whole numbers is called irrational. Rational and irrational numbers, taken together, make up the system of real numbers. Any number, such as $3 + \sqrt{3}$, which contains a radical sign that cannot be removed also is considered irrational. Roots of quadratic equations are real if a minus sign does not occur under a radical. For example, $x = 5$ is a real root—roots such as $x = \frac{3 + \sqrt{8}}{2}$ or $x = \frac{3 - \sqrt{8}}{2}$ are real, but irrational.

b. One important fact to be remembered when using the quadratic formula is that the expression under the radical sign, $b^2 - 4ac$, must be regarded as a whole before the square root can be taken. The quantity $b^2 - 4ac$ is called the *discriminant* of the quadratic equation. Many things can be learned about a quadratic equation merely by inspecting the discriminant. If the value of the discriminant is positive, real roots will be obtained when the equation is solved. These roots are either rational or irrational—rational when the discriminant is a perfect square, irrational when it is not. The roots are equal only when the value of $b^2 - 4ac$ is zero. When $b^2 - 4ac$ is negative, the square root will be that of a negative number and the roots will be imaginary.

c. In summary, a quadratic equation always has two solutions. The solutions will be:

Real and equal.....if $b^2 - 4ac$ equals 0.

Unequal but real.....if $b^2 - 4ac$ is positive.

Real and rational.....if $b^2 - 4ac$ is a perfect square.

Imaginary.....if $b^2 - 4ac$ is negative.

94. Review Problems—Quadratic Equations

a. Solve by factoring.

(1) $2x^2 + 3x = 0$

(2) $(x - 4)x = 0$

(3) $(x + 3)\frac{x}{3} = 0$

(4) $\frac{1}{2}x^2 + \frac{1}{2}x = 0$

(5) $2x^2 - 128 = 0$

(6) $\frac{1}{2}x^2 - 2 = 1$

(7) $3x^2 - 25 = 2$

(8) $3x(x - 2) + 2x(3 - x) = 16$

(9) $x^2 - x - 42 = 0$

(10) $x^2 - 13x + 12 = 0$

b. Solve by completing the square.

(1) $x^2 + 3x - 1 = 0$

(2) $y^2 + 6y - 10 = 0$

(3) $E^2 - 4E + 1 = 0$

(4) $2E^2 + 8E - 3 = 0$

(5) $8H^2 - 8H = 5$

(6) $5L^2 - 5 = 2L^2 - 10L$

(7) $14r^2 - 28r - 42 = 0$

(8) $\frac{1}{v^2} - \frac{4}{v} = 2$

(9) $y^2 - 5 = 2y$

(10) $8x^2 - 8x = 8$

c. Solve by using the quadratic formula.

(1) $a^2 + 2a + 1 = 0$

(2) $12y^2 - 6 + y = 0$

(3) $0 = 1 + 5E + 3E^2$

(4) $6I^2 + I - 12 = 0$

(5) $2c^2 + 4c - 6 = 0$

(6) $15R^2 = 22R + 5$

(7) $\frac{Z - 2}{Z} = 1 - Z$

(8) $\frac{3}{r - 2} = 1 + \frac{2}{r + 3}$

(9) $\frac{3x + 2}{2x + 4} = \frac{x + 2}{2x}$

(10) $0 = 6 - \frac{b - 2}{b + 2} + \frac{b - 1}{b + 1}$

CHAPTER 6

GRAPHS

Section I. BASIC CHARACTERISTICS OF GRAPHS

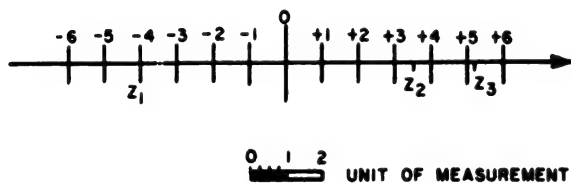
95. General

A graph is a pictorial representation of the relation between two or more quantities. In many instances, problems are more clearly understood when solved graphically than when solved by other methods. Numerical data taken from an experiment or calculations derived from a formula require interpretation, and a curve on a graph depicting such data will provide a picture that shows at a glance how one factor or function depends on another.

96. The Number Line

a. In figure 15, on a straight line of indeterminate length, a point 0 has been chosen from which to measure distances. The point 0 is called the origin. A unit of measurement also has been chosen, and positive and negative integers have been marked off and labeled. The usual choice for a positive direction is shown by the arrow. On the number line, Z_1 corresponds to -4 , Z_2 corresponds to $3\frac{1}{2}$, and Z_3 corresponds to 5.2 .

b. Consider a number x as corresponding to a point a distance of x units from 0. If x is positive, the point will be in the direction of the arrow from 0; if x is negative, the point will be in the opposite direction from 0. The relative size of two numbers is indicated graphically by the relative positions on the number



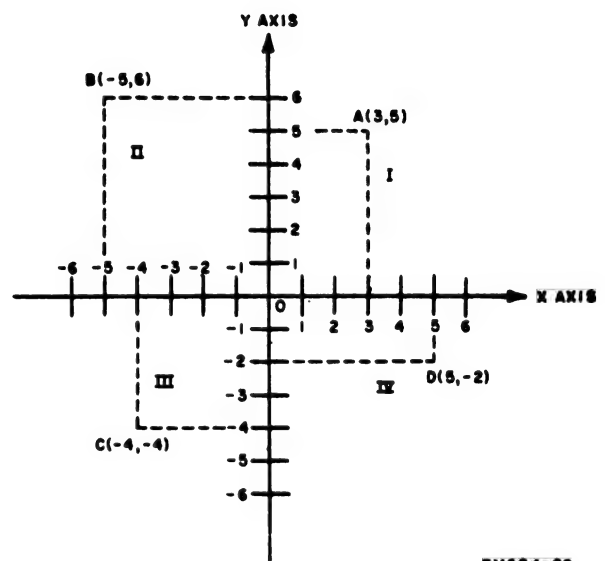
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Figure 15. The number line.

line of points corresponding to the two numbers. For example, if x is greater than w , the point corresponding to x will be to the right of the point corresponding to w ; if x is less than w , the point corresponding to x will be to the left of the point corresponding to w . The number of units from the origin to the point representing a certain number, regardless of direction, is the absolute value (par. 35) of the number.

97. Rectangular Coordinates

a. In the preceding paragraph, a relationship was given between numbers and points on a straight line. A similar relationship can be established between a pair of numbers and a point on a plane. In figure 16, two number lines are drawn perpendicular to each other at their origins for form a set of axes. The horizontal axis is commonly called the x axis;



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Figure 16. Rectangular coordinates.

the vertical axis is commonly called the y axis. Any point on the plane can be located with reference to the two axes: It must lie a certain number of units to the left (negative) or to the right (positive) of the y axis; and it must lie a certain number of units above (positive) or below (negative) the x -axis. To locate a point with reference to the set of axes, it is necessary only to know the x value and the y value of the point. These two values are known as the *coordinates* of the point. The x value, called the *abscissa*, is written first; the y value, called the *ordinate*, follows. The two numbers are separated by a comma and are usually enclosed in parentheses. Thus, in figure 16, the correct notation for the coordinates at point A is (3,5), because the x value is 3 and the y value 5.

b. The axes divide the graph into four sections, or *quadrants*, identified by the Roman numerals I, II, III, and IV in figure 16. The signs of the abscissa and the ordinate in each of the quadrants are given in the chart below.

Quadrant	Abscissa	Ordinate
I	+	+
II	—	+
III	—	—
IV	+	—

98. Plotting Points

The procedure for locating points by their coordinates is called *plotting* the points. To plot the point D (5, —2) in figure 16, for example, erect a perpendicular on the x axis five units to the right of the y axis; then erect a perpendicular to the y axis two units below the

x axis; the point of intersection of these two perpendiculars is the point D (5,—2).

99. Review Problems—Plotting Points

a. Plot each of the following points and state the quadrant, if any, in which each lies:

- (1) (4,2)
- (2) (4,—2)
- (3) (—1,3)
- (4) (6,—1)
- (5) (3,0)
- (6) (0,—3)
- (7) (—15,—27)
- (8) ($3\frac{1}{2}$, $4\frac{1}{2}$)
- (9) (5.6,—6.5)

b. Plot the points in the following chart and connect them by straight segments in the order of increasing values of x :

x	—3	—2	—1	0	1	2	3	4
y	18	8	2	0	2	8	18	32

c. Plot the points in the following chart and sketch a smooth curve passing through them in the order of increasing values of x :

x	—3	—2	—1	0	1	2	3
y	—37	—8	5	8	7	7	17

d. If $y = 2x - 3$, plot the points for which $x = 4, 2, 1, 0, -1, -2$, and -4 after finding the corresponding values of y .

e. Draw the triangle of which the vertices are (—2,6), (3,2), and (0,—3).

f. Draw the quadrilateral of which the vertices, connected in the order given, are (1,3), (—3,4), (—2,—5), and (3,—2).

Section II. GRAPHING EQUATIONS

100. Graphing Linear Equations

a. *General.* An equation in the first degree in two unknowns is called a *linear equation* since its graph is a straight line. For example, $x + y = 5$, $2x + y = 12$, and $x - 6y = 6$ are linear equations. An equation is said to be of the first degree in two unknowns if only the first power of either unknown is involved and

if neither of the unknowns appears in a denominator.

b. *Plotting Graphs of Linear Equations.*

- (1) The first step in plotting the graph of a linear equation (or of any other equation or formula) is to set up a table of values for both unknowns that will satisfy the equation. In the equa-

tion $x + y = 5$, for example, it is apparent that there are a number of values for x and y that will satisfy the equation. For any number assigned to x , there is a corresponding number for y which will satisfy the equation. Consider that 4 and -4 will be the maximum plus and minus values for x . Using the values 4, 3, 2, 1, 0, -1 , -2 , -3 , and -4 for x , the equation is solved for y at each value of x . These are arranged in tabular form as shown on figure 17.

- (2) Each of these pairs of values gives a point on a graph. Consider each of the corresponding points as coordinates—the value of x the abscissa and the value of y the ordinate. The line joining these points (fig. 17) is the graph of the equation $x + y = 5$. Note that the coordinates for any two points are sufficient to determine its graph. Therefore, plotting the coordinates for any two points is sufficient to determine the graph of a first degree equation. Plotting a third point, however, will serve as a check, for if the three points are not on the same straight line, one of them is in error.

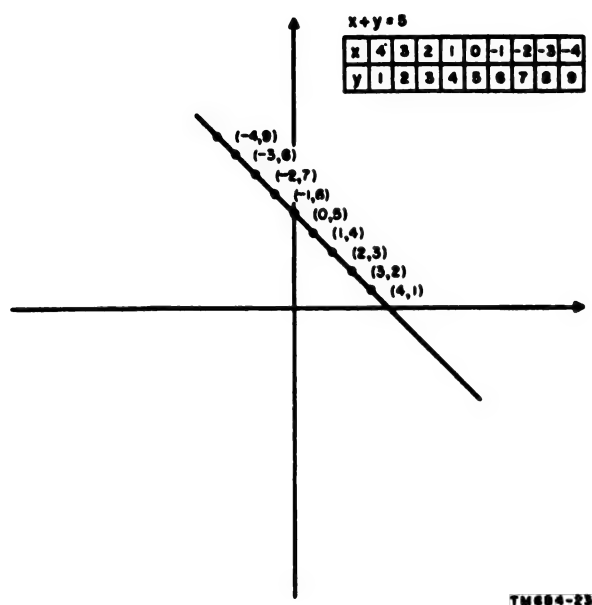


Figure 17. Graph of linear equation.

101. Graphical Solution of Simultaneous Linear Equations

a. When two *independent* linear equations contain the same two related unknowns, there will be an unlimited number of solutions for each equation. However, *there can be only one set of values that will satisfy both equations*. Determining the one set of values is known as the simultaneous solution of the two independent equations.

b. Graphically, the two equations can be solved simultaneously by plotting them on the same graph and locating their point of intersection (if there is one). For example, consider the graphical solution of the equations $3x - 2y = 0$ and $3x + 2y = 6$. Selecting 6 and -6 as the maximum plus and minus values for x and using $x = 4$ as a checkpoint, the coordinates for both equations are determined. For the equation $3x - 2y = 0$, these coordinates are (6, 9), (4, 6), and $(-6, -9)$; for the equation $3x + 2y = 6$, (6, -6), (4, -3), and $(-6, 12)$. These coordinates are plotted on an axis and a line is drawn joining the plotted points of each equation (fig. 18). The graphs of the two independent linear equation cross at point P, where $x = 1$ and $y = 1.5$. To check the graphical solution of the equations, substitute these values for x and y in the original equations. Since they satisfy both equation, the graphical solution is correct.

c. If two *dependent* equations are plotted on a graph, their lines will coincide. For example, the equations $x + y = 4$ and $2x + 2y = 8$

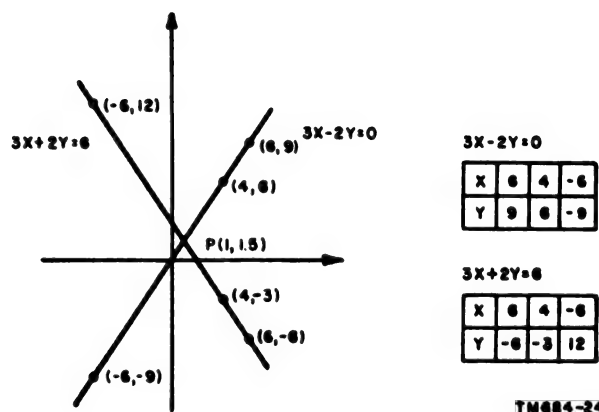


Figure 18. Graphical solution of simultaneous linear equations.

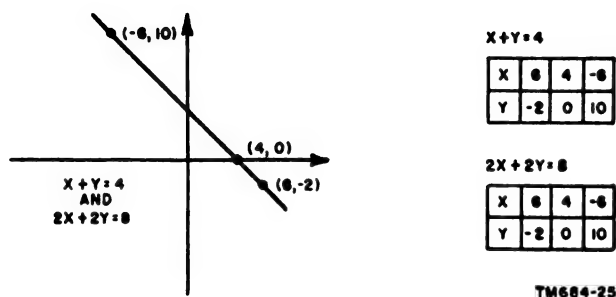


Figure 19. Graph of dependent simultaneous linear equations.

are dependent, since they can be reduced to identical forms. Selecting the same plus and minus values for x and the same checkpoint as in *b* above, the coordinates for both equations are found to be $(6, -2)$, $(4, 0)$, and $(-6, 10)$. Plotted on a graph, both equations form a single line (fig. 19).

d. Simultaneous equations that have no common solution are called *inconsistent*. No solution is possible for the equations $x + y = 3$ and $x + y = 5$, because there are no values for x and y which, when added together to make 3, will also equal 5. Using 6 and -6 as maximum plus and minus values for x , and using $x = 4$ as a checkpoint, the coordinates for equation $x + y = 3$ are found to be $(6, -3)$, $(4, -1)$, and $(-6, 9)$; the coordinates for $x + y = 5$ are $(6, -1)$, $(4, 1)$, and $(-6, 11)$. Plotted on a graph, these equations form parallel lines (fig. 20).

102. Graphing Quadratic Equations

a. *The Dependent Variable.* In graphing a quadratic equation, only two values, or points, for plotting the equation can be obtained by finding the roots of the equation (par. 88). These values do not give a complete picture of the equation. To get a continuous graph, a *dependent variable* is introduced. This variable, usually identified by the letter y , gets its name from the fact that it depends on another quantity for its value. For example, in the equation $y = x^2 - 6x + 5$, the value of y depends on the value of x ; therefore, y is a dependent variable. The quantity on which y depends is called the *independent variable*. A more accurate designation for the dependent variable is $f(x)$, meaning *function of x* . Using

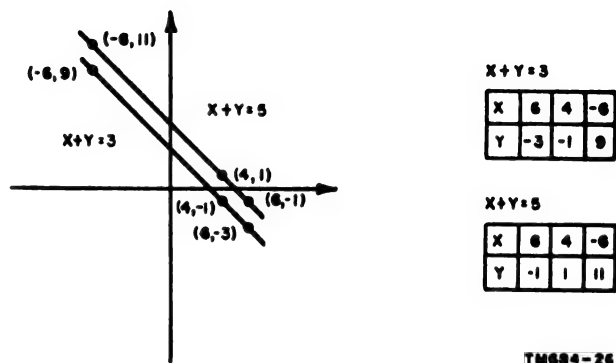


Figure 20. Graph of inconsistent simultaneous linear equations.

this designation, the equation given above would be written $f(x) = x^2 - 6x + 5$. If the independent variable in the equation were z , the equation would be written $f(z) = z^2 - 6z + 5$.

b. *Graphical Solution of Quadratic Equations.* In the original equation $f(x) = x^2 - 6x + 5$, different values are substituted for the unknown to find the corresponding values of the function; thus if x equals -1 , the equation becomes $f(-1) = (-1)^2 - 6(-1) + 5 = 12$; if x equals zero, the equation becomes $f(0) = 0 - 0 + 5 = 5$; if x equals 1, the equation becomes $f(1) = (1)^2 - 6(1) + 5 = 0$, etc. Compile a table of enough values to make it possible to plot the equation, as shown in figure 21. The graph of the function crosses the x -axis at two points, 1 and 5, which give a graphical solution of the equation $x^2 - 6x + 5 = 0$. The equation also may be solved by factoring, as follows:

$$\begin{aligned}(x-1)(x-5) &= 0 \\ x-1 &= 0 \text{ and } x-5 = 0 \\ x &= 1 \text{ and } x = 5\end{aligned}$$

Thus, the solutions or the roots of the equation are obtained when $f(x) = 0$. These roots represent the points where the graph of $f(x) = x^2 - 6x + 5$ crosses the x -axis.

c. *Properties of Functions.* In addition to the original equation, $f(x) = x^2 - 6x + 5$, consider three equations that differ in one respect—their constant terms are not the same. For example:

$$\begin{aligned}f(x) &= x^2 - 6x + 8 \\ f(x) &= x^2 - 6x + 9 \\ f(x) &= x^2 - 6x + 12\end{aligned}$$

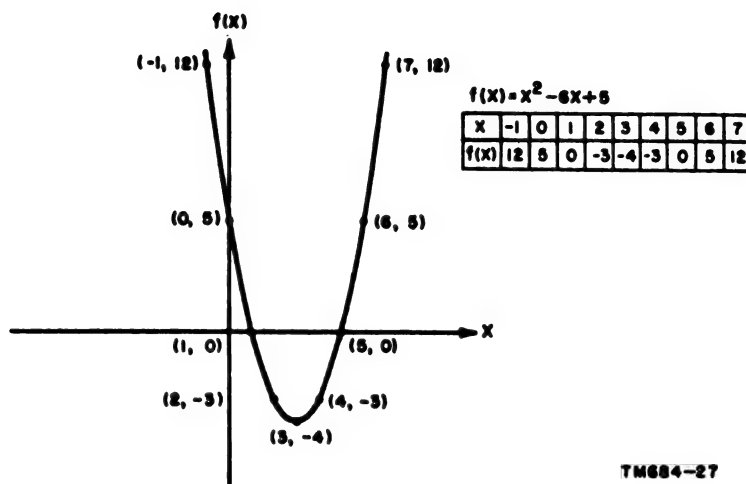


Figure 21. Graph of function of quadratic equation.

The graphs of the four corresponding functions have interesting properties and can be studied more advantageously when plotted on the same graph, as shown in figure 22.

- (1) The function of $x^2 - 6x + 5$ crosses the horizontal or x -axis at two points, 1 and 5. These points indicate that the roots of the equation are, $x = 1$ and $x = 5$. To compare this information with the discussion on quadratic equations in chapter 5, the discriminant of the equation must be investigated. The discriminant of $x^2 - 6x + 5$ is $(b^2 - 4ac) = (36 - 4 \cdot 1 \cdot 5) = 36 - 20 = 16$. Referring to the summary of the character of roots in paragraph 93, the roots are real and rational. To prove this, substitute the value of the discriminant in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{16}}{2}$$

$$x = \frac{6 + 4}{2} = 5 \text{ or } \frac{6 - 4}{2} = 1$$

Thus, the discriminant is a perfect square and the roots are real and rational.

- (2) The function of $x^2 - 6x + 8$ crosses the horizontal axis at 2 and 4, indicating that the roots are $x = 2$ and $x = 4$. Calculating the discriminant,

$(b^2 - 4ac) = (36 - 4 \cdot 2 \cdot 2) = 36 - 32 = 4$. Thus, the discriminant is a perfect square and will give real and rational roots.

- (3) The function of $x^2 - 6x + 9$ touches the x -axis at only one point, 3. Thus, both roots of the equation are $x = 3$. Calculating the discriminant, $(b^2 - 4ac) = (36 - 4 \cdot 9) = 0$, which indicates that the roots are real and equal. Check the graph of this equation (fig. 22); it will be seen that the curve just touches the x -axis at one point. Thus, the root $x = 3$ must be counted twice and may be called a double root.
- (4) The equation $f(x) = x^2 - 6x + 12$ has a discriminant equal to $(36 - 4 \cdot 12) = -12$. Solving for the roots of this equation,

$$x = \frac{6 \pm \sqrt{-12}}{2} = 3 \pm \sqrt{-3}.$$

This is imaginary, but the meaning becomes apparent when the graph of the function of the equation is inspected. The plot does not cross the x -axis and, therefore, both roots must be imaginary.

d. Minimum Value of a Quadratic.

- (1) The minimum value of a quadratic function will occur at $x = \frac{-b}{2a}$ when

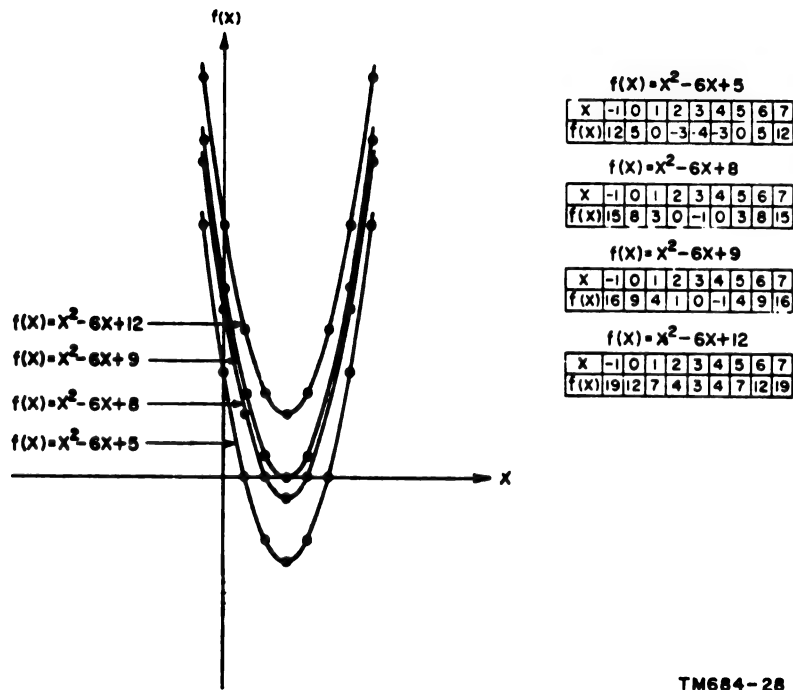


Figure 22. Properties of functions.

the general quadratic equation $ax^2 + bx + c = y$ (par. 91) defines the coefficients a and b . This relation can be checked by calculating the value of x at which the minimum value of the function $x^2 - 6x + 5$ occurs and comparing this calculated value with the plot of the equation (fig. 21 or 22). Thus,

$$x = \frac{-b}{2a} = -\frac{(-6)}{2(1)} = \frac{6}{2} = 3,$$

and the minimum value of the function $x^2 - 6x + 5$ occurs at $x = 3$. Checking the graph verifies this statement. The minimum value of the functions $x^2 - 6x + 8$, $x^2 - 6x + 9$, and $x^2 - 6x + 12$ also occurs at $x = 3$.

- (2) To find the value of the function at the minimum point, substitute for x . The minimum occurs at $x = \frac{-b}{2a}$;

therefore, substitute $\frac{-b}{2a}$ for x in the function of the general quadratic equation.

$$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a\left(\frac{-b}{2a}\right)^2 + b\left(\frac{-b}{2a}\right) + c \\ &= \frac{b^2}{4a} - \frac{b^2}{2a} + c = \frac{b^2}{4a} - \frac{2b^2}{4a} + c \\ &= \frac{-b^2}{4a} + c \end{aligned}$$

Thus, to find the value of the function $f(x) = x^2 - 6x + 5$ at the minimum point:

$$\begin{aligned} f(x) &= \frac{-b^2}{4a} + c = \frac{-36}{4} + 5 = \\ &= -9 + 5 = -4 \end{aligned}$$

This method can be used to find the minimum value of the function if the value of x at which the minimum occurs is *not* known. However if it is known that the minimum value occurs at $x = 3$, merely substitute this value for x in the original equation.

$$\begin{aligned} f(x) &= x^2 - 6x + 5 \\ &= 9 - 6 \cdot 3 + 5 \\ &= 14 - 18 \\ f(x)_{\min} &= -4 \end{aligned}$$

- (3) Note that in all cases where the word *minimum* is used, the word *maximum* is applicable if the equation $y = f(x)$ is such that its graph has a maximum instead of a minimum. If the equation were $f(x) = 3 + 6x - x^2$, the minus sign preceding the term x^2 would indicate that the curve has a maximum.

e. Practical Application. The methods of analysis presented in *c* and *d* above can be used for some very important relationships in applied electricity and electronics. It may be used, for example, to find the load resistance of a circuit in terms of the circuit components necessary to obtain maximum power transfer (par. 216).

103. Review Problems—Graphs

a. Plot the graphs of the following linear equations:

- (1) $2x - 5 = y$
- (2) $5 - 2x = y$
- (3) $y = 5x$
- (4) $3x + 2y = 18$
- (5) $5x - 5y = 20$
- (6) $3x + y + 14 = 0$

b. Plot the graphs of the following sets of simultaneous equations:

- (1) $2x + 3y = 12$
 $3x - y = 7$
- (2) $x + y = 9$
 $5x + y = 17$
- (3) $x + 5y = 22$
 $3x - 2y = -2$
- (4) $3x - 2y = 0$
 $x - 5y = 13$
- (5) $6x + 2y = 12$
 $4y + 2y = 10$
- (6) $x - 2y = 0$
 $y = 1 + x$

c. Find the roots of the following quadratic equations to the nearest tenth by plotting their graphs:

- (1) $y^2 - 2y - 2 = 0$
- (2) $x^2 - 1 + x = 0$
- (3) $9 - t^2$
- (4) $x^2 - 2x + 2 = 0$
- (5) $x^2 - 5x + 3 = 0$
- (6) $10 - 3x - x^2 = 0$

CHAPTER 7

POWERS OF 10

104. General

The technique of using powers of 10 can greatly simplify mathematical calculations. A number containing many zeros to the right or to the left of the decimal point can be dealt with much more readily when put in the form of powers of 10. For example, $.0000037 \times .000021$ can be handled more easily when put in the form $3.7 \times 10^{-5} \times 2.1 \times 10^{-5}$.

105. Table of Powers of 10

The table below gives some of the values of the powers of 10. In a whole number, the exponent is positive and equals the number of zeros following the 1; in decimals, the exponent is negative and equals one more than the number of zeros immediately following the decimal point.

Number	Power of 10	Number	Power of 10
.000001	10^{-6}	1	10^0
.00001	10^{-5}	10	10^1
.0001	10^{-4}	100	10^2
.001	10^{-3}	1,000	10^3
.01	10^{-2}	10,000	10^4
.1	10^{-1}	100,000	10^5
		1,000,000	10^6

106. Expressing Numbers in Scientific Notation

Any number written as the product of an integral power of 10 and a number between 1 and 10 is said to be expressed in *scientific notation*.

Example 1: $81,000,000 = 8.1 \times 10,000,000 = 8.1 \times 10^7$

Example 2: $600,000,000 = 6 \times 100,000,000 = 6 \times 10^8$

Example 3: $.000,000,000,9 = 9 \times .000,000,000,1 = 9 \times 10^{-10}$

107. Addition and Subtraction of Numbers in Scientific Notation

Numbers expressed in scientific notation can only be added or subtracted if the powers of 10 are the same. For example, 3×10^5 can be added to 2×10^5 to get 5×10^5 ; however, 3×10^5 cannot be added to 2×10^6 because the powers of 10 are not the same. The number 3×10^5 can be changed to 30×10^4 , however, and it can then be added to 2×10^5 to obtain 32×10^4 . The answers to problems solved by using scientific notation can be left in the exponential form. In the examples below, however, the answers are converted to the decimal form to aid in understanding this technique.

Example 1: Add 450,000 and 763,000.

$$\begin{aligned} 450,000 + 763,000 &= 45 \times 10^4 + 76.3 \times 10^4 \\ &= 121.3 \times 10^4 \\ &= 1,213,000 \end{aligned}$$

Example 2: Add .000,068,25 and .000,007,54.

$$\begin{aligned} .000,068,25 + .000,007,54 &= 6825 \times 10^{-8} + 754 \times 10^{-8} \\ &= 7579 \times 10^{-8} \\ &= .000,075,79 \end{aligned}$$

Example 3: Subtract .000,004,33 from .000,05.

$$\begin{aligned} .000,05 - .000,004,33 &= 5000 \times 10^{-8} - 433 \times 10^{-8} \\ &= 4567 \times 10^{-8} \\ &= .000,045,67 \end{aligned}$$

108. Multiplication of Numbers in Scientific Notation

The general rules covering the multiplication of radicals (par. 74) also apply in the multiplication of numbers that are expressed in scientific notation.

Example 1: Multiply 100,000 by 1,000.

$$100,000 \times 1,000 = 10^5 \times 10^3 = 10^{5+3} = 10^8 = 100,000,000$$

Example 2: Multiply 25,000 by 5,000.

$$\begin{aligned} 25,000 \times 5,000 &= 2.5 \times 10^4 \times 5 \times 10^3 = 2.5 \times 5 \times 10^{4+3} \\ &= 12.5 \times 10^7 \\ &= 125,000,000 \end{aligned}$$

Example 3: Multiply 1,800, .000015, 300, and .0048.

$$\begin{aligned} 1,800 \times .000015 \times 300 \times .0048 \\ &= 1.8 \times 10^3 \times 1.5 \times 10^{-5} \times 3 \times 10^2 \times 4.8 \times 10^{-4} \\ &= 1.8 \times 1.5 \times 3 \times 4.8 \times 10^{3-5+2-4} \\ &= 38.88 \times 10^{-3} \\ &= .03888 \end{aligned}$$

109. Division of Numbers in Scientific Notation

The general rules covering the division of radicals (par. 75) also apply in the division of numbers that are expressed in scientific notation.

Example 1: Divide 75,000 by .0005.

$$\frac{75,000}{.0005} = \frac{75 \times 10^3}{5 \times 10^{-4}} = \frac{75}{5} \times 10^{3+4} = 15 \times 10^7 = 150,000,000$$

Example 2: Divide 14,400,000 by 1,200,000.

$$\frac{14,400,000}{1,200,000} = \frac{144 \times 10^5}{12 \times 10^5} = \frac{144}{12} = 12$$

Example 3: Divide 98,100 by .0025, 180, and 1,090,000.

$$\begin{aligned} &\frac{98,100}{.0025 \times 180 \times 1,090,000} \\ &= \frac{9.81 \times 10^4}{2.5 \times 10^{-3} \times 1.8 \times 10^2 \times 1.09 \times 10^6} \\ &= \frac{9.81 \times 10^4}{2.5 \times 1.8 \times 1.09 \times 10^{-3+2+6}} \\ &= \frac{9.81 \times 10^4}{4.905 \times 10^5} \\ &= 2 \times 10^{-1} \\ &= .2 \end{aligned}$$

110. Finding the Power or Root of a Number in Scientific Notation

The general rules covering powers and roots (pars. 71 and 72) also apply to numbers expressed in scientific notation.

Example 1: Find the square root of 144,000,000.

$$\begin{aligned} \sqrt[2]{144,000,000} &= \sqrt[2]{144 \times 10^6} \\ &= 12 \times 10^3 \\ &= 12,000 \end{aligned}$$

Example 2: Find the cube root of .000,008.

$$\begin{aligned}\sqrt[3]{.000,008} &= \sqrt[3]{8 \times 10^{-6}} \\ &= 2 \times 10^{-2} \\ &= .02\end{aligned}$$

Example 3: Square 15,000.

$$\begin{aligned}(15,000)^2 &= (15 \times 10^3)^2 \\ &= 225 \times 10^6 \\ &= 225,000,000\end{aligned}$$

Example 4: Find the square root of $(160,000)^3$.

$$\begin{aligned}\sqrt[3]{160,000^3} &= (160,000)^{3/2} \\ &= (16 \times 10^4)^{3/2} \\ &= 64 \times 10^6 \\ &= 64,000,000\end{aligned}$$

Example 5: Find the square root of $\frac{86,900}{3,560,000}$.

$$\begin{aligned}\sqrt{\frac{86,900}{3,560,000}} &= \sqrt{\frac{8.69 \times 10^4}{3.56 \times 10^6}} \\ &= \sqrt{2.44 \times 10^{-2}} \\ &= 1.56 \times 10^{-1} \\ &= .156\end{aligned}$$

111. Review Problems—Powers of 10

In the following problems, leave the answer in powers of ten:

a. Convert the following numbers to powers of 10 and add:

- (1) $1,245,000 + 368,000$
- (2) $79,000 + 421,000$
- (3) $.000,007,66 + .000,054$

b. Convert the following numbers to powers of 10 and subtract:

- (1) $333,400 - 22,500$
- (2) $.000,068 - .000,049$
- (3) $.000,004,89 - .000,000,398$

c. Convert the following numbers to powers of 10 and multiply:

- (1) $446,000 \times 200$

$$(2) 7,700 \times .003,2$$

$$(3) .000,096 \times .000,33$$

$$(4) .003,66 \times 4,000,000$$

d. Convert the following numbers to powers of 10 and divide:

$$(1) 668,000 \div 4,000$$

$$(2) 88,445,000 \div .000,55$$

$$(3) .000,963 \div .000,009$$

$$(4) .006,93 \div 21$$

e. Convert the following numbers to powers of 10 and perform the indicated operations:

$$(1) \sqrt[3]{64,000,000}$$

$$(2) \sqrt[3]{.000,169}$$

$$(3) .003^3$$

$$(4) 27,000^{2/3}$$

CHAPTER 8

LOGARITHMS

112. General

Many lengthy mathematical operations may be accomplished more easily through the use of logarithms. With logarithms (also called logs), multiplication of numbers is reduced to a simple process of addition, division becomes a process of subtraction, raising a number to a power becomes simple multiplication, and extraction of roots is done by simple division.

113. Definition

The logarithm of a given number is the power to which another number (called the base) must be raised to equal the given number. The word "logarithm" has the same meaning as the word "exponent."

Example: Find the logarithm of 1,000 to the base 10.

From the definition, the logarithm of a number (1,000) is the power (x) to which another number called the base (10) must be raised to equal the given number (1,000).

Thus, $10^x = 1,000$. Since $10^3 = 1,000$, then:

$10^x = 10^3$ and by inspection:

$$x = 3$$

Therefore, the logarithm of 1,000 to the base 10 equals 3 or $\log_{10} 1,000 = 3$.

114. Types of Logarithms

a. Common Logarithms. Common logarithms use the number 10 as a base. They are so universally used that the 10 usually is omitted; the answer in paragraph 113 could be $\log 1,000 = 3$. Some values of common logarithms are included in the table below. The common logarithm of any number between

these values consists of the logarithm of the smaller number plus a decimal. For example, the log of a number between 100 and 1,000, such as 157, consists of the log of the smaller number (10) plus a decimal. The log of 157 is 2.1959.

$\log 1 = 0$	$\log .1 = -1$
$\log 10 = 1$	$\log .01 = -2$
$\log 100 = 2$	$\log .001 = -3$
$\log 1,000 = 3$	$\log .0001 = -4$
$\log 10,000 = 4$	

b. Natural Logarithms. Natural logarithms are based upon the irrational number e , and are written both as $\log_e$ and $\ln$. Natural logarithms are used in special applications and as such are not explained further in this text.

115. Parts of Logarithms

a. Logarithms are divided into two parts, the integral and the decimal. The integral part is known as the characteristic, and the decimal part is called the mantissa.

- (1) *The characteristic of any number is one less than the number of digits to the left of the decimal point. Thus, the characteristic for the number 3 is 1 — 1 or zero, since there is one number to the left of the decimal point. The characteristic for 30, with two numbers to the left of the decimal point, is 2 — 1 or 1. Similarly, the characteristic for 300 is 2, and the characteristic for 3,000 is 3. The characteristic of the log of a decimal is negative and is based upon the position of the first rational number to the right of the decimal point. If there are no numbers to the left of the decimal point, the characteristic is negative. In the number .327, for example, the first*

rational number is in the first decimal place and the characteristic is -1 ; in the number .03, the first rational number is in the second decimal place and the characteristic is -2 . Similarly, the characteristic for .003 is -3 , and the characteristic for .0003 is -4 .

- (2) The mantissa is always the same for a given sequence of integers, regardless of where the decimal point appears among them. Thus, the *mantissa* is the same for 1570, 157, 15.7, 1.57, .157, and .0157, and the logs of these numbers differ only in respect to their characteristics. Their logarithms, respectively, are 3.1959, 2.1959, 1.1959, 0.1959, -1.1959 and -2.1959 .

b. The mantissa is always positive—even when the characteristic is negative. This fact poses a problem of notation, and also complicates the addition and subtraction of logarithms.

- (1) In the notation of logarithms, to say that $\log .157$ is -1.1959 is not strictly true, for what we mean to say is -1 plus .1959. To overcome this problem, the minus sign is generally written above the characteristic, and is made long enough to cover the entire negative portion of the logarithm. More properly, therefore, $\log .157$ is written $\bar{1}.1959$.
- (2) In the addition and subtraction of logarithms, the complication can be removed by expressing the negative characteristic in a positive manner; more precisely, by adding a large enough number to the characteristic and by subtracting the same number from the entire logarithm. Thus, the log of .157 is written 9.1959-10, and the log of .0157 is written 8.1959-10.

116. Finding a Logarithm

A table of common logarithms is given in appendix III. Note, however, that the table contains only the mantissas of logarithms. The characteristic must be obtained, in each in-

stance, by following the rules given in paragraph 115a(1).

Example 1: Find the logarithm of 333.

Determine the characteristic of 333. The characteristic is $3 - 1$, or 2.

Determine the mantissa of 333.

In the table of common logarithms, look down the N column for the number 33. The mantissa for 333 is in this horizontal row in the column headed by the number 3. The mantissa is .5224.

$\log 333 = 2.5224$.

Example 2: Find the logarithm of .127.

Determine the characteristic of .127. The characteristic is -1 or 9. ----- -10 .

Determine the mantissa of .127.

In the table of common logarithms, look down the N column for 12. The mantissa for 127 is in this horizontal row in the column headed by the number 7. The mantissa is .1038.

$\log .127 = 9.1038-10$.

117. Logarithmic Interpolation

The table of common logarithms given in appendix III is adequate if the given number has three or less integers. If it has four or more integers, however, it is necessary to interpolate—that is, to find the proportional part of the difference between the logarithms shown in the table.

Example 1: Find the logarithm of 2.369.

Step 1. The characteristic of 2.369 is 0. Since the mantissa for this number cannot be found in the table, it is necessary to interpolate. Look for the mantissas of the numbers next lower and higher than 2369. The mantissa of the number 2360 is .3729 and the mantissa of the number 2370 is .3747. Since 2369 lies between 2360 and 2370, the mantissa of

2369 must lie between .3729 and .3747. This may be written:

$$\begin{aligned}\log 2360 &= .3729 \\ \log 2369 &= .3729 + x \\ \log 2370 &= .3747\end{aligned}$$

Step 2. Set up the proportions. The difference between 2369 and 2360 is 9. The difference between 2370 and 2360 is 10. Therefore, the desired mantissa is $\frac{9}{10}$ of the difference between these two. Let the difference between the mantissa of 2369 and 2360 equal x . The difference between .3747 and .3729 is .0018. The proportion is $\frac{x}{.0018}$.

Step 3. Solve the problem.

$$\begin{aligned}\frac{9}{10} &= \frac{x}{.0018} \\ 10x &= .0162 \\ x &= .0016\end{aligned}$$

Step 4. Since the value of x is .0016, the mantissa of 2369 is .3729 + .0016 or .3745. Therefore, $\log 2369 = 0.3745$.

Example 2: Find the logarithm of .017234.

Step 1. The characteristic of .017234 is —2 or 8. —10. The numbers in the table lower and higher than 17234 are 17200 and 17300. The mantissa of 17200 is .2355; the mantissa of 17300 is .2380. The difference between 17234 and 17200 is 34; the difference between 17300 and 17200 is 100; the difference between .2380 and .2355 is .0025. This may be written:

$$\begin{aligned}\log 17200 &= .2355 \\ \log 17234 &= .2355 + x \\ \log 17300 &= .2380\end{aligned}$$

Step 2. Let the difference between the mantissas of 17234 and 17200 equal x . The equation is as follows:

$$\begin{aligned}\frac{34}{100} &= \frac{x}{.0025} \\ 100x &= .0850 \\ x &= .00085 = .0009\end{aligned}$$

Step 3. Since the value of x is .0009, the mantissa of 17234 is .2355 + .0009 or .2364. Therefore, $\log .017234 = 8.2364-10$.

118. Reading Antilogarithms

The process of finding the antilogarithm (also called antilog), consists of determining the number from which the logarithm was derived. This process is essentially the reverse of finding the logarithm (par. 116). Consequently, the location of the decimal point is determined from the characteristic, and the numerical value of the number is determined from the mantissa.

Example 1: Find the antilog of 1.8954.

Step 1. Since the characteristic of the logarithm is 1, there will be two digits to the left of the decimal point in the number.

Step 2. Look in the table for the mantissa, .8954. The number given for .8954 is 786.

Step 3. Count off two digits from the left and insert the decimal point. The antilog of 1.8954 is 78.6.

Example 2: Find the antilog of 7.0828—10.

Step 1. Since the characteristic of the logarithm is —3, the first significant figure will be in the third decimal place.

Step 2. Look for the mantissa .0828 in the table. The number given for .0828 is 121.

Step 3. Add two zeros to the right of the decimal point and before the first significant figure. Thus, the antilog of 7.0828—10 is .0021.

119. Antilogarithmic Interpolation

If the mantissa of a logarithm does not appear in the table, it is necessary to interpolate.

Example 1: Find the antilog of 2.7654.

Step 1. Since the characteristic of the logarithm is 2, there will be three digits to the left of the decimal point in the number.

Step 2. The mantissa in the table lower than .7654 is .7649. The num-

- ber with .7649 as a mantissa is 582.
- Step 3.** The mantissa higher than .7654 is .7657. The number with .7657 as a mantissa is 583.
- Step 4.** Set up the proportions. The difference between .7654 and .7649 is .0005; the difference between .7657 and .7649 is .0008. The proportional difference is $\frac{.0005}{.0008}$ or $\frac{5}{8}$. The difference between 583 and 582 is 1. This can be written:
- $$\begin{aligned}\text{antilog } .7649 &= 582 \\ \text{antilog } .7654 &= 582 + x \\ \text{antilog } .7657 &= 583\end{aligned}$$
- Step 5.** Let x equal the difference between the number represented by the mantissa .7654 and the number 582. The equation is as follows:
- $$\begin{aligned}\frac{5}{8} &= \frac{x}{1} \\ 8x &= 5 \\ x &= .625\end{aligned}$$
- Step 6.** The number is $582 + .625$. Since there are three digits to the left of the decimal point, the antilog of 2.7654 is 582.625.
- Example 2:** Find the antilog of 6.7166—10.
- Step 1.** Since the characteristic of the logarithm is —4, the first rational number will be in the fourth decimal place.
- Step 2.** The mantissa in the table lower than .8166 is .8162; the number with .8162 as a mantissa is 655.
- Step 3.** The mantissa in the table higher than .8166 is .8169; the number with .8169 as a mantissa is 656.
- Step 4.** The difference between .8162 and .8166 is .0004; the difference between .8169 and .8162 is .0007. The proportional difference is $\frac{.0004}{.0007}$ or $\frac{4}{7}$. The difference between 656 and 655 is 1. This may be written:

$$\begin{aligned}\text{antilog } .8162 &= 655 \\ \text{antilog } .8166 &= 655 + x \\ \text{antilog } .8169 &= 656\end{aligned}$$

- Step 5.** Let x equal the difference between the number represented by the mantissa .8166 and the number 655. The equation is as follows:

$$\begin{aligned}\frac{4}{7} &= \frac{x}{1} \\ 7x &= 4 \\ x &= .57\end{aligned}$$

- Step 6.** The number is $655 + .57$. Since the first rational figure is in the fourth decimal place, the antilog of 6.7166—10 is .00065557.

120. Addition and Subtraction of Logarithms

Logarithms are added and subtracted arithmetically. Since every mantissa is positive (par. 115b), however, every negative characteristic should be expressed as a positive (par. 115b).

Example 1: Add the logarithms 3.7493 and 2.4036.

$$\begin{array}{r} 3.7493 \\ + 2.4036 \\ \hline 6.1529 \end{array}$$

Example 2: Add the logarithms 3.4287 and 6.3982.

$$\begin{array}{r} 3.4287 \\ + 4.3982-10 \\ \hline 7.8269-10 \end{array}$$

Example 3: Add the logarithms 8.9324—10, 7.2812—10, 5.4138—10, and 9.9918—10.

$$\begin{array}{r} 8.9324-10 \\ 7.2812-10 \\ 5.4138-10 \\ + 9.9918-10 \\ \hline 31.6192-40 \\ -(30 \quad -30) \\ \hline 1.6192-10 \end{array}$$

Example 4: Subtract the logarithm 9.1245 from the logarithm 6.3058.

To subtract a larger logarithm from a smaller logarithm, add 10 or a multiple of 10 to the smaller logarithm, and subtract the same number from the loga-

rithm by writing that number with a minus sign to the right of the logarithm. The number chosen for this purpose should be the least that will cause the smaller logarithm to exceed the larger.

$$\begin{array}{r} 16.3058-10 \\ - 9.1245 \\ \hline 7.1813-10 \end{array}$$

Example 5: Subtract the logarithm 3.7980—10 from 2.8686. When subtracting a negative logarithm from a positive logarithm, where that part of the characteristic of the negative logarithm to the left of the mantissa is larger than the characteristic of the positive logarithm, add 10 or a multiple of 10 to the characteristic of the positive logarithm, and subtract that same amount from the right of the positive logarithm.

$$\begin{array}{r} 12.8686-10 \\ - 3.7980-10 \\ \hline 9.0706 \end{array}$$

121. Multiplication by Use of Logarithms

The logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers. Thus, $\log (2 \times 6) = \log 2 + \log 6$; and $\log (12 \times 8) = \log 12 + \log 8$.

Example 1: Multiply 68.2 by 40.8 by using logarithms.

$$\begin{array}{l} \log (68.2 \times 40.8) = \log 68.2 + \log 40.8. \\ \log 68.2 = 1.8338 \\ \log 40.8 = 1.6107 \\ \log (68.2 \times 40.8) = 3.4445 \\ \text{antilog } .4440 = 278 \\ \text{antilog } .4445 = 278 + x \\ \text{antilog } .4455 = 279 \\ \frac{5}{15} = \frac{x}{1} \\ 15x = 5 \\ x = .33 \\ \text{antilog } .4445 = 2783 \\ 68.2 \times 40.8 = 2,783 \end{array}$$

Example 2: Find the product of 2.11 and 41.3 by using logarithms.

$$\begin{array}{l} \log (2.11 \times 41.3) = \log 2.11 + \log 41.3. \\ \log 2.11 = 0.3243 \\ \log 41.3 = 1.6160 \\ \log (2.11 \times 41.3) = 1.9403 \\ \text{antilog } .9400 = 871 \\ \text{antilog } .9403 = 871 + x \\ \text{antilog } .9405 = 872 \\ \frac{3}{5} = \frac{x}{1} \\ 5x = 3 \\ x = .6 \\ \text{antilog } 1.9403 = 87.16 \\ 2.11 \times 41.3 = 87.16 \end{array}$$

122. Division by Use of Logarithms

The logarithm of the quotient of two numbers is equal to the difference between the logarithms of the numbers. Thus, $\log (75 \div 83) = \log 75 - \log 83$, and $\log (8 \div 2) = \log 8 - \log 2$.

Example 1: Divide 785 by 329 by using logarithms.

$$\begin{array}{l} \log (785 \div 329) = \log 785 - \log 329. \\ \log 785 = 2.8949 \\ \log 329 = 2.5172 \\ \log (785 \div 329) = 0.3777 \\ \text{antilog } .3766 = 238 \\ \text{antilog } .3777 = 238 + x \\ \text{antilog } .3784 = 239 \\ \frac{11}{18} = \frac{x}{1} \\ 18x = 11 \\ x = .611 \\ \text{antilog } 0.3777 = 2.386 \\ 785 \div 329 = 2.386 \end{array}$$

Example 2: Find the value of $\frac{3}{7}$ by using logarithms.

$$\begin{array}{l} \log \frac{3}{7} = \log 3 - \log 7. \\ \log 3 = 0.4771 \\ \log 7 = 0.8451 \\ \text{Since the logarithm of 7 is greater than the logarithm of 3, it is necessary to add 10.} \\ \text{-----} \\ \text{—10 to the logarithm of 3 before subtracting the logarithm of 7.} \end{array}$$

$$\begin{aligned}\log 3 &= 10.4771-10 \\ \log 7 &= 0.8451 \\ \hline \log (3 \div 7) &= 9.6320-10 \\ \text{antilog } .6314 &= 428 \\ \text{antilog } .6320 &= 428 + x \\ \text{antilog } .6325 &= 429\end{aligned}$$

$$\begin{aligned}\frac{6}{11} &= \frac{x}{1} \\ 11x &= 6 \\ x &= .55 \\ \text{antilog } 9.6320-10 &= 42855 \\ 3 \div 7 &= .42855\end{aligned}$$

123. Finding the Power of a Number by Logarithms

The logarithm of a number raised to a power is equal to the logarithm of the number multiplied by the power.

Example 1: Evaluate $(18.7)^3$.

$$\begin{aligned}\log (18.7)^3 &= 3 \log 18.7 \\ &= 3 \times 1.2718 \\ &= 3.8154 \\ \text{antilog } .8149 &= 653 \\ \text{antilog } .8154 &= 653 + x \\ \text{antilog } .8156 &= 654 \\ \frac{5}{7} &= \frac{x}{1} \\ 7x &= 5 \\ x &= .7 \\ \text{antilog } 3.8154 &= 6537 \\ (18.7)^3 &= 6,537\end{aligned}$$

Example 2: Evaluate $(.03625)^4$.

$$\begin{aligned}\log (.03625)^4 &= 4 \log .03625 \\ \log 3620 &= .5587 \\ \log 3625 &= .5587 + x \\ \log 3630 &= .5599 \\ \frac{5}{10} &= \frac{x}{.0012} \\ x &= .0006 \\ \log (.03625)^4 &= 4 (8.5593-10) \\ &= 34.2372-40 \\ (\text{Subtract}) &\quad \frac{30.0000-30}{4.2372-10} \\ \text{antilog } .2355 &= 172 \\ \text{antilog } .2372 &= 172 + x \\ \text{antilog } .2380 &= 173 \\ \frac{17}{25} &= \frac{x}{1} \\ 25x &= 17 \\ x &= .68 = .7\end{aligned}$$

$$\begin{aligned}\text{antilog } 4.2372-10 &= .000001727 \\ (.03625)^4 &= .000001727\end{aligned}$$

Example 3: Evaluate $(2.13)^{\frac{1}{3}}$.

$$\begin{aligned}\log (2.13)^{\frac{1}{3}} &= \frac{1}{3} \log 2.13 \\ &= \frac{1}{3} \times 0.3284 \\ &= 0.2189 \\ \text{antilog } .2175 &= 165 \\ \text{antilog } .2189 &= 165 + x \\ \text{antilog } .2201 &= 166 \\ \frac{14}{26} &= \frac{x}{1} \\ 26x &= 14 \\ x &= .5 \\ \text{antilog } 0.2189 &= 1.655 \\ (2.13)^{\frac{1}{3}} &= 1.655\end{aligned}$$

124. Finding the Root of a Number by Logarithms

The logarithm of the root of a number is equal to the logarithm of the number divided by the root.

Example 1: Evaluate $\sqrt[4]{34987}$.

$$\begin{aligned}\log \sqrt[4]{34987} &= \frac{\log 34987}{4} \\ \log 34900 &= .5428 \\ \log 34987 &= .5428 + x \\ \log 35000 &= .5441 \\ \frac{87}{100} &= \frac{x}{.0013} \\ 100x &= .1131 \\ x &= .0011 \\ &= \frac{4.5439}{4} \\ &= 1.135975 = 1.1360 \\ \text{antilog } .1335 &= 136 \\ \text{antilog } .1360 &= 136 + x \\ \text{antilog } .1367 &= 137 \\ \frac{25}{32} &= \frac{x}{1} \\ 32x &= 25 \\ x &= .78 \\ \text{antilog } 1.1360 &= 13.678 \\ \sqrt[4]{34987} &= 13.678\end{aligned}$$

Example 2: Evaluate $\sqrt[3]{76.24}$.

$$\begin{aligned}\log \sqrt[3]{76.24} &= \frac{\log 76.24}{3} \\ \log 7620 &= .8820 \\ \log 7624 &= .8820 + x \\ \log 7630 &= .8825 \\ \frac{4}{10} &= \frac{x}{.0005}\end{aligned}$$

$$\begin{aligned}
 10x &= .0020 \\
 x &= .0002 \\
 &= \frac{1.8822}{3} \\
 &= 0.6274
 \end{aligned}$$

$$\text{antilog } 0.6274 = 4.24$$

$$\sqrt[3]{76.24} = 4.24$$

Example 3: Evaluate $\sqrt[3]{.0073573}$.

$$\log \sqrt[3]{.0073573} = \frac{\log .0073573}{3}$$

$$\log 73500 = .8663$$

$$\log 73573 = .8663 + x$$

$$\log 73600 = .8669$$

$$\frac{73}{100} = \frac{x}{.0006}$$

$$100x = .0438$$

$$x = .0004$$

$$= \frac{7.8667-10}{3}$$

The quotient of 7.8667—10 divided by 3 is 2.6222—3 $\frac{1}{3}$. By adding 20.0000—20 to 7.8667—10, the sum, 27.8667—30, can be divided by 3 and the quotient will be a workable logarithm.

$$\begin{array}{r}
 \log .0073573 = 7.8667-10 \\
 \text{add } 20.0000-20 \\
 \hline
 27.8667-30
 \end{array}$$

$$\frac{27.8667-30}{3} = 9.2889-10$$

$$\text{antilog } .2878 = 194$$

$$\text{antilog } .2889 = 194 + x$$

$$\text{antilog } .2900 = 195$$

$$\frac{11}{22} = \frac{x}{1}$$

$$22x = 11$$

$$x = .5$$

$$\text{antilog } 9.2889-10 = .1945$$

$$\sqrt[3]{.0073573} = .1945$$

125. Cologarithms

The *cologarithms* of a number is the logarithm of the reciprocal of the number. For example, $\text{colog } N = \log \frac{1}{N}$. However,

$$\log \frac{1}{N} = \log 1 - \log N$$

$$= 0 - \log N$$

$$\log \frac{1}{N} = -\log N$$

Therefore, $\text{colog } N = \log \frac{1}{N} = -\log N$. Thus the cologarithm of a number is the logarithm of the number subtracted from the logarithm of 1 (0.0000 or, to avoid a negative mantissa, 10.0000—10).

Example 1: Evaluate the cologarithm of 373.

$$\text{colog } 373 = \log \frac{1}{373}$$

$$\log 1 = 10.0000-10$$

$$\log 373 = 2.5717$$

$$\text{colog } 373 = 7.4283-10$$

Example 2: Evaluate $\frac{2.37}{3.61}$.

$$\log \frac{2.37}{3.61} = \log 2.37 - \log 3.61$$

$$= \log 2.37 + \text{colog } 3.61$$

$$\log 1 = 10.0000-10$$

$$\log 3.61 = 0.5575$$

$$\text{colog } 3.61 = 9.4425-10$$

$$\log 2.37 = 0.3747$$

$$9.8172-10$$

$$\text{antilog } 9.8172-10 = .65643$$

126. Computation by Logarithms

In performing logarithmic computations, follow the principles given in paragraphs 117 through 125. When negative quantities are involved (in multiplication and division), disregard the minus sign when making logarithmic calculations. After calculating the antilog, the sign is determined in accordance with the algebraic law of signs for multiplication and division.

Example 1: Evaluate $\sqrt[3]{\frac{(94.7)^2 (.00789)}{(3.71)^3 (.345)}}$.

$$\begin{aligned}
 \log (94.7)^2 &= 2 \log 94.7 \\
 &= 2 \times 1.9763 \\
 &= 3.9526 \\
 \log (.00789) &= \frac{7.8971-10}{1} \\
 \log (94.7)^2 + \log (.00789) &= \frac{11.8497-10}{1} = 1.8497 \\
 \log (3.71)^3 &= 3 \log 3.71 \\
 &= 3 \times 0.5694 \\
 &= 1.7082 \\
 \log (.345) &= \frac{9.5378-10}{1} \\
 \log (3.71)^3 + \log (.345) &= \frac{11.2460-10}{1} = 1.2460 \\
 \log (94.7)^2 (.00789) &= 1.8497 \\
 \log (3.71)^3 (.345) &= \frac{1.2460}{1} = 0.6037 \\
 \log \sqrt[3]{\frac{(94.7)^2 (.00789)}{(3.71)^3 (.345)}} &= \frac{0.6037}{3} \\
 &= .2012 \\
 \text{antilog } .2012 &= 1.5892
 \end{aligned}$$

Example 2: Evaluate $\sqrt[4]{\frac{(6.484)^2 \cdot \sqrt[3]{7.667}}{(12.35)^2 \cdot \sqrt[3]{3007}}}$.

$$\begin{aligned}
 \log (6.484)^2 &= 2 \log 6.484 \\
 &= 2 \times 0.8118 \\
 &= 1.6236 \\
 \log \sqrt[3]{7.667} &= \frac{\log 7.667}{3} \\
 &= \frac{0.8846}{3} \\
 &= 0.2949 \\
 \log (6.484)^2 + \log \sqrt[3]{7.667} &= 1.6236 + .2949 \\
 &= 1.9185 \\
 \log (12.35)^2 &= 2 \log 12.35 \\
 &= 2 \times 1.0917 \\
 &= 2.1834 \\
 \log \sqrt[3]{3007} &= \frac{\log 3007}{3} \\
 &= \frac{3.4782}{3} \\
 &= 1.1594 \\
 \log (12.35)^2 + \log \sqrt[3]{3007} &= 2.1834 + 1.1594 \\
 &= 3.3428 \\
 \log (6.484)^2 \sqrt[3]{7.667} &= 11.9185-10 \\
 \log (12.35)^2 \sqrt[3]{3007} &= \frac{3.3428}{1} = 3.3428 \\
 &= \frac{11.9185-10}{1} = 8.5757-10
 \end{aligned}$$

$$\log \sqrt[4]{\frac{(6.484)^2 \sqrt[3]{7.667}}{(12.35)^2 \sqrt[3]{3007}}} = \frac{38.5757 - 40}{4}$$

$$= 9.6439 - 10$$

$$\text{antilog } 9.6439 - 10 = .4405$$

127. Review Problems—Logarithms

a. Find the logarithms of the following numbers to the base 10:

- (1) 785
- (2) 3.57
- (3) .0345
- (4) .000476
- (5) 49.6
- (6) 273.5
- (7) 760.1
- (8) 7.234
- (9) .009875
- (10) .00005254

b. Find the antilogs of the following logarithms:

- (1) 4.8457
- (2) 2.4330
- (3) 9.5453—10
- (4) 6.8299—10
- (5) 0.6010
- (6) 2.5690
- (7) 5.4343—10
- (8) 5.6994
- (9) 0.2018
- (10) 4.5372—10

c. Using logarithms, find the products of the following to four significant figures:

- (1) 6.93×23.7
- (2) 186×215
- (3) 64.3×21.4
- (4) $.089 \times .076$
- (5) 135×42.3

d. Using logarithms, find the quotients of the following to four significant figures:

- (1) $148 \div 297$
- (2) $\frac{251}{648}$

$$(3) 14.9 \div 37.4$$

$$(4) 47.38 \div 63.29$$

$$(5) \frac{1.06}{4.35}$$

e. Using logarithms, evaluate the following:

$$(1) (.0293)^4$$

$$(2) (1.756)^7$$

$$(3) (7.953)^{\frac{1}{2}}$$

$$(4) (69.37)^{\frac{1}{7}}$$

$$(5) (27.98)^{\frac{1}{3}}$$

$$(6) \sqrt[3]{.01325}$$

$$(7) \sqrt[4]{815}$$

$$(8) \sqrt{7698}$$

$$(9) \sqrt[3]{8.942}$$

$$(10) \sqrt[4]{.000079911}$$

f. Using logarithms, compute the following:

$$(1) \frac{3.8 \times 2.6}{4.3}$$

$$(2) \sqrt{\frac{.541 \times 47.3}{.0157}}$$

$$(3) \frac{44.1 \times 1.82}{10.27 \times .32}$$

$$(4) \frac{85.21 \times \sqrt[3]{4651}}{\sqrt{46.82} \times 6.230}$$

$$(5) \left(\frac{31.21}{40.70}\right)^{\frac{1}{2}}$$

$$(6) \sqrt{\frac{.5720^{\frac{1}{2}}}{(31.42)^{\frac{1}{3}}}}$$

$$(7) \sqrt{\frac{.08152 \times 1.953}{95.27}}$$

$$(8) \sqrt{\frac{.8531}{9.327}} \times \sqrt[3]{\frac{518.2}{61.52}}$$

$$(9) \frac{48.19 \times \sqrt{56.02}}{431.6 \times \sqrt[3]{46.25} \times \sqrt{16.34}}$$

$$(10) \sqrt{\frac{.008150 \times .08532}{.01234 \times \sqrt[3]{.09156}}}$$

CHAPTER 9

PLANE GEOMETRY

128. Introduction

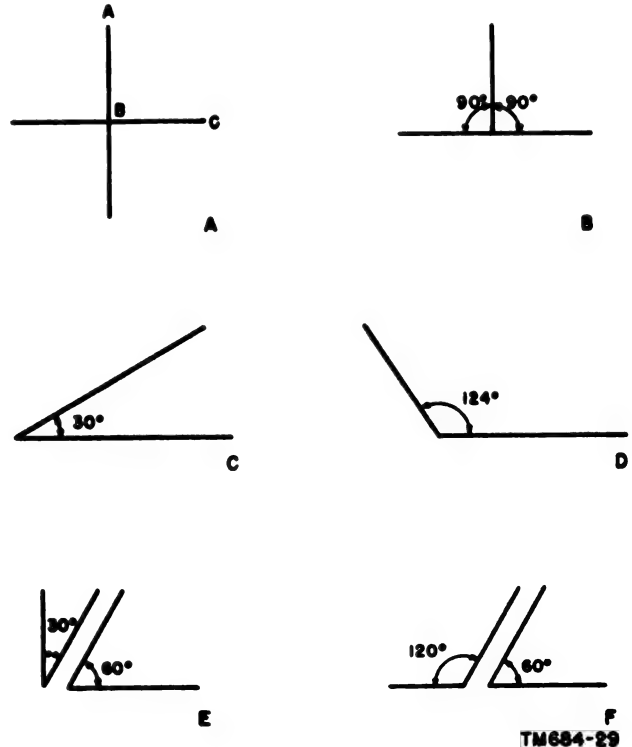
Plane geometry is that part of geometry which deals with plane figures. In electronics, as in many other fields, it is necessary to know how to deal with areas of common plane figures. This chapter presents the formulas for finding the areas of triangles, quadrilaterals (plane figures having four sides and four angles), and circles. No effort has been made to cover the entire field of geometry. Only those principles and proofs are presented that are of value in practical work.

129. Definitions

a. *Lines.* A line has length, but no width or thickness. What is drawn on paper and called a line has thickness and breadth because of the material used to draw it—however, this mark only *represents* the actual line.

b. *Angles.* An angle, such as ABC in A, figure 23, is formed by the intersection of two lines. An angle, therefore, is the measure of the difference in direction of two straight lines that meet. The lines which form the angle, AB or BC , are called the *sides* of the angle, and the point of meeting, B , the vertex. The symbol $\angle$ is used to indicate angles. Angles usually are measured in *degrees*. A complete circle or rotation consists of 360 degrees. The symbol $^\circ$ is used to indicate degrees; it is written to the right and slightly above the number. For example, 30 degrees is written 30° . Each degree consists of 60 *minutes*, and each minute is further broken down into 60 *seconds*. The symbol $'$ is used to indicate minutes; the symbol $"$ indicates seconds. For example, 20 minutes is written $20'$; 15 seconds is written $15''$.

- (1) When one straight line is *perpendicular* to another straight line, the angle formed is a right angle (90°) (B, fig. 23).



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Figure 23. Angles.

- (2) Two right angles, added together, form a *straight angle*. A straight angle, therefore, is an angle of 180° .
- (3) Any angle less than a right angle is an *acute angle* (C, fig. 23).
- (4) Any angle greater than a right angle and less than 180° is an *obtuse angle* (D, fig. 23).
- (5) Two angles whose sum is one right angle are called *complementary angles* (E, fig. 23).
- (6) Two angles whose sum is a straight angle are called *supplementary angles* (F, fig. 23).

130. Basic Principles of Geometric Construction

a. *Reproducing Angles.* To draw an angle equal to a given angle BAC (fig. 24)—

- (1) Draw a line, $A'C'$.
- (2) With A as the center, use a compass to strike an arc that cuts the sides of the given angle at X and Y . Using the same radius, strike a similar arc, $X'Y'$, on the line, $A'C'$.
- (3) Measure the opening of the given angle by setting one point of the compass at Y and the other at X . With the compass at this distance and with Y' as the center, strike an arc as shown in figure 24. This will cut the first arc at point X' .
- (4) Draw a line, $A'B'$, through X' . The new angle, $B'A'C'$, is the same size as angle BAC .

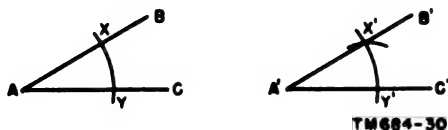


Figure 24. Reproducing an angle.

b. *Finding the Midpoint of a Straight Line Segment.* To find the midpoint of any straight line segment, such as AB in figure 25—

- (1) Use a radius greater than half the length of AB . Using point A as the center, draw arcs CD and $C'D'$. With point B as the center, and using the same radius, draw arcs EF and $E'F'$.
- (2) Draw a straight line to connect the points where the arcs intersect. Point X , where this line intersects AB , is the midpoint of straight line segment AB .

c. *Constructing a Perpendicular.* To construct a perpendicular to a straight line at a given point—

- (1) On the straight line, such as AB in figure 26, mark point P at which the perpendicular is to be constructed.
- (2) Set a compass for a radius less than the shorter of the two segments, AP

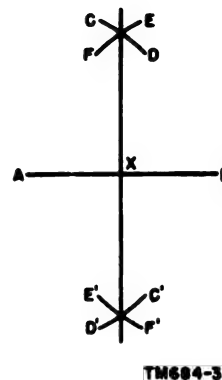


Figure 25. Bisecting a straight line segment.

or PB . With P as a center, draw arcs, cutting line AB at points X and Y .

- (3) Set the compass for a radius greater than PX . With X as a center, draw an arc above point P (fig. 26). Keep the compass at the same setting and, with Y as a center, draw another arc intersecting the one drawn with X as a center. (The two arcs may be drawn to intersect below point P instead of above.)
- (4) Draw a straight line from the point where the two arcs intersect to point P . The line is perpendicular to AB .
- (5) To construct the perpendicular bisector of a straight line segment, first find the midpoint of the line segment (b above), and construct the perpendicular at that point.

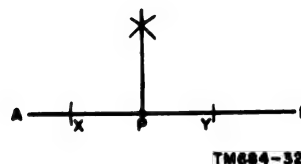


Figure 26. Constructing a perpendicular to a straight line at a point on the line.

d. *Constructing a Perpendicular to a Straight Line from a Point Not on the Line.* To draw a perpendicular to a straight line from a point outside the line, such as point P in figure 27—

- (1) With point P as the center, draw an arc cutting line AB at points X and Y .

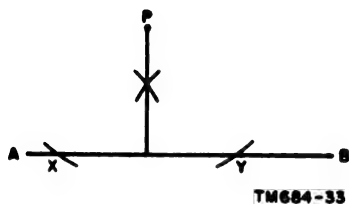


Figure 27. Constructing a perpendicular to a straight line from a point not on the line.

- (2) Using a radius greater than one-half the distance between X and Y and, with points X and Y as centers, draw arcs that intersect.
- (3) Draw a straight line from point P , through the point where the two arcs intersect, to line AB . The line is perpendicular to AB .

e. Finding the Center of a Circle.

- (1) Draw any two chords, such as AB and AC in figure 28.
- (2) Construct the perpendicular bisector of each chord (c above). Point X , where the two perpendicular bisectors meet, is the center of the circle.

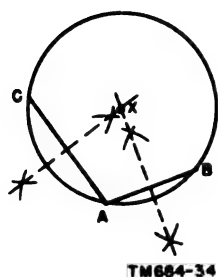


Figure 28. Finding the center of a circle.

f. Bisecting an Angle. Any angle, such as angle CAB in figure 29, can be divided into two equal angles. An angle, thus divided, is said to be bisected. To bisect an angle—

- (1) Using A as a center, draw an arc cutting the sides of angle CAB at X and Y .
- (2) With X and Y as centers, draw intersecting arcs.
- (3) Draw a straight line from A through the point where the arcs intersect. The line divides angles CAB into two

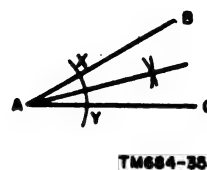


Figure 29. Bisecting an angle.

equal angles and is called the bisector of angle CAB .

131. Triangles

a. General. A triangle is a plane figure bounded by three straight lines. There are several different kinds of triangles.

- (1) An *equilateral triangle* (A, fig. 30) has three equal sides and three equal angles; each angle equals 60° .
- (2) An *isosceles triangle* has two equal

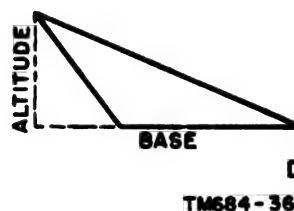
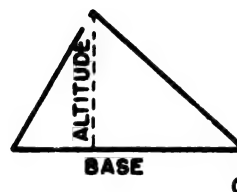
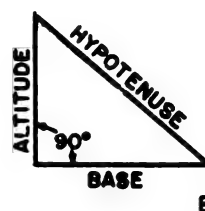
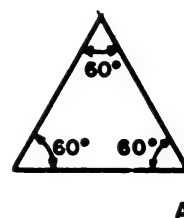


Figure 30. Triangles.

sides and two equal angles. The equal angles are opposite the equal sides.

(3) A *right triangle* (B, fig. 30) has one right angle.

(4) An *oblique triangle* (C and D, fig. 30) is one that does not contain a right angle. Thus, all except right triangles are oblique triangles.

b. Base. The base of a triangle is the side on which the triangle is supposed to stand. However, any side of a triangle may be used as the base.

c. Altitude. The altitude is the perpendicular line distance from the vertex of the triangle to the base or the base extended. In B, figure 30, the altitude of a right triangle is shown, in C, figure 30, the altitude of an acute triangle, and in D, figure 30, the altitude of an obtuse triangle. Note that in an obtuse triangle, it is necessary to extend the base of the triangle to find the altitude.

d. Area. The area of a triangle is the entire surface within the perimeter.

e. Hypotenuse. The side opposite the right angle of any right triangle is the hypotenuse (B, fig. 30).

132. Law of Angles of Any Triangle

The sum of the angles of any triangle is equal to 180° . When given any two of three angles of a triangle, the third angle can be found by subtracting the sum of the given angles from 180° .

Example 1:

If two angles of a triangle are 90° and 45° , what is the size of the third angle?

$$90^\circ + 45^\circ = 135^\circ$$

$$180^\circ - 135^\circ = 45^\circ$$

Therefore, the third angle is 45° .

Example 2:

Angle A of triangle ABC is 100° ; angle B is 30° . What is the size of angle C?

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A = 100^\circ$$

$$\angle B = 30^\circ$$

$$\angle A + \angle B = 130^\circ$$

$$\angle C = 180^\circ - 130^\circ$$

$$\angle C = 50^\circ$$

133. Law of Right Triangles

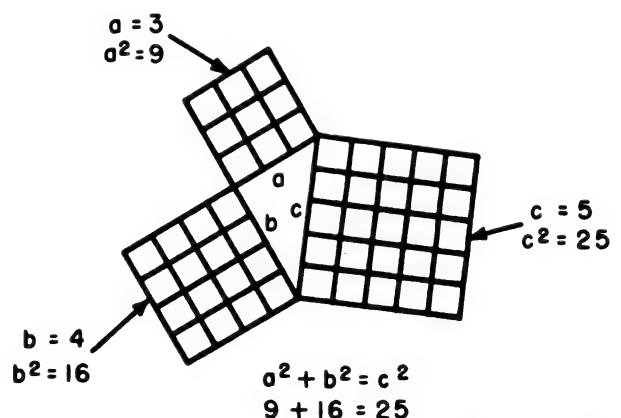
a. The Pythagorean Theorem. This theorem, which applies to any right triangle, states that *the square of the hypotenuse is equal to the sum of the squares of the other two sides*. The Pythagorean theorem is of prime importance in trigonometry (ch. 10) since the value of one side of a right triangle can be found if the other two sides are known. Thus, in figure 31:

$$c^2 = a^2 + b^2 \text{ or } 25 = 16 + 9$$

$$a^2 = c^2 - b^2 \text{ or } 16 = 25 - 9$$

$$b^2 = c^2 - a^2 \text{ or } 9 = 25 - 16$$

Example 1: Find the hypotenuse of a right triangle if the sides are 3 and 4 inches long, respectively.



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Figure 31. The Pythagorean theorem.

$$\begin{aligned}
c^2 &= a^2 + b^2 \\
c^2 &= 9 + 16 \\
c^2 &= 25 \\
c &= \sqrt{25} \\
c &= 5 \text{ inches}
\end{aligned}$$

Example 2: The hypotenuse of a right triangle is 13 inches long and one side is 5 inches long. Find the length of the other side.

$$\begin{aligned}
c^2 &= a^2 + b^2 \\
13^2 &= 5^2 + b^2 \\
b^2 &= 169 - 25 \\
b^2 &= 144 \\
b &= \sqrt{144} \\
b &= 12 \text{ inches}
\end{aligned}$$

Example 3: Given the right triangle *ABC* (fig. 31), find *c* if *a* = 7 and

$$\begin{array}{rcl}
b &= 6. & 9.22 \\
c^2 &= a^2 + b^2 & \sqrt{85.00\ 00} \\
c^2 &= 49 + 36 & 81 \\
c^2 &= 85 & 182 \quad 400 \\
c &= \sqrt{85} & 364 \\
c &= 9.22- & 1842 \quad 3600 \\
& & 3684
\end{array}$$

Example 4: Given the right triangle *ABC* (fig. 31), find *b* if *a* = 9 and

$$\begin{array}{rcl}
c &= 12. & 7.93 \\
b^2 &= c^2 - a^2 & \sqrt{63.00\ 00} \\
b^2 &= 144 - 81 & 49 \\
b^2 &= 63 & 149 \quad 1400 \\
b &= \sqrt{63} & 1341 \\
b &= 7.93+ & 1583 \quad 5900 \\
& & 4749
\end{array}$$

Example 5: Given the right triangle *ABC* (fig. 31), find *a* if *b* = 6 and

$$\begin{array}{rcl}
c &= 13. & 1\ 1.5\ 3 \\
a^2 &= c^2 - b^2 & \sqrt{01\ 33.00\ 00} \\
a^2 &= 169 - 36 & 21 \quad 33 \\
a^2 &= 133 & 225 \quad 1200 \\
a &= \sqrt{133} & 1125 \\
a &= 11.53+ & 2303 \quad 7500 \\
& & 6909
\end{array}$$

b. Special Right Triangles. The two right triangles in examples 1 and 2 of *a* above are special right triangles with sides that have whole numbers. These triangles are called the 3-4-5 right triangle and the 5-12-13 right triangle, although their sides may also be multiples of these numbers. For example, a triangle having sides of 6, 8, and 10 inches is also a 3-4-5 right triangle, because its sides are multiples of 3, 4, and 5. When determining the unknown side of a right triangle, the process is greatly simplified if the triangle is a 3-4-5 or 5-12-13 right triangle. In these cases, the unknown side can often be determined by inspection.

Example 1: The hypotenuse of a right triangle is 15 inches long, and one side is 12 inches long. Find the other side.

Since 15 and 12 can be divided by 3 to give 5 and 4, the triangle is a 3-4-5 right triangle. The third side, therefore, is equal to 3 times 3, or

9 inches. The answer can be checked by the Pythagorean theorem.

Example 2: The two sides of a triangle are 10 and 24 feet long. Find the length of the hypotenuse.

Dividing 10 and 24 by 2 gives 5 and 12, the two sides of a 5-12-13 right triangle. Therefore, the hypotenuse is 2 times 13, or 26 inches.

134. Area of Any Triangle

The area of any triangle is equal to one-half the product of its base and altitude. The formula for finding the area is $A = \frac{bh}{2}$ where b is the base of the triangle and h is the altitude.

Example 1:

What is the area of a triangle with a base of 15 inches and an altitude of 10 inches?

$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{15 \times 10}{2} \\ &= \frac{150}{2} \\ &= 75 \text{ square inches} \end{aligned}$$

Example 2:

Find the area of a right triangle if the base measures 7 feet and the hypotenuse 25 feet.

$$\begin{aligned} c^2 - b^2 &= a^2 \\ a^2 &= 25^2 - 7^2 = 625 - 49 \\ a^2 &= 576 \\ a &= \sqrt{576} = 24 \text{ feet altitude} \\ A &= \frac{bh}{2} \\ &= \frac{7 \times 24}{2} = \frac{168}{2} \\ &= 84 \text{ square feet} \end{aligned}$$

135. Quadrilaterals

A quadrilateral is a plane figure bounded by four straight lines.

a. A *parallelogram* (A, fig. 32) is a quadrilateral having both pairs of opposite sides parallel.

b. A *rectangle* (B, fig. 32) is a parallelogram that has four right angles.

c. A *square* (C, fig. 32) is a rectangle, all four sides of which are equal.

d. A *trapezoid* (D, fig. 32) is a quadrilateral with two sides (called bases) parallel and unequal.

136. Area of Any Parallelogram

The area of any parallelogram is equal to the product of the base by the altitude. The formula for finding the area is $A = bh$ where b is the base and h is the height or altitude.

Example 1: Find the area of a square, each side of which is 15 inches.

$$\begin{aligned} A &= bh \\ &= 15 \times 15 \\ &= 225 \text{ square inches} \end{aligned}$$

Example 2: What is the area of a rectangle with a base of 12 inches and an altitude of 7 inches?

$$\begin{aligned} A &= bh \\ &= 12 \times 7 \\ &= 84 \text{ square inches} \end{aligned}$$

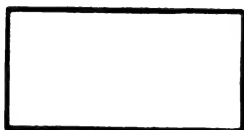
137. Area of Trapezoid

The area of a trapezoid is determined by multiplying one-half the sum of the bases by the altitude of the trapezoid.

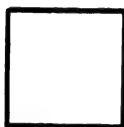
$$\text{Thus, } A = \left(\frac{B + b}{2} \right) h.$$



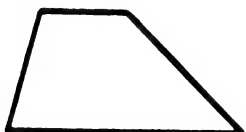
A



B



C



D

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Figure 32. Quadrilaterals.

Example: Find the area of a trapezoid the bases of which are 16 and 10 inches long and the altitude is 8 inches.

$$\begin{aligned}
 A &= \left(\frac{B + b}{2} \right) h \\
 &= \left(\frac{16 + 10}{2} \right) 8 \\
 &= \frac{26}{2} \times 8 \\
 &= 104 \text{ square inches}
 \end{aligned}$$

138. Circles

a. General. A circle is a plane figure bounded by a closed curve, every point of which is equidistant from the center.

b. Circumference. The circumference is the curved line that bounds a circle (A, fig. 33).

c. Chord. A chord is a straight line drawn through a circle and terminated at its intersections with the circumference (B, fig. 33).

d. Diameter. The diameter of a circle is a chord that passes through the center of the circle (A, fig. 33).

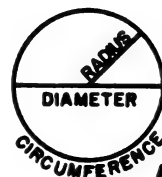
e. Radius. The radius of a circle is a straight line from the center to a point on the circumference (A, fig. 33). All radii of the same circle are of equal length, one-half of the diameter.

f. Arc. An arc is any part of the circumference of a circle.

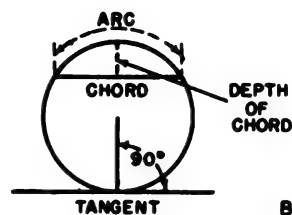
g. Segment. A segment is that area of a circle bounded by a chord and the arc subtended by that chord (C, fig. 33).

h. Sector. A sector is the area between an arc and two radii drawn to the ends of the arc (C, fig. 33).

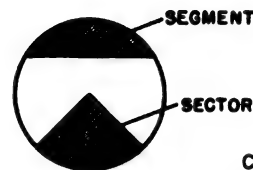
i. Tangent. A tangent is a straight line that touches the circumference of a circle at only one point and is perpendicular to the radius drawn to the point of contact (B, fig. 33). This



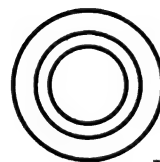
A



B



C



D

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Figure 33. Circles.

point is called the *point of tangency* or the *point of contact*.

j. *Concentric Circles*. Concentric circles are circles having a common center (D, fig. 33).

k. *Pi (π)*. The Greek letter π is used to represent the relationship of the circumference of any circle to its diameter. Roughly, it equals $\frac{22}{7}$. More approximately, it equals 3.1416. In many applications, it is rounded off to 3.14.

139. Circumference of Any Circle

The circumference of any circle is π times the diameter; therefore, $C = \pi D$.

Example 1: Find the circumference of a circle if the diameter is $6\frac{1}{2}$ inches.

$$\begin{aligned} C &= \pi D \\ &= 3.14 \times 6.5 \\ &= 20.42 \text{ inches} \end{aligned}$$

Example 2: Find the diameter of a circular tank having a circumference of $31\frac{1}{2}$ inches.

When the circumference of a circle is given, the diameter is calculated by dividing the circumference by π — $D = \frac{C}{\pi}$.

$$\begin{aligned} D &= \frac{C}{\pi} \\ &= \frac{31.5}{3.1416} \\ &= 10.03 \text{ inches} \end{aligned}$$

140. Area of Any Circle

a. The area of any circle is equal to π multiplied by the radius squared; therefore, $a = \pi r^2$.

Example 1: Find the area of a circle having a diameter of 5 feet 6 inches.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \left(\frac{5.5}{2}\right)^2 \\ &= \pi (2.75)^2 \\ &= 3.14 \times 7.56 \\ &= 23.76 \text{ square feet} \end{aligned}$$

Example 2: What is the diameter of a circle the area of which is 78.54 square rods?

$$A = \pi r^2 \text{ and } r = \frac{D}{2}$$

$$A = \pi \left(\frac{D}{2}\right)^2$$

$$A = \frac{\pi D^2}{4}$$

Transposing:

$$D^2 = \frac{4A}{\pi}$$

$$D = \sqrt{\frac{4A}{\pi}}$$

$$D = \sqrt{\frac{4}{\pi} A}$$

Substituting and solving for D :

$$D = \sqrt{\frac{4 \times 78.54}{3.1416}}$$

$$D = \sqrt{100}$$

$$D = 2 \times 5$$

$$D = 10 \text{ rods}$$

b. The area of any circle also is equal to one-half the product of the circumference and the radius.

Example: If the diameter of a circle is 10 inches, and the circumference of the circle is 31.416 inches, what is the area of the circle?

$$\begin{aligned} A &= \frac{1}{2} Cr \\ r &= \frac{1}{2} D \text{ or } r = 5 \\ A &= \frac{1}{2} (31.416 \times 5) \\ &= \frac{157.08}{2} \\ &= 78.54 \text{ square inches} \end{aligned}$$

141. Area of Ring

A ring is the area between the circumferences of two concentric circles. The area of a ring may be found by subtracting the area of the small circle from the area of the large circle. If R is the radius of the large circle and r is the radius of the small circle, a simplified formula for the area of the ring can be developed as follows:

$$\begin{aligned} \text{Area of ring} &= \text{area of large circle} - \text{area of small circle} \\ &= \pi R^2 - \pi r^2 \\ &= \pi (R^2 - r^2) \end{aligned}$$

By factoring $(R^2 - r^2)$ into $(R + r)(R - r)$, the formula also can be written:

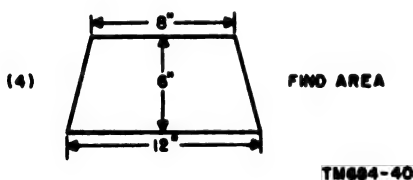
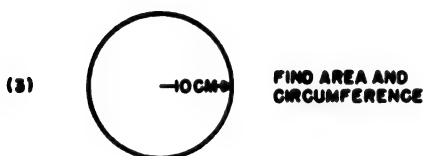
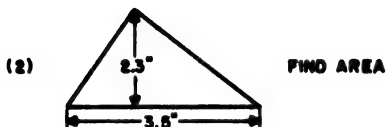
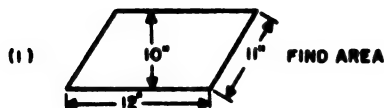
$$A = \pi (R + r)(R - r)$$

Example: Find the area of a ring having an inside diameter of 8 inches and an outside diameter of 12 inches.

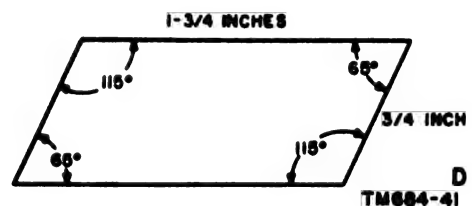
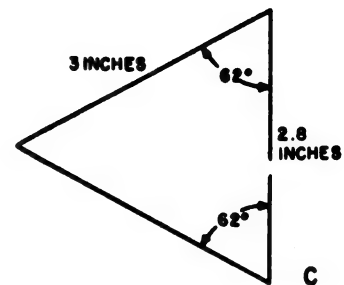
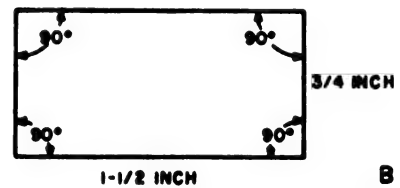
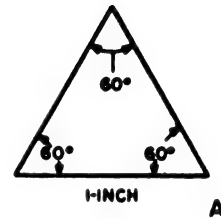
$$\begin{aligned} A &= \pi(R + r)(R - r) \\ &= 3.14(6 + 4)(6 - 4) \\ &= 3.14 \times 10 \times 2 \\ &= 62.8 \text{ square inches} \end{aligned}$$

142. Review Problems—Plane Geometry

- Find the area of a rectangle having a base of 12 inches and an altitude of 8 inches.
- What is the area of a square, each side of which is 6 inches?
- Find the area of a triangle of which the altitude is 5 inches and the base is 10 inches.
- Find the area of a triangle having an altitude of 15 inches and a base of 2 inches.
- What is the hypotenuse of a right triangle the sides of which are 12 and 8 inches?
- Find the third side of a right triangle if one side is 7 inches and the hypotenuse is 9 inches.
- Identify the following figures, give the formulas, and solve for the required quantity.



k. What are the perimeters of the following figures?



i. Find the area of the largest circle that can be cut from a square piece of sheet metal with sides of 10 inches.

j. If the height of an antenna is 80 feet, how far from its top is an object on the ground 60 feet from the base of the pole?

k. How many square feet of lumber are needed to build 10 boxes 18 inches by 16 inches by 9 inches?

l. A metal plate is in the shape of an equilateral triangle. If the altitude is 14 inches, what is the perimeter?

CHAPTER 10

TRIGONOMETRY

Section I. BASIC TRIGONOMETRIC THEORY

143. Introduction

a. Definition. Trigonometry deals with the relationships between the sides and angles of triangles. It uses the theories of basic mathematics—the numbers of arithmetic, the equations of algebra, and the theorems of geometry—to aid in the measurement of the sides and angles of triangles.

b. Application. The ability to use angles and their trigonometric relationships in electrical calculations is especially important in the study of alternating current (ac). Most effects of ac circuit components can be studied or described only in terms of the part of a cycle by which a current lags behind a corresponding voltage, or vice versa. A large percentage of the problems relating to the analysis of ac circuits and communication networks involves the solution of the right triangle in some form. Certain facts about right triangles are familiar (ch 9)—namely, that the square of the hypotenuse is equal to the sum of the squares of the other two sides ($c^2 = a^2 + b^2$), that the sum of the acute angles of a right triangle is 90° , and that the sum of the interior angles of any triangle is 180° . However, it would be impossible to solve certain problems with only this information. After learning other relationships between the sides and angles of triangles, it will be found that trigonometry is an easy and accurate method of solving many problems in ac electricity (ch 15).

144. Trigonometric Functions

a. General. Trigonometry is based on the six trigonometric functions involved in the study of the right angle. If the value of one quantity depends on the value of a second quantity, the first quantity is said to be a function of the second. The six trigonometric functions—sine (sin), cosine (cos), tangent (tan), co-

tangent (cot), secant (sec), and cosecant (csc)—are derived from the ratios of the sides of a right triangle to each other.

b. The Right Triangle. Figure 34 shows a right triangle, with the angles labeled A , B , and C ; C is the right angle. The sides of the triangle are labeled a , b , and c , with the side opposite each angle given the same letter as the angle. The following are the trigonometric ratios of the sides of a triangle:

$$\sin = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\cot = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec = \frac{\text{hypotenuse}}{\text{adjacent side}}$$

$$\csc = \frac{\text{hypotenuse}}{\text{opposite side}}$$

c. Angle A. Refer again to figure 34. Using the acute angle A , a is the opposite side, b is the adjacent side, and c , which is the side opposite the right angle, is the hypotenuse. Therefore,

$$\sin A = \frac{a}{c}$$

$$\cos A = \frac{b}{c}$$

$$\tan A = \frac{a}{b}$$

$$\cot A = \frac{b}{a}$$

$$\sec A = \frac{c}{b}$$

$$\csc A = \frac{c}{a}$$

d. Angle B. Using the acute angle *B* in figure 34, *b* is the opposite side, *a* is the adjacent side, and *c* is the hypotenuse. Therefore,

$$\sin B = \frac{b}{c}$$

$$\cos B = \frac{a}{c}$$

$$\tan B = \frac{b}{a}$$

$$\cot B = \frac{a}{b}$$

$$\sec B = \frac{c}{a}$$

$$\csc B = \frac{c}{b}$$

e. Angle C. Right angle *C* is the angle which establishes the relationship between the other sides and other angles and thus may be called a constant. Although it is possible to obtain functions for angle *C*, they are not covered here because they are not needed in solving problems of this type.

Example:

Determine the values of the trigonometric functions of a right triangle with sides as follows: $a = 3$, $b = 4$, $c = 5$ (fig. 35).

Functions of angle *A*:

$$\sin A = \frac{a}{c} = \frac{3}{5}$$

$$\cos A = \frac{b}{c} = \frac{4}{5}$$

$$\tan A = \frac{a}{b} = \frac{3}{4}$$

$$\cot A = \frac{b}{a} = \frac{4}{3}$$

$$\sec A = \frac{c}{b} = \frac{5}{4}$$

$$\csc A = \frac{c}{a} = \frac{5}{3}$$

Functions of angle *B*:

$$\sin B = \frac{b}{c} = \frac{4}{5}$$

$$\cos B = \frac{a}{c} = \frac{3}{5}$$

$$\tan B = \frac{b}{a} = \frac{4}{3}$$

$$\cot B = \frac{a}{b} = \frac{3}{4}$$

$$\sec B = \frac{c}{a} = \frac{5}{3}$$

$$\csc B = \frac{c}{b} = \frac{5}{4}$$

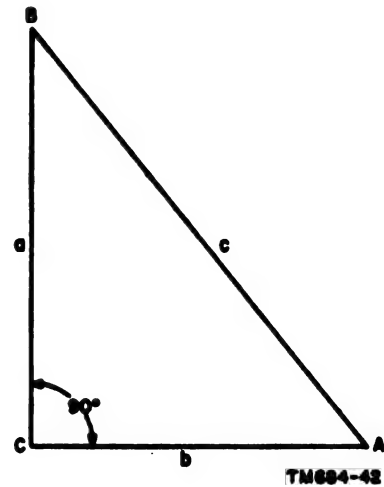


Figure 34. Trigonometric functions of the right triangle.

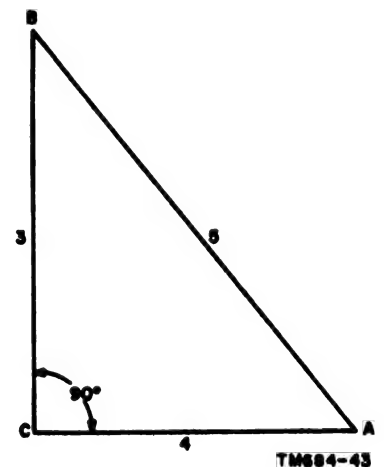


Figure 35. Right triangle with sides known.

145. Reciprocal Relations of Trigonometric Functions

From the definitions of the six trigonometric functions (par. 144), the reciprocal relations (listed below) can be determined. The cosecant, secant, and cotangent should always be thought of as the reciprocals of the sine, cosine, and tangent, respectively.

$$\sin A = \frac{a}{c} = \frac{1}{\frac{c}{a}} = \frac{1}{\csc A}$$

$$\cos A = \frac{b}{c} = \frac{1}{\frac{c}{b}} = \frac{1}{\sec A}$$

$$\tan A = \frac{a}{b} = \frac{1}{\frac{b}{a}} = \frac{1}{\cot A}$$

$$\csc A = \frac{c}{a} = \frac{1}{\frac{a}{c}} = \frac{1}{\sin A}$$

$$\sec A = \frac{c}{b} = \frac{1}{\frac{b}{c}} = \frac{1}{\cos A}$$

$$\cot A = \frac{b}{a} = \frac{1}{\frac{a}{b}} = \frac{1}{\tan A}$$

146. Functions of Complementary Angles

a. The function of an acute angle is equal to the cofunction of its complementary angle. Apply the definitions of the trigonometric functions (par. 144) to angles A and B to obtain the following relations:

$$\sin B = \frac{b}{c} = \cos A$$

$$\tan B = \frac{b}{a} = \cot A$$

$$\sec B = \frac{c}{a} = \csc A$$

$$\cos B = \frac{a}{c} = \sin A$$

$$\cot B = \frac{a}{b} = \tan A$$

$$\csc B = \frac{c}{b} = \sec A$$

b. With angle B equal to $90^\circ - A$, these relations may be written:

$$\sin (90^\circ - A) = \cos A$$

$$\tan (90^\circ - A) = \cot A$$

$$\sec (90^\circ - A) = \csc A$$

$$\cos (90^\circ - A) = \sin A$$

$$\cot (90^\circ - A) = \tan A$$

$$\csc (90^\circ - A) = \sec A$$

147. Solving for Unknown Functions

If one trigonometric function of a right triangle is known, the other trigonometric functions can be determined. This is done by using the Pythagorean theorem (par. 133).

Example 1: Given the right triangle ABC (fig. 23): side a is 4; side C is 9. Since $\sin A = \frac{4}{9}$, find the other trigonometric functions of angle A .

$$\sin A = \frac{a}{c}; \text{ also, } \sin A = \frac{4}{9}.$$

$$\text{Therefore, } a = 4, c = 9$$

$$b^2 = c^2 - a^2$$

$$b^2 = 81 - 16$$

$$b^2 = 65$$

$$b = \sqrt{65}$$

$$b = 8.06$$

$$\begin{array}{r} 8.06 \\ \sqrt{65.00\ 00} \\ 64 \\ \hline 1606 \quad 10000 \\ \quad 9636 \\ \hline \end{array}$$

$$\begin{array}{ll}\sin A = \frac{4}{9} & \cot A = \frac{8.06}{4} \\ \cos A = \frac{8.06}{9} & \sec A = \frac{9}{8.06} \\ \tan A = \frac{4}{8.06} & \csc A = \frac{9}{4}\end{array}$$

Example 2: Given the right triangle ABC (fig. 23): side A is $\sqrt{3}$; side b is 7. Since $\tan A = \frac{\sqrt{3}}{7}$ or $\frac{1}{7}\sqrt{3}$, find the other trigonometric functions of angle A .

$$\tan A = \frac{a}{b}; \text{ also, } \tan A = \frac{1}{7}\sqrt{3} = \frac{\sqrt{3}}{7}.$$

Therefore,

$$a = \sqrt{3}, b = 7$$

$$c^2 = a^2 + b^2$$

$$c^2 = 3 + 49$$

$$c^2 = 52$$

$$c = \sqrt{52}$$

$$c = \sqrt{4} \cdot \sqrt{13}$$

$$c = 2\sqrt{13}$$

$$\sin A = \frac{\sqrt{3}}{2\sqrt{13}}$$

$$\cot A = \frac{7}{\sqrt{3}}$$

$$\cos A = \frac{7}{2\sqrt{13}}$$

$$\sec A = \frac{2\sqrt{13}}{7}$$

$$\tan A = \frac{\sqrt{3}}{7}$$

$$\csc A = \frac{2\sqrt{13}}{\sqrt{3}}$$

148. Solving for Sides and Trigonometric Functions When One Side and One Function Are Given

When one side and one function of an angle of a right triangle are given, the two other sides and the remaining trigonometric functions of the given angle can be found. These are determined by use of the Pythagorean theorem.

Example 1: Given the right triangle ABC (fig. 34): if the hypotenuse is 30 inches and $\sec A = 5$, solve for sides a and b and the trigonometric functions of angle A .

$$\sec A = \frac{c}{b}; \text{ also, } \sec A = \frac{30}{b}; \text{ but } \sec A = 5 \text{ or } \frac{5}{1}$$

$$\text{Therefore, } \frac{30}{b} = \frac{5}{1}$$

$$5b = 30$$

$$b = 6 \text{ inches}$$

$$a^2 = c^2 - b^2$$

$$a^2 = 900 - 36$$

$$a^2 = 864$$

$$a = \sqrt{864}$$

$$a = \sqrt{144} \sqrt{6}$$

$$a = 12\sqrt{6} \text{ inches, } b = 6 \text{ inches, } c = 30 \text{ inches}$$

$$\sin A = \frac{12\sqrt{6}}{30} = \frac{12}{30}\sqrt{6} = \frac{2}{5}\sqrt{6}$$

$$\cos A = \frac{\sqrt{6}}{30} = \frac{1}{5}$$

$$\tan A = \frac{12\sqrt{6}}{6} = 2\sqrt{6}$$

$$\cot A = \frac{6}{12\sqrt{6}} = \frac{1}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{(2)(6)} = \frac{\sqrt{6}}{12} = \frac{1}{12}\sqrt{6}$$

$$\sec A = \frac{30}{6} = 5$$

$$\csc A = \frac{30}{12\sqrt{6}} = \frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{(2)(6)} = \frac{5\sqrt{6}}{12} = \frac{5}{12}\sqrt{6}$$

Example 2: Given the right triangle ABC (fig. 34): solve for sides b and c and the trigonometric functions of angle A when side a is 21.2 inches and $\sin A = \frac{4}{7}$.

$$\sin A = \frac{a}{c}; \text{ also, } \sin a = \frac{21.2}{c}, \text{ but } \sin A = \frac{4}{7}.$$

$$\text{Therefore, } \frac{21.2}{c} = \frac{4}{7}$$

$$4c = 148.4$$

$$c = 37.1 \text{ inches}$$

$$b^2 = c^2 - a^2$$

$$b^2 = 1376.41 - 449.44$$

$$b^2 = 926.97$$

$$b = \sqrt{926.97}$$

$$b = 30.4 \text{ inches, } a = 21.2 \text{ inches, } c = 37.1 \text{ inches}$$

$$\sin A = \frac{21.2}{37.1} = \frac{4}{7} \qquad \cot A = \frac{30.4}{21.2} = \frac{7.6}{5.3}$$

$$\cos A = \frac{30.4}{37.1} \qquad \sec A = \frac{37.1}{30.4}$$

$$\tan A = \frac{21.2}{30.4} = \frac{5.3}{7.6} \qquad \csc A = \frac{37.1}{21.2} = \frac{7}{4}$$

149. Constructing an Acute Angle of Right Triangle When One Trigonometric Function Is Known

When the trigonometric function of an acute angle is given, the angle may be constructed geometrically. Use the definition given for the given function.

Example: Construct the acute angle A of right triangle ABC if $\tan A = \frac{1}{4}$.

Step 1. Let $a = 1$ unit and $b = 4$ units.

Step 2. Erect perpendicular lines AC and BC . Use cross-sectional paper if available.

Step 3. Measure off 1 unit along BC and 4 units along AC (A, fig. 36).

Step 4. Join A and B , thus forming the right triangle ABC (B, fig. 36).

Step 5. $\tan A = \frac{1}{4}$; therefore, A is the required angle. Measuring angle A with a protractor shows it to

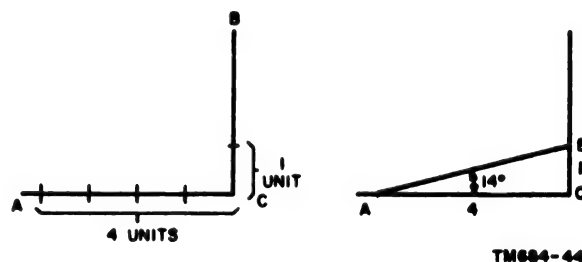


Figure 36. Constructing an angle when one function is known.

be an angle of approximately 14° .

150. Common Trigonometric Functions

a. General. There are two special-case right triangles that are commonly used in solving mathematical problems. These are the right isosceles triangle (par. 131a) with equal acute angles of 45° (fig. 37) and the right triangle with acute angles of 30° and 60° . The functions of these angles are tabulated in appendix III.

b. Trigonometric Functions of 45° . Draw the right triangle ABC (fig. 37) with angle A equal to 45° . Because the acute angles of a right triangle are complementary, angle A plus angle B equals 90° . Thus, angle B is also 45° . Since sides opposite equal angles are equal, side a is equal to side b .

Let $a = 1$ and $b = 1$.

$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 1$$

$$c^2 = 2$$

$$c = \sqrt{2}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$$

$$\tan 45^\circ = \frac{1}{1} = 1$$

$$\cot 45^\circ = \frac{1}{1} = 1$$

$$\sec 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

c. Trigonometric Functions of 30° and 60° . Draw the equilateral triangle ABX (fig. 38). The angles of any equilateral triangle are 60° and the sides are equal (par. 131a). Drop a perpendicular BC to the center of the base AX . Right angles ACB and BCX are formed by the perpendicular and the base. The angles ABC and XBC are 30° angles. Since the sides of the equilateral triangle are equal, the perpendicular bisecting the base makes the base AC of the right triangle ABC one-half the length of the base AX of the equilateral triangle. Thus, the side opposite the right angle in a right triangle

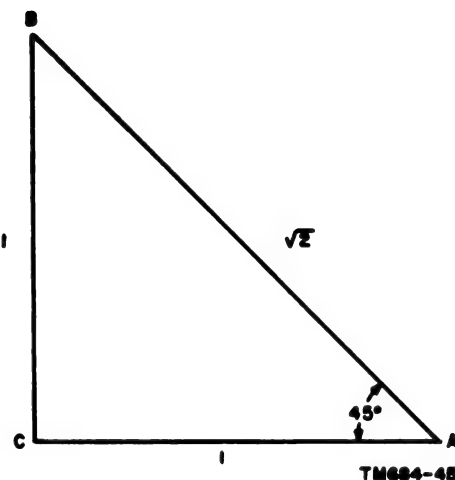


Figure 37. Right isosceles triangle—trigonometric functions of 45° .

is twice the length of the side opposite the 30° angle.

Let $b = 1$ and $c = 2$.

$$a^2 = c^2 - b^2$$

$$a^2 = 4 - 1$$

$$a^2 = 3$$

$$a = \sqrt{3}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\cot 60^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3}$$

$$\sec 60^\circ = \frac{2}{1} = 2$$

$$\csc 60^\circ = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{2}{3}\sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3}$$

$$\cot 30^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\sec 30^\circ = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{2}{3}\sqrt{3}$$

$$\csc 30^\circ = \frac{2}{1} = 2$$

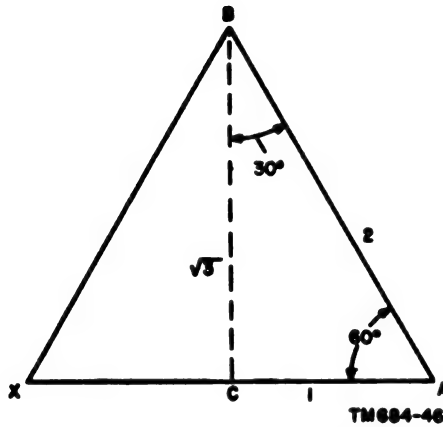


Figure 38. Equilateral right triangle—trigonometric functions of a right triangle with angles of 30° and 60°

151. Solving for Sides of 45° – 45° – 90° or 30° – 60° – 90° Triangles When One Side Is Given

In special cases, right triangles can be solved when only one side is given. These are the 45° – 45° – 90° isosceles triangle and the 30° – 60° – 90° triangle.

Example 1: Solve for the unknown sides of right triangle ABC if angle $A = 60^\circ$ and $b = 4$ inches.

$$\tan 60^\circ = \frac{a}{b} = \frac{a}{4}; \text{ however, } \tan 60^\circ = \sqrt{3}.$$

Therefore,

$$\frac{a}{4} = \frac{\sqrt{3}}{1}$$

$$a = 4\sqrt{3} \text{ inches}$$

$$\cos 60^\circ = \frac{b}{c} = \frac{4}{c}; \text{ however, } \cos 60^\circ = \frac{1}{2}.$$

Therefore,

$$\frac{4}{c} = \frac{1}{2}$$

$$c = 8 \text{ inches}$$

Thus, $a = 4\sqrt{3}$ inches, $b = 4$ inches, $c = 8$ inches.

Example 2: Solve for the unknown sides of right triangle ABC if angle $A = 45^\circ$ and $c = 6$ inches.

$$\sin 45^\circ = \frac{a}{c} = \frac{a}{6}; \text{ however, } \sin 45^\circ = \frac{\sqrt{2}}{2}.$$

Therefore;

$$\frac{a}{6} = \frac{\sqrt{2}}{2}$$

$$2a = 6\sqrt{2}$$

$$a = 3\sqrt{2}$$

$$\cos 45^\circ = \frac{b}{c} = \frac{b}{6}; \text{ however, } \cos 45^\circ = \frac{\sqrt{2}}{2}.$$

Therefore,

$$\frac{b}{6} = \frac{\sqrt{2}}{2}$$

$$2b = 6\sqrt{2}$$

$$b = 3\sqrt{2} \text{ inches}$$

Thus, $a = 3\sqrt{2}$ inches, $b = 3\sqrt{2}$ inches, $c = 6$ inches.

152. Calculations Involving Angles

a. Addition. To add angles, arrange the degrees, minutes, and seconds in separate columns and add each column separately. If the sum of the seconds column is 60 or more, subtract 60 or a multiple of 60 from that column, and add 1 minute or the same multiple of 1 minute to the minutes column. If the sum of the minutes column is 60 or more, subtract 60 from that column and add 1° to the degree column.

Example 1: Add $20^{\circ} 40' 25''$, $8^{\circ} 35' 5''$, and $30^{\circ} 58' 51''$.

$$\begin{array}{r} 20^{\circ} 40' 25'' \\ 8^{\circ} 35' 5'' \\ 30^{\circ} 58' 51'' \\ \hline 58^{\circ} 133' 81'' \end{array}$$

Subtract 60'' from 81'' and add 1' to 133'.

$$\begin{array}{r} 58^{\circ} 133' 81'' \\ + 1' -60'' \\ \hline 58^{\circ} 134' 21'' \end{array}$$

Subtract 120' from 134' and add 2° to 58°.

$$\begin{array}{r} 58^{\circ} 134' 21'' \\ + 2^{\circ} -120' \\ \hline 60^{\circ} 14' 21'' \end{array}$$

Example 2: Add $15^{\circ} 44' 36''$ and $12^{\circ} 38' 35''$.

$$\begin{array}{r} 15^{\circ} 44' 36'' \\ 12^{\circ} 38' 35'' \\ \hline 27^{\circ} 82' 71'' = 27^{\circ} 83' 11'' = 28^{\circ} 23' 11''. \end{array}$$

b. Subtraction. To subtract angles, arrange the degrees, minutes, and seconds in separate columns with the larger angle on top. Then, subtract the individual columns. If the upper number in a column is too small to allow subtraction, one unit must be taken away from the preceding column and 60 units added to the insufficient number to make subtraction possible.

Example 1: Subtract $14^{\circ} 51' 30''$ from $86^{\circ} 45' 10''$.

$$\begin{array}{r} 86^{\circ} 45' 10'' \\ -14^{\circ} 51' 30'' \\ \hline \end{array}$$

Subtraction cannot be performed in either the seconds or minutes columns. Subtract 1' from 45' leaving 44', and add 60'' to 10'' for a total of 70''.

$$\begin{array}{r} 86^{\circ} 44' 70'' \\ -14^{\circ} 51' 30'' \\ \hline \end{array}$$

Subtraction still cannot be performed in the minutes column. Subtract 1° from 86°, leaving 85°, and add 60' to 44' for a total of 104'.

$$\begin{array}{r} 85^{\circ} 104' 70'' \\ -14^{\circ} 51' 30'' \\ \hline 71^{\circ} 53' 40'' \end{array}$$

Example 2: Subtract $10^{\circ} 35' 42''$ from $19^{\circ} 20' 20''$.

$$\begin{array}{r} 19^{\circ} 20' 20'' \\ -10^{\circ} 35' 42'' \\ \hline \end{array}$$

Subtraction cannot be performed in either the minutes or seconds columns. Therefore, change $19^\circ 20' 20''$ to $18^\circ 79' 80''$ and subtract.

$$\begin{array}{r} 18^\circ 79' 80'' \\ -10^\circ 35' 42'' \\ \hline 8^\circ 44' 38'' \end{array}$$

c. **Multiplication.** To multiply an angle by a given number, multiply each column by the number. If the answer in the seconds or minutes column is greater than 60, reduce as in the addition of angles (a above).

Example 1: Multiply $15^\circ 21' 40''$ by 3.

$$\begin{array}{r} 15^\circ 21' 40'' \\ \times 3 \\ \hline 45^\circ 63' 120'' = 45^\circ 65' 0'' = 46^\circ 5' \end{array}$$

Example 2: Multiply $12^\circ 14' 36''$ by 5.

$$\begin{array}{r} 12^\circ 14' 36'' \\ \times 5 \\ \hline 60^\circ 70' 180'' = 60^\circ 73' = 61^\circ 13' \end{array}$$

d. **Division.** To divide an angle by a given number, divide each column by the number (beginning with the degrees column). Change the remainder in degrees, if any, into minutes and add it to the minutes column; then, perform division on the numbers in the minutes column. Change the remainder in minutes, if any, to seconds and add it to the seconds column; then, perform division on the numbers in the seconds column.

Example 1: Divide $71^\circ 22' 21''$ by 3.

$$\begin{array}{r} 23^\circ \quad 47' \quad 27'' \\ \sqrt{71^\circ \quad 22' \quad 21''} \\ \underline{69} \\ 2^\circ = \frac{120'}{2} \\ \underline{142} \\ 141' \\ \underline{1'} = \frac{60''}{1} \\ 81'' \\ \underline{81''} \end{array}$$

Example 2: Divide $166^\circ 17' 36''$ by 6.

$$\begin{array}{r} 27^\circ \quad 42' \quad 56'' \\ \sqrt{166^\circ \quad 17' \quad 36''} \\ \underline{162^\circ} \\ 4^\circ = \frac{240'}{4} \\ \underline{257} \\ 252' \\ \underline{5'} = \frac{300''}{5} \\ 336'' \\ \underline{336''} \end{array}$$

153. Review Problems—Basic Trigonometry

Notes. In the following problems, angle C is the right angle and equals 90° .

a. Find the third side of each of the following right triangles ABC , if two sides are:

- (1) $a = 5, b = 7$
- (2) $b = 18, c = 19$
- (3) $a = 17, c = 43$
- (4) $a = 3b$
- (5) $a = 2m, c = m^2 + 1$

b. Given the right triangle ABC , solve for the trigonometric functions of angle A in each of the following cases:

- (1) $\sin A = \frac{4}{7}$
- (2) $\tan A = \frac{2}{3}$
- (3) $\cos A = \frac{\sqrt{3}}{2}$
- (4) $\csc A = 2.4$
- (5) $\cot A = \frac{1}{y}$
- (6) $\sec A = 2\frac{2}{3}$

c. Solve each of the right triangles (ABC) for the two unknown sides:

- (1) $\sin A = \frac{1}{2}, a = 17$
- (2) $\tan A = \frac{3}{4}, b = 12$

$$(3) \cos A = \frac{4}{5}, c = 20$$

$$(4) \csc A = \frac{15}{7}, c = 37.5$$

$$(5) \cot A = \frac{3}{5}, a = 10$$

$$(6) \sec A = \frac{9}{4}, b = 18.4$$

d. Solve each of the following right triangles (ABC) for the unknown sides:

$$(1) A = 30^\circ, a = 10$$

$$(2) B = 45^\circ, b = 7$$

$$(3) A = 60^\circ, c = 8$$

$$(4) B = 30^\circ, a = 9$$

$$(5) B = 60^\circ, c = 25$$

Section II. NATURAL TRIGONOMETRIC FUNCTIONS

154. Tables and Their Uses

For convenience in computing, trigonometric functions are arranged in tables similar to the tables of logarithms. The ratios themselves are called *natural* sines, cosines, tangents, cotangents, etc. The tables in appendix III give the sines and cosines, the tangents and cotangents, and the secants and cosecants of the angles from 0° to 90° . Angles less than 45° are read down the page; the degrees are at the top of the page and the minutes are on the left. Angles greater than 45° are read up the page; the degrees are at the bottom of the page and the minutes are on the right. As with logarithms, it is necessary to interpolate to find the function of an angle which does not reduce to an integral number of minutes. When working with the sine and tangent, which are increasing in size from 0° to 90° , it is necessary to add in interpolation. When working with the cosine and cotangent, which are decreasing in size from 0° to 90° , it is necessary to subtract.

155. Finding the Function of an Angle From the Table

To find the function of an angle from the table, proceed much the same as with the table of logarithms. This is illustrated by the following examples:

a. When an Angle Is Given in the Table.

Example 1: Find the cosine of $44^\circ 27'$

- Step 1. Turn to the table of sines and cosines.
- Step 2. Locate the 44° column at the top of the page.
- Step 3. Locate the $27'$ at the left of the page.
- Step 4. Read .71386 in the column headed Cosin.
- Step 5. $\cos 44^\circ 27' = .71386$.

Example 2: Find the tangent of $86^\circ 18'$.

- Step 1. Turn to the table of tangents and cotangents.
- Step 2. Locate the 86° column at the bottom of the page.
- Step 3. Locate the $18'$ at the right of the page.
- Step 4. Read 15.4638 in the column headed Tang.
- Step 5. $\tan 86^\circ 18' = 15.4638$.

b. When an Angle Is Not Given in the Table.

Example 1: Find the sine of $32^\circ 46' 36''$.

$$\begin{aligned} \sin 32^\circ 46' &= .54122 \\ \sin 32^\circ 46' 36'' &= .54122 + \text{ } \\ \sin 32^\circ 47' &= .54146 \end{aligned}$$

$$\begin{array}{r}
 \sin 32^\circ 46' 36'' \\
 \underline{-32^\circ 46'} \\
 36''
 \end{array}
 \qquad
 \begin{array}{r}
 32' 47' \\
 \underline{-32^\circ 46'} \\
 1' = 60''
 \end{array}$$

$$\text{ratio} = \frac{36}{60} = \frac{6}{10} = \frac{3}{5}$$

$$.54146 - .54122 = .00024$$

$$\text{ratio} = \frac{x}{.00024}$$

$$\frac{3}{5} = \frac{x}{.00024}$$

$$5x = .00072$$

$$x = .000144$$

$$\sin 32^\circ 46' 36'' = .54122 + .000144 = .54136$$

Example 2: Find the tangent of $56^\circ 43' 27''$.

$$\begin{array}{rcl}
 \tan 56^\circ 43' & = & 1.52332 \\
 \tan 56^\circ 43' 27'' & = & 1.52332 + x \\
 \tan 56^\circ 44' & = & 1.52429
 \end{array}$$

$$\frac{27}{60} \text{ or } \frac{9}{20} = \frac{x}{.00097}$$

$$20x = .00873$$

$$x = .000436 \text{ or } .00044$$

$$\tan 56^\circ 43' 27'' = 1.52332 + .00044 = 1.52376$$

156. Finding an Angle When the Trigonometric Function Is Given

The procedure for using the table to find an angle corresponding to a function is similar to that of logarithms. This is illustrated in the examples in *a* and *b* below.

a. When the Function Is Given in the Table.

Example: Find the value of angle A if $\sin A = .27284$.

Step 1. Find .27284 in the Sine column of the Sines and Cosines table.

Step 2. Reading 15° at the top of the column and $50'$ in the minutes column on the left, angle $A = 15^\circ 50'$.

b. When the Function Is Not Given in the Table.

Example 1: Find the value of angle A when $\sin A = .78112$.

$$\begin{array}{r}
 .78098 = \sin 51^\circ 21' \\
 .78112 = \sin 51^\circ 21' + x \\
 .78116 = \sin 51^\circ 22'
 \end{array}$$

$$\begin{array}{r}
 .78112 \\
 \underline{.78098} \\
 .00014
 \end{array}
 \qquad
 \begin{array}{r}
 .78116 \\
 \underline{.78098} \\
 .00018
 \end{array}$$

$$\text{ratio} = \frac{.00014}{.00018} = \frac{14}{18} = \frac{7}{9}$$

$$51^\circ 22' - 51^\circ 21' = 1' = 60''$$

$$\text{ratio} = \frac{x}{60}$$

$$\frac{7}{9} = \frac{x}{60}$$

$$9x = 420$$

$$x = 47$$

$$\text{angle } A = 51^\circ 21' 47''$$

Examp. Find the value of angle A when $\cot A = .33820$.

$$\begin{aligned} .33848 &= \cot 71^\circ 18' \\ .33820 &= \cot 71^\circ 18' + x \\ .33816 &= \cot 71^\circ 19' \end{aligned}$$

$$\frac{28}{32} \text{ or } \frac{7}{8} = \frac{x}{60}$$

$$8x = 420$$

$$x = 53$$

$$\text{angle } A = 71^\circ 18' 53''$$

157. Solving a Right Triangle When an Acute Angle and the Hypotenuse Are Given

To solve for the unknowns in a right triangle when an acute angle and the hypotenuse are given, proceed as in *a* and *b* below. In both examples, angle C is the right angle; therefore, angle $C = 90^\circ$.

Example 1: Find the unknown sides a and b , and the value of angle B in right triangle ABC (fig. 39) if angle A is $33^\circ 15'$ and the hypotenuse, c is 9 inches.

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B = 180^\circ - \angle A - \angle C$$

$$\angle B = 180^\circ - 33^\circ 15' - 90^\circ$$

$$\angle B = 56^\circ 45'$$

$$\sin A = \frac{a}{c}$$

$$\sin 33^\circ 15' = \frac{a}{9}$$

$$a = 9 \sin 33^\circ 15'$$

$$a = 9 \times .54829 = 4.93461$$

$$a = 4.93461$$

$$\cos A = \frac{b}{c}$$

$$\cos 33^\circ 15' = \frac{b}{9}$$

$$b = 9 \cos 33^\circ 15'$$

$$b = 9 \times .83629$$

$$b = 7.52661$$

$$\text{Therefore, } \angle A = 33^\circ 15'$$

$$\angle B = 56^\circ 45'$$

$$\angle C = 90^\circ$$

$$a = 4.93461 \text{ inches}$$

$$b = 7.52661 \text{ inches}$$

$$c = 9 \text{ inches}$$

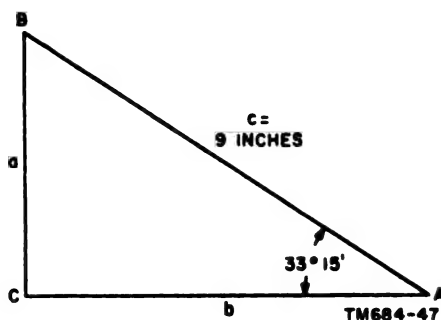


Figure 39. Solving a right triangle when an acute angle ($33^\circ 15'$) and the hypotenuse are given.

Example 2: Solve for the unknown sides a and b , and the value of angle B in right triangle ABC (fig. 40) if angle A is $24^\circ 35' 36''$ and the hypotenuse, c , is 12 inches.

$$\angle B = 180^\circ - \angle A - \angle C$$

$$\angle B = 180^\circ - 24^\circ 35' 36'' - 90^\circ$$

$$\angle B = 65^\circ 24' 24''$$

$$\sin A = \frac{a}{c}$$

$$\sin 24^\circ 35' 36'' = \frac{a}{12}$$

$$a = 12 \sin 24^\circ 35' 36''$$

$$\sin 24^\circ 35' = .41602$$

$$\sin 24^\circ 35' 36'' = .41602 + x$$

$$\sin 24^\circ 36' = .41628$$

$$\frac{36}{60} \text{ or } \frac{3}{5} = \frac{x}{.00026}$$

$$5x = .00078$$

$$x = .00016$$

$$\sin 24^\circ 35' 36'' = .41602 + .00016 = .41618$$

$$a = 12 \times .41618$$

$$a = 4.99416$$

$$\cos A = \frac{b}{c}$$

$$\cos 24^\circ 35' 36'' = \frac{b}{12}$$

$$b = 12 \cos 24^\circ 35' 36''$$

$$\cos 24^\circ 35' = .90936$$

$$\cos 24^\circ 35' 36'' = .90936 - x$$

$$\cos 24^\circ 36' = .90924$$

$$\frac{36}{60} \text{ or } \frac{3}{5} = \frac{x}{.00012}$$

$$5x = .00036$$

$$x = .00007$$

$$\cos 24^\circ 35' 36'' = .90936 - .00007 = .90929$$

$$b = 12 \times .90929$$

$$b = 10.91148$$

Therefore, $\angle A = 24^\circ 35' 36''$

$$a = 4.99416 \text{ inches}$$

$\angle B = 65^\circ 24' 24''$

$$b = 10.91148 \text{ inches}$$

$\angle C = 90^\circ$

$$c = 12 \text{ inches}$$

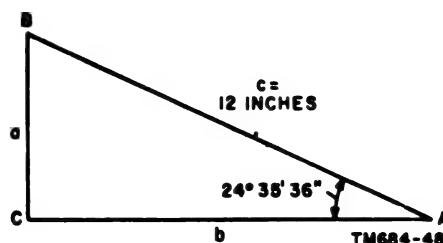


Figure 40. Solving a right triangle when an acute angle ($24^\circ 35' 36''$) and the hypotenuse are given.

158. Solving a Right Triangle When an Acute Angle and the Adjacent Side Are Given

To solve a right triangle when an acute angle and the adjacent side are given, proceed as shown in the example below. Angle C is the right angle.

Example: Find the unknown sides a and c and the value of angle B in the right triangle ABC (fig. 41) if angle A is $37^\circ 42' 42''$ and the side adjacent to angle A is 8 inches.

$$\angle B = 180^\circ - 90^\circ - 37^\circ 42' 42''$$

$$\angle B = 52^\circ 17' 18''$$

$$\cos A = \frac{b}{c}$$

$$\cos 37^\circ 42' 42'' = \frac{8}{c}$$

$$c (\cos 37^\circ 42' 42'') = 8$$

$$\cos 37^\circ 42' = .79122$$

$$\cos 37^\circ 42' 42'' = .79122 - x$$

$$\cos 37^\circ 43' = .79105$$

$$\frac{42}{60} \text{ or } \frac{7}{10} = \frac{x}{.00017}$$

$$10x = .00119$$

$$x = .00012$$

$$\cos 37^\circ 42' 42'' = .79122 - .00012 = .79110$$

$$.79110c = 8$$

$$c = \frac{8}{.79110}$$

$$c = 10.11$$

$$\tan A = \frac{a}{b}$$

$$\tan 37^\circ 42' 42'' = \frac{a}{8}$$

$$a = 8 \tan 37^\circ 42' 42''$$

$$\tan 37^\circ 42' = .77289$$

$$\tan 37^\circ 42' 42'' = .77289 + x$$

$$\tan 37^\circ 43' = .77335$$

$$\frac{42}{60} \text{ or } \frac{7}{10} = \frac{x}{.00046}$$

$$10x = .00322$$

$$x = .00032$$

$$\tan 37^\circ 42' 42'' = .77289 + .00032 = .77321$$

$$a = 8 \times .77321$$

$$a = 6.18568$$

Therefore, $\angle A = 37^\circ 42' 42''$

$$\angle B = 52^\circ 17' 18''$$

$$\angle C = 90^\circ$$

$$a = 6.18568 \text{ inches}$$

$$b = 8 \text{ inches}$$

$$c = 10.11 \text{ inches}$$

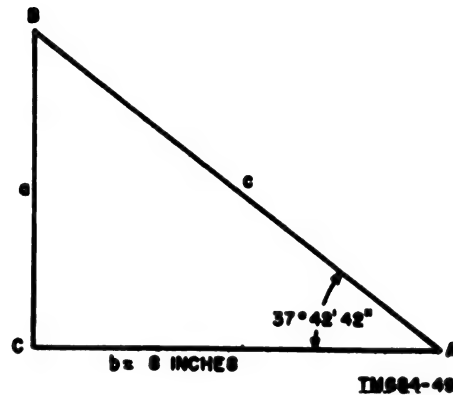


Figure 41. Solving a right triangle when an acute angle and the adjacent side are given.

159. Solving a Right Triangle When Hypotenuse and One Side Are Given

Given the hypotenuse and one other side of a right triangle, solve for the unknown angles and side as illustrated in the example below.

Example: Find the unknown angles A and B , and side c of right triangle ABC (fig. 42) if the hypotenuse is 12 inches and the side opposite angle A is 8 inches.

$$\begin{aligned} b^2 &= c^2 - a^2 \\ b^2 &= 12^2 - 8^2 \\ b^2 &= 144 - 64 \\ b^2 &= 80 \\ b &= \sqrt{80} \\ b &= 8.94 \end{aligned}$$

$$\sin A = \frac{a}{c}$$

$$\sin A = \frac{8}{12} = \frac{2}{3}$$

$$\sin A = .66667$$

$$.66653 = \sin 41^\circ 48'$$

$$.66667 = \sin 41^\circ 48' + x$$

$$.66675 = \sin 41^\circ 49'$$

$$\frac{14}{22} = \frac{x}{60}$$

$$22x = 840$$

$$x = \frac{840}{22} = 38$$

$$.66667 = \sin 41^\circ 48' 38''$$

$$\text{angle } A = 41^\circ 48' 38''$$

$$\angle B = 180^\circ - \angle C - \angle A$$

$$\angle B = 180^\circ - 90^\circ - 41^\circ 48' 38''$$

$$\angle B = 48^\circ 11' 22''$$

$$\text{Therefore, } \angle A = 41^\circ 48' 38''$$

$$\angle B = 48^\circ 11' 22''$$

$$\angle C = 90^\circ$$

$$a = 8 \text{ inches}$$

$$b = 8.94 \text{ inches}$$

$$c = 12 \text{ inches}$$

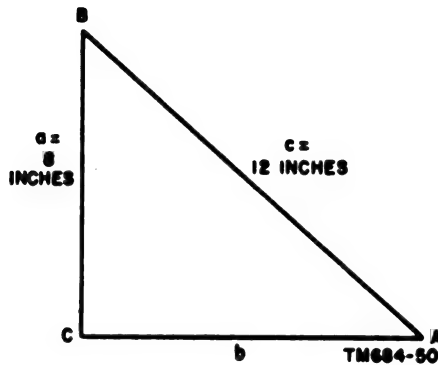


Figure 48. Solving a right triangle, when the hypotenuse and one side are given.

160. Solving a Right Triangle When Two Sides Are Given

When two sides of a right triangle are given, solve for the unknown angles and the hypotenuse as shown in the example below.

Example: Find the unknown angles A and B and side c in right triangle ABC (fig. 48) if side a is 8 inches and side b is 10 inches.

$$c^2 = a^2 + b^2$$

$$c^2 = 64 + 100$$

$$c^2 = 164$$

$$c = \sqrt{164}$$

$$c = 12.8$$

$$\tan A = \frac{a}{b}$$

$$\tan A = \frac{8}{10}$$

$$\tan A = .80000$$

$$.79972 = \tan 38^\circ 39'$$

$$.80000 = \tan 38^\circ 39' + x$$

$$.80020 = \tan 38^\circ 40'$$

$$\frac{28}{48} \text{ or } \frac{7}{12} = \frac{x}{60}$$

$$12x = 420$$

$$x = 35$$

$$.80000 = \tan 38^\circ 39' 35''$$

$$\text{angle } A = 38^\circ 39' 35''$$

$$\angle B = 180^\circ - \angle C - \angle A$$

$$\angle B = 180^\circ - 90^\circ - 38^\circ 39' 35''$$

$$\angle B = 51^\circ 20' 25''$$

$$\text{Therefore, } \angle A = 38^\circ 39' 35''$$

$$\angle B = 51^\circ 20' 25''$$

$$\angle C = 90^\circ$$

$$a = 8 \text{ inches}$$

$$b = 10 \text{ inches}$$

$$c = 12.8 \text{ inches}$$

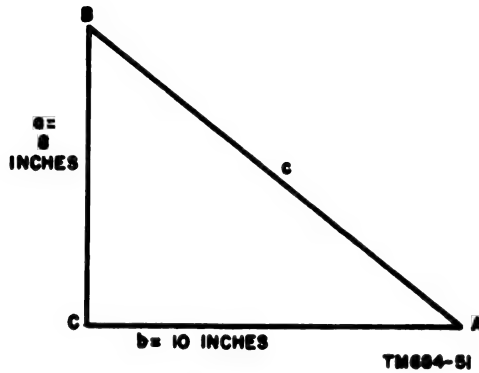


Figure 43. Solving a right triangle when two sides are given.

161. Solving a 30° - 60° - 90° Triangle When One Side Is Given

In a 30° - 60° - 90° triangle, the side opposite the 30° angle is equal to one-half the hypotenuse. Refer to paragraph 150c for the derivation of the trigonometric functions. Solve for the unknown sides as shown in the example below.

Example: Find the unknown sides b and c of 30° - 60° - 90° triangle ABC (fig. 44) if the side opposite the 60° angle is 6 inches.

$$\sin 60^\circ = \frac{\sqrt{3}}{2}; \text{ also, } \sin 60^\circ = \frac{a}{c} = \frac{6}{c}$$

$$\frac{\sqrt{3}}{2} = \frac{6}{c}$$

$$\sqrt{3}c = 12$$

$$c = \frac{12}{\sqrt{3}}$$

Eliminate $\sqrt{3}$ in the denominator by multiplying $\frac{12}{\sqrt{3}}$ by $\frac{\sqrt{3}}{\sqrt{3}}$:

$$c = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{\sqrt{9}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$c = 4\sqrt{3} = 4 \times 1.7321 = 6.9284$$

$$\tan 60^\circ = \frac{\sqrt{3}}{1}; \text{ also, } \tan 60^\circ = \frac{a}{b} = \frac{6}{b}$$

$$\frac{\sqrt{3}}{1} = \frac{6}{b}$$

$$\sqrt{3}b = 6$$

$$b = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{9}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

$$b = 2\sqrt{3} = 2 \times 1.7321 = 3.4642$$

Therefore, $a = 6$ inches

$b = 3.4642$ inches

$c = 6.9284$ inches

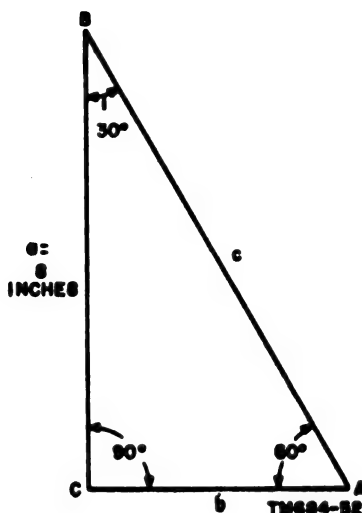


Figure 44. Solving a 30°-60°-90° triangle when one side is given.

162. Solving a 45°-45°-90° Triangle When One Side Is Given

In a 45°-45°-90° triangle, the sides opposite the equal angles are equal. Refer to paragraph 150b for the derivation of the trigonometric functions. Solve for the unknown sides as shown in the example below.

Example: Find the unknown sides a , b , and c of 45°-45°-90° triangle ABC (fig. 45) if the side opposite acute angle A is 5 inches.

$$\sin 45^\circ = \frac{1}{\sqrt{2}}; \text{ also, } \sin A = \frac{a}{c} = \frac{5}{c}$$

$$\frac{1}{\sqrt{2}} = \frac{5}{c}$$

$$c = 5\sqrt{2}$$

$$c = 5 \times 1.4142 = 7.0710$$

$$\tan 45^\circ = \frac{1}{1}; \text{ also, } \tan A = \frac{a}{b} = \frac{5}{b}$$

$$\frac{1}{1} = \frac{5}{b}$$

$$[b = 5]$$

Therefore, $a = 5$ inches

$b = 5$ inches

$c = 7.071$ inches

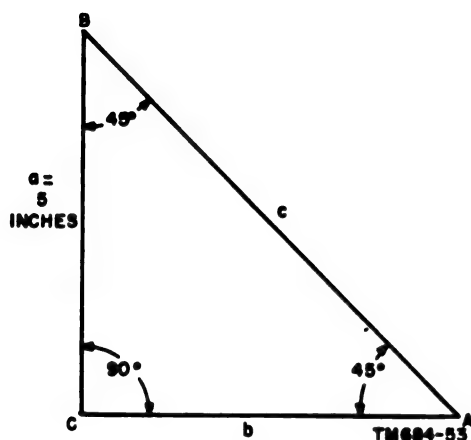


Figure 45. Solving a 45°-45°-90° triangle when one side is given.

163. Angles of Elevation and Depression

When an object is higher than the observer's eye, the angle between the horizontal and the line of sight to the object is called the *angle of elevation* (A, fig. 46). When an object is lower than the observer's eye, the angle between the line of sight to the object and the horizontal is called the *angle of depression* (B, fig. 46).

Example:

A television antenna mast is 450 feet high (fig. 47). Find to the nearest second the angle of elevation to its top at a point 200 feet from the base of the mast.

$$\tan A = \frac{a}{b}$$

$$\tan A = \frac{450}{200}$$

$$\tan A = 2.2500$$

$$2.2496 = \tan 66^\circ 2'$$

$$2.2500 = \tan 66^\circ 2' + x$$

$$2.2513 = \tan 66^\circ 3'$$

$$\frac{4}{17} = \frac{x}{60}$$

$$17x = 240$$

$$x = 14$$

$$2.2500 = \tan 66^\circ 2' 14''$$

$$A = 66^\circ 2' 14''$$

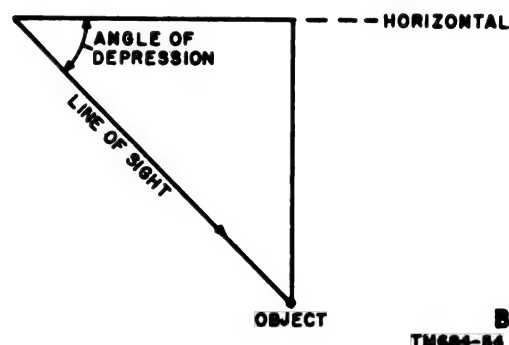
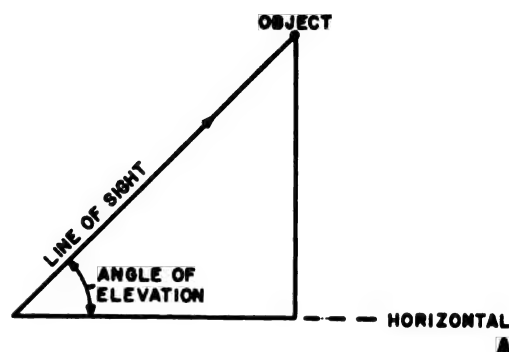


Figure 46. Angles of elevation and depression.

164. Review Problems—Natural Trigonometric Functions

a. Find the sine, cosine, tangent, and cotangent of the following angles:

- (1) $1^\circ 30'$
- (2) $15^\circ 25'$
- (3) $32^\circ 10'$
- (4) $36^\circ 39'$
- (5) $44^\circ 59'$
- (6) $44^\circ 59' 45''$
- (7) $35^\circ 12' 15''$
- (8) $54^\circ 27' 32''$
- (9) $48^\circ 25' 37''$
- (10) $67^\circ 33' 42''$

b. Solve for the values of the following angles in degrees, minutes and seconds:

- (1) $\sin A = .25737$
- (2) $\cot A = .43279$

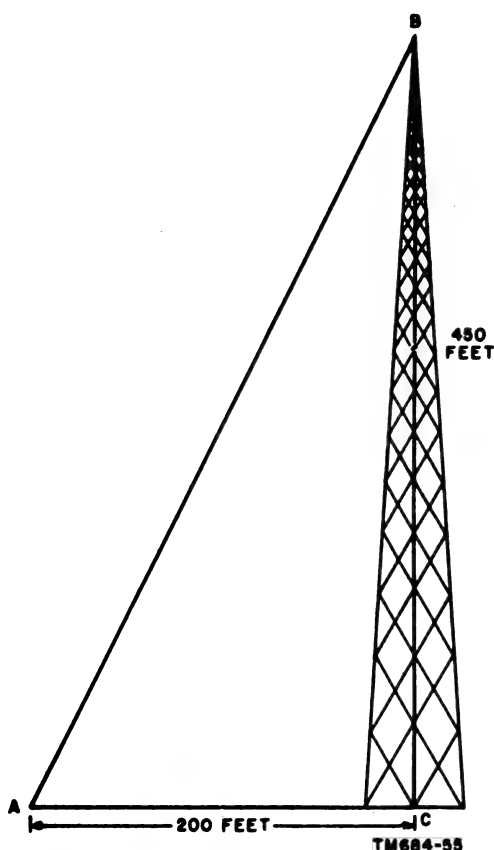


Figure 47. Finding the angle of elevation to top of an antenna mast.

- (3) $\cos A = .94000$
- (4) $\tan A = .47237$
- (5) $\cot A = 1.17529$
- (6) $\cos A = .36243$
- (7) $\sin A = .37778$
- (8) $\tan A = .67676$
- (9) $\tan A = 1.29000$
- (10) $\cot A = .79553$

c. Solve for the following (angle $C = 90^\circ$):

- (1) Angle A in right triangle ABC when $a = 19$ and $c = 27$.
- (2) Side a in right triangle ABC when $A = 37^\circ 15'$ and $c = 17$.
- (3) Side c in right triangle ABC when $A = 42^\circ 37' 15''$ and $a = 22$.
- (4) Side B in right triangle ABC when $A = 37^\circ 45' 42''$ and $c = 25$.

- (5) Side c in right triangle ABC when $A = 14^\circ 35'$ and $b = 12$.
- (6) Angle A in right triangle ABC when $b = 7$ and $c = 12$.
- (7) Side a in right triangle ABC when $A = 47^\circ 22' 52''$ and $b = 31$.
- (8) Side b in right triangle ABC when $A = 56^\circ 31' 25''$ and $a = 25$.
- (9) Angle A in right triangle ABC when $a = 17$ and $b = 23$.
- (10) Side b in right triangle ABC when $A = 7^\circ 32' 54''$ and $a = 17$.
- (11) Side c in right triangle ABC when $a = 15$ and $b = 27$.
- (12) Angle A in right triangle ABC when $a = 15$ and $b = 27$.

d. Solve the following problems:

- (1) Over a distance of 300 feet, the angle of elevation of a road is $8^\circ 24' 30''$. What is the rise in feet?
- (2) The angle of elevation to the top of an antenna mast is $34^\circ 17' 50''$. If the distance from the transit to the center of the mast is 110 feet, how high is the mast? The transit is 5 feet high.
- (3) If a ladder 15 feet long just touches the top of a wall and subtends an angle of $35^\circ 24' 16''$ with the ground, how far is the lower end of the ladder from the wall and how high is the wall?
- (4) A captive balloon is anchored by 950 feet of cable. A man observes that the angle of elevation from his point of observation to the bottom of the balloon is $16^\circ 47' 12''$. How far is he from the balloon anchor?
- (5) An excavation is 33 feet wide. The angle of depression from the top of one side to the bottom of the other side is $19^\circ 34' 24''$. How deep is the excavation?
- (6) The angle of elevation from a given

- point to the top of a tower is $17^{\circ} 37' 15''$. Moving back 40 feet in a direct line, the angle of elevation from this point to the top of the tower is $15^{\circ} 35' 20''$. Find the height of the tower.
- (7) To determine the height of a tower, two sights are taken on a straight line perpendicular to the tower. If the distance between the points of observation is 60 feet and the angles of elevation are $32^{\circ} 30' 15''$ and $28^{\circ} 15' 30''$, respectively, what is the height of the tower?
- (8) From a point in an open field a man sights on two mileposts along the side of a highway. The angles formed by an imaginary line perpendicular to the highway and the sights on the mileposts are $33^{\circ} 20'$ and $39^{\circ} 17' 30''$. How far is the man from the closest point on the highway?
- (9) An airplane is flying between two towns at an altitude of 5,000 feet. Measured with respect to the horizontal, at a given moment, the angle to the outskirts of one town is $50^{\circ} 26' 14''$, while the angle to the outskirts of the other town is $64^{\circ} 44' 12''$. How far apart, in a direct line, are the two towns?
- (10) A radio antenna on top of a building is 10 feet high. The angle of elevation to the base of the pole is $37^{\circ} 17' 20''$; the angle of elevation to the top of the antenna is $40^{\circ} 30' 15''$. How high is the building?
- (11) In a 45° - 45° - 90° right triangle the hypotenuse is 2 inches long. Find the length of the other two sides.
- (12) In a 30° - 60° - 90° right triangle the hypotenuse is 6 inches long. Find the length of the other two sides.

Section III. TRIGONOMETRIC LAWS

165. Solving Oblique Triangles

An oblique triangle is one in which one of the angles is a right angle. The formulas in this section are used primarily to solve oblique triangles, but may also be used to solve right triangles. In the solution of triangles by trigonometric laws, the four following cases arise:

- When any side and any two angles are given.
- When any two sides and the angle opposite one of them are given.

c. When any two sides and the angle included between them are given.

d. When the three sides are given.

166. Law of Sines

In any triangle, the sides are proportional to the sines of the opposite angles.

$$\text{Thus, } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

a. *Two Angles and One Side Given.*

Example: Solve for the unknowns in oblique triangle ABC (fig. 48) when angle $A = 35^{\circ} 47' 36''$, angle $B = 68^{\circ} 42' 27''$, and the side opposite angle A is 15 inches.

$$\angle C = 180^{\circ} - \angle A - \angle B$$

$$\angle C = 180^{\circ} - 35^{\circ} 47' 36'' - 68^{\circ} 42' 27''$$

$$\angle C = 75^{\circ} 29' 57''$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$b \sin A = a \sin B$$

$$b = \frac{a \sin B}{\sin A}$$

$$b = \frac{15 \sin 68^{\circ} 42' 27''}{\sin 35^{\circ} 47' 36''}$$

$$\sin 68^\circ 42' = .98169$$

$$\sin 68^\circ 42' 27'' = .98169 + x$$

$$\sin 68^\circ 48' = .98180$$

$$\frac{27}{60} \text{ or } \frac{9}{20} = \frac{x}{.00011}$$

$$20x = .00099$$

$$x = .000049 = .00005$$

$$\sin 68^\circ 42' 27'' = .98169 + .00005 = .98174$$

$$\sin 35^\circ 47' = .58472$$

$$\sin 35^\circ 47' 36'' = .58472 + x$$

$$\sin 35^\circ 48' = .58496$$

$$\frac{36}{60} \text{ or } \frac{3}{5} = \frac{x}{.00024}$$

$$5x = .00072$$

$$x = .00014$$

$$\sin 35^\circ 47' 36'' = .58472 + .00014 = .58486$$

$$b = \frac{15 \times .98174}{.58486}$$

$$b = \frac{13.97610}{.58486}$$

$$b = 23.89$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c \sin A = a \sin C$$

$$c = \frac{a \sin C}{\sin A}$$

$$c = \frac{15 \sin 75^\circ 29' 57''}{\sin 35^\circ 47' 36''}$$

$$\sin 75^\circ 29' = .96807$$

$$\sin 75^\circ 29' 57'' = .96807 + x$$

$$\sin 75^\circ 30' = .96815$$

$$\frac{57}{60} \text{ or } \frac{19}{20} = \frac{x}{.00008}$$

$$20x = .00152$$

$$x = .000076 = .00008$$

$$\sin 75^\circ 29' 57'' = .96807 + .00008 = .96815$$

$$c = \frac{15 \times .96815}{.58486}$$

$$c = \frac{14.52225}{.58486}$$

$$c = 24.83$$

Therefore, $\angle A = 35^\circ 47' 36''$

$\angle B = 68^\circ 42' 27''$

$\angle C = 75^\circ 29' 57''$

$a = 15$ inches.

$b = 23.89$ inches

$c = 24.83$ inches

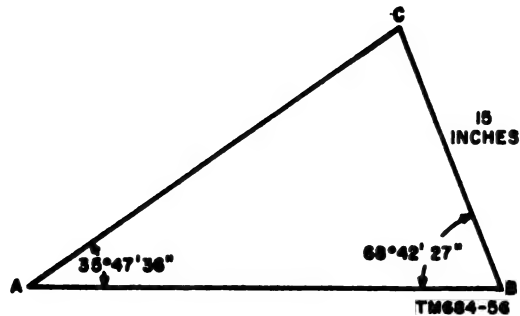


Figure 48. Solving an oblique triangle by the law of sines when two angles and a side are given.

b. Two Sides and One Angle Given.

Example: Find the unknowns in oblique triangle ABC (fig. 49) when angle $A = 53^\circ 35' 40''$, the side opposite angle A is 10 inches, and the side opposite angle B is 12 inches.

$$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} \\ a \sin B &= b \sin A \\ \sin B &= \frac{b \sin A}{a} \\ \sin B &= \frac{12 \sin 53^\circ 35' 40''}{10} \\ \sin 53^\circ 35' &= .80472 \\ \sin 53^\circ 35' 40'' &= .80472 + x \\ \sin 53^\circ 36' &= .80489 \\ \frac{40}{60} \text{ or } \frac{2}{3} &= \frac{x}{.00017} \\ 8x &= .00084 \\ x &= .00011 \\ \sin 53^\circ 35' 40'' &= .80472 + .00011 = .80483 \\ \sin B &= \frac{12 \times .80483}{10} \\ \sin B &= \frac{4.82898}{5} \\ \sin B &= .965796 = .96580 \\ .96578 &= \sin 74^\circ 58' \\ .96580 &= \sin 74^\circ 58' + x \\ .96585 &= \sin 74^\circ 59' \\ \frac{2}{7} &= \frac{x}{60} \\ 7x &= 120 \\ x &= 17 \\ .96580 &= \sin 74^\circ 58' 17'' \\ \angle B &= 74^\circ 58' 17'' \\ \angle C &= 180^\circ - \angle A - \angle B \end{aligned}$$

$$\angle C = 180^\circ - 58^\circ 35' 40'' - 74^\circ 58' 17''$$

$$\angle C = 51^\circ 26' 3''$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c \sin A = a \sin C$$

$$c = \frac{a \sin C}{\sin A}$$

$$c = \frac{10 \sin 51^\circ 26' 3''}{\sin 58^\circ 35' 40''}$$

$$\sin 51^\circ 26' = .78188$$

$$\sin 51^\circ 26' 3'' = .78188 + x$$

$$\sin 51^\circ 27' = .78206$$

$$\frac{3}{60} \text{ or } \frac{1}{20} = \frac{x}{.00018}$$

$$20x = .00018$$

$$x = .000009 = .00001$$

$$\sin 51^\circ 26' 3'' = .78188 + .00001 = .78189$$

$$c = \frac{10 \times .78189}{.80488}$$

$$c = \frac{7.8189}{.80488}$$

$$c = 9.71$$

Therefore, $\angle A = 58^\circ 35' 40''$

$\angle B = 74^\circ 58' 17''$

$\angle C = 51^\circ 26' 3''$

$a = 10$ inches

$b = 12$ inches

$c = 9.71$ inches

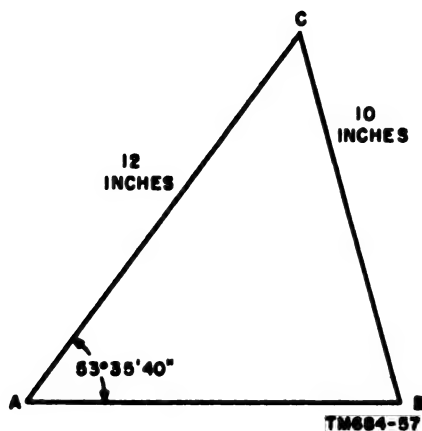


Figure 49. Solving an oblique triangle by the law of sines when two sides and an angle are given.

167. Law of Cosines

In any triangle, the square of any side equals the sum of the squares of the other two sides minus twice the product of these two sides times the cosine of the angle between them.

Thus, $a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

Example: Find the unknowns in oblique triangle ABC (fig. 50) when angle $C = 56^\circ 45' 24''$, the side opposite angle A is 6 inches, and the side opposite angle B is 8 inches.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 6^2 + 8^2 - 2(6)(8) \cos 56^\circ 45' 24''$$

$$c^2 = 36 + 64 - 96 \cos 56^\circ 45' 24''$$

$$c^2 = 100 - 96 \cos 56^\circ 45' 24''$$

$$\cos 56^\circ 45' = .54829$$

$$\cos 56^\circ 45' 24'' = .54829 - x$$

$$\cos 56^\circ 46' = .54805$$

$$\frac{24}{60} \text{ or } \frac{2}{5} = \frac{x}{.00024}$$

$$5x = .00048$$

$$x = .000096 \text{ or } .00010$$

$$\cos 56^\circ 45' 24'' = .54829 - .00010 = .54819$$

$$c^2 = 100 - 96(.54819)$$

$$c^2 = 100 - 52.62624$$

$$c^2 = 47.37376$$

$$c = \sqrt{47.37376}$$

$$c = 6.882$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c \sin A = a \sin C$$

$$\sin A = \frac{a \sin C}{c}$$

$$\sin A = \frac{6 \sin 56^\circ 45' 24''}{6.882}$$

$$\sin 56^\circ 45' = .83629$$

$$\sin 56^\circ 45' 24'' = .83629 + x$$

$$\sin 56^\circ 46' = .83645$$

$$\frac{24}{60} \text{ or } \frac{2}{5} = \frac{x}{.00016}$$

$$5x = .00082$$

$$x = .000064 = .00006$$

$$\sin 56^\circ 45' 24'' = .83629 + .00006 = .83635$$

$$\sin A = \frac{6(.83635)}{6.882}$$

$$\sin A = \frac{5.01810}{6.882}$$

$$\sin A = .72916$$

$$.72897 = \sin 46^\circ 48'$$

$$.72916 = \sin 46^\circ 48' + x$$

$$.72917 = \sin 46^\circ 49'$$

$$\frac{19}{20} = \frac{x}{60}$$

$$20x = 1140$$

$$x = 57$$

$$.72917 = \sin 46^\circ 48' 57''$$

$$\angle A = 46^\circ 48' 57''$$

$$\angle B = 180^\circ - \angle C - \angle A$$

$$\angle B = 180^\circ - 56^\circ 45' 24'' - 46^\circ 48' 57''$$

$$\angle B = 76^\circ 25' 39''$$

$$\text{Therefore, } \angle A = 46^\circ 48' 57''$$

$$\angle B = 76^\circ 25' 39''$$

$$\angle C = 56^\circ 45' 24''$$

$$a = 6 \text{ inches}$$

$$b = 8 \text{ inches}$$

$$c = 6.882 \text{ inches}$$

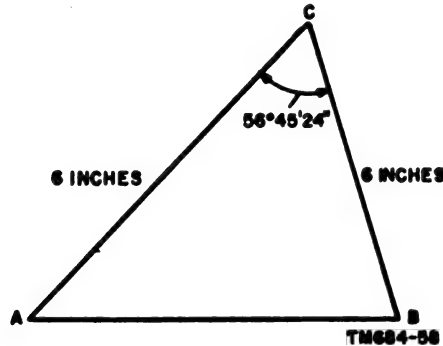


Figure 50. Solving an oblique triangle by the law of cosines when an angle and two sides are given.

168. Law of Tangents

The law of tangents is expressed by the formula $\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}$ where a and b are any two sides and A and B are the angles opposite these sides.

Example: Find the unknowns in oblique triangle ABC (fig. 51) when two sides of the triangle are 9 and 11 inches, respectively, and angle C , the angle included between these two sides, is $40^\circ 40' 40''$.

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle B + 40^\circ 40' 40'' = 180^\circ$$

$$\angle A + \angle B = 180^\circ - 40^\circ 40' 40''$$

$$\angle A + \angle B = 139^\circ 19' 20''$$

$$\frac{1}{2}(\angle A + \angle B) = \frac{139^\circ 19' 20''}{2}$$

$$\frac{1}{2}(\angle A + \angle B) = 69^\circ 39' 40''$$

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(\angle A - \angle B)}{\tan \frac{1}{2}(\angle A + \angle B)}$$

$$\frac{11-9}{11+9} \text{ or } \frac{2}{20} = \frac{\tan \frac{1}{2}(\angle A - \angle B)}{\tan 69^\circ 39' 40''}$$

$$20 \tan \frac{1}{2}(\angle A - \angle B) = 2 \tan 69^\circ 39' 40''$$

$$10 \tan \frac{1}{2}(\angle A - \angle B) = \tan 69^\circ 39' 40''$$

$$\tan \frac{1}{2}(\angle A - \angle B) = \frac{\tan 69^\circ 39' 40''}{10}$$

$$\tan 69^\circ 39' = 2.69612$$

$$\tan 69^\circ 39' 40'' = 2.69612 + x$$

$$\tan 69^\circ 40' = 2.69853$$

$$\frac{40}{60} \text{ or } \frac{2}{3} = \frac{x}{.00241}$$

$$3x = .00482$$

$$x = .00161$$

$$\tan 69^\circ 39' 40'' = 2.69612 + .00161 = 2.69773$$

$$\tan \frac{1}{2}(A - B) = \frac{2.69773}{10}$$

$$\tan \frac{1}{2}(A - B) = .26977$$

$$.26951 = \tan 15^\circ 5'$$

$$.26977 = \tan 15^\circ 5' + x$$

$$.26982 = \tan 15^\circ 6'$$

$$\frac{26}{31} = \frac{x}{60}$$

$$31x = 1560$$

$$x = 50$$

$$.26977 = \tan 15^\circ 5' 50''$$

$$\frac{1}{2}(A - B) = 15^\circ 5' 50''$$

$$\frac{1}{2}(A + B) = \frac{1}{2}A + \frac{1}{2}B = 69^\circ 39' 40''$$

$$\frac{1}{2}(A - B) = \frac{1}{2}A - \frac{1}{2}B = 15^\circ 5' 50''$$

$$\begin{array}{r} \text{(add)} \\ \hline A = 84^\circ 44' 90'' \end{array}$$

$$\angle A = 84^\circ 45' 30''$$

$$\frac{1}{2}(A + B) = \frac{1}{2}A + \frac{1}{2}B = 69^\circ 38' 100''$$

$$\frac{1}{2}(A - B) = \frac{1}{2}A - \frac{1}{2}B = 15^\circ 5' 50''$$

$$\begin{array}{r} \text{(subtract)} \\ \hline B = 54^\circ 33' 50'' \end{array}$$

$$\angle B = 54^\circ 33' 50''$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c \sin A = a \sin C$$

$$c = \frac{a \sin C}{\sin A}$$

$$c = \frac{11 \sin 40^\circ 40' 40''}{\sin 84^\circ 45' 30''}$$

$$\sin 40^\circ 40' = .65166$$

$$\sin 40^\circ 40' 40'' = .65166 + x$$

$$\sin 40^\circ 40' 41'' = .65188$$

$$\frac{40}{60} \text{ or } \frac{2}{3} = \frac{x}{.00022}$$

$$3x = .00044$$

$$x = .000146 = .00015$$

$$\sin 40^\circ 40' 40'' = .65166 + .00015 = .65181$$

$$\sin 84^\circ 45' = .99580$$

$$\sin 84^\circ 45' 30'' = .99580 + x$$

$$\sin 84^\circ 46' = .99583$$

$$\frac{30}{60} \text{ or } \frac{1}{2} = \frac{x}{.00003}$$

$$2x = .00003$$

$$x = .000015 = .00002$$

$$\sin 84^\circ 45' 30'' = .99580 + .00002 = .99582$$

$$c = \frac{11 \sin 40^\circ 40' 40''}{\sin 84^\circ 45' 30''}$$

$$c = \frac{11 \times .65181}{.99582}$$

$$c = \frac{7.16991}{.99582}$$

$$c = 7.2$$

Therefore, $\angle A = 84^\circ 45' 30''$ $a = 11$ inches
 $\angle B = 54^\circ 33' 50''$ $b = 9$ inches
 $\angle C = 40^\circ 40' 40''$ $c = 7.2$ inches

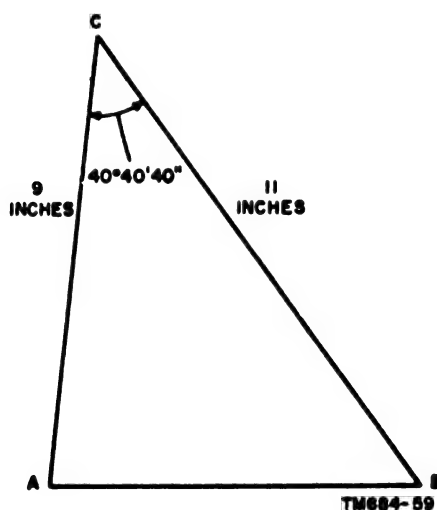


Figure 51. Solving an oblique triangle by the law of tangents when an angle and two sides are given.

169. Finding an Angle When Three Sides Are Given

The following formulas are used to find the angles of a triangle when three sides of the triangle are given:

$$\sin \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\sin \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}}$$

$$\sin \frac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

In these formulas, a , b , and c are the sides of the triangle, and $s = \frac{1}{2}(a + b + c)$.

Example: Find the angles of an oblique triangle if $a = 5$ inches, $b = 8$ inches, and $c = 11$ inches.

$$s = \frac{1}{2}(a + b + c)$$

$$s = \frac{1}{2}(5 + 8 + 11)$$

$$s = \frac{1}{2}(24)$$

$$s = 12$$

$$\sin \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\sin \frac{1}{2}A = \sqrt{\frac{(12-8)(12-11)}{(8)(11)}}$$

$$\sin \frac{1}{2}A = \sqrt{\frac{(4)(1)}{88}}$$

$$\sin \frac{1}{2}A = \sqrt{\frac{4}{88}} = \sqrt{\frac{1}{22}}$$

$$\sin \frac{1}{2}A = \sqrt{.0454545}$$

$$\begin{aligned}
\sin \frac{1}{2}A &= .21819 \\
.21303 &= \sin 12^\circ 18' \\
.21319 &= \sin 12^\circ 18' + x \\
.21331 &= \sin 12^\circ 19' \\
\frac{16}{28} \text{ or } \frac{4}{7} &= \frac{x}{60} \\
7x &= 240 \\
x &= 34 \\
.21319 &= \sin 12^\circ 18' 34'' \\
\frac{1}{2}A &= 12^\circ 18' 34'' \\
\angle A &= 24^\circ 36' 68'' \text{ or } 24^\circ 37' 8'' \\
\sin \frac{1}{2}B &= \sqrt{\frac{(s-a)(s-c)}{ac}} \\
\sin \frac{1}{2}B &= \sqrt{\frac{(12-5)(12-11)}{(5)(11)}} \\
\sin \frac{1}{2}B &= \sqrt{\frac{(7)(1)}{55}} \\
\sin \frac{1}{2}B &= \sqrt{\frac{7}{55}} \\
\sin \frac{1}{2}B &= \sqrt{.1272727} \\
\sin \frac{1}{2}B &= .35675 \\
.35674 &= \sin 20^\circ 54' \\
.35675 &= \sin 20^\circ 54' + x \\
.35701 &= \sin 20^\circ 55' \\
\frac{1}{27} &= \frac{x}{60} \\
27x &= 60 \\
x &= 2 \\
.35675 &= \sin 20^\circ 54' 2'' \\
\frac{1}{2}B &= 20^\circ 54' 2'' \\
\angle B &= 40^\circ 108' 4'' \text{ or } 41^\circ 48' 4'' \\
\angle C &= 180^\circ - \angle A - \angle B \\
\angle C &= 180^\circ - 24^\circ 37' 8'' - 41^\circ 48' 4'' \\
\angle C &= 180^\circ - 66^\circ 25' 12'' \\
\angle C &= 113^\circ 34' 48'' \\
\text{Therefore, } \angle A &= 24^\circ 37' 8'' \\
\angle B &= 41^\circ 48' 4'' \\
\angle C &= 113^\circ 34' 48''
\end{aligned}$$

170. Finding the Area of a Triangle When Two Sides and the Included Angle Are Given

The formula for finding the area of a triangle when two sides and the included angle are given is $S = \frac{1}{2}ab \sin C$ where S is the area of the triangle, a and b are the given sides, and C is the included angle.

Example: Find the area of oblique triangle ABC (fig. 52) when two sides are 7 and 8 inches, respectively, and the included angle is $50^\circ 50' 50''$.

$$\begin{aligned}
S &= \frac{1}{2}ab \sin C \\
S &= \frac{1}{2} \times 7 \times 8 \times \sin 50^\circ 50' 50'' \\
\sin 50^\circ 50' &= .77531 \\
\sin 50^\circ 50' 50'' &= .77531 + x
\end{aligned}$$

$$\begin{aligned}
 \sin 50^\circ 51' &= .77550 \\
 \frac{50}{60} \text{ or } \frac{5}{6} &= \frac{x}{.00019} \\
 6x &= .00095 \\
 x &= .00016 \\
 \sin 50^\circ 50' 50'' &= .77581 + .00016 = .77547 \\
 S &= \frac{1}{2} \times 7 \times 8 \times .77547 = 21.71316 \\
 S &= 21.71316 \text{ square inches}
 \end{aligned}$$

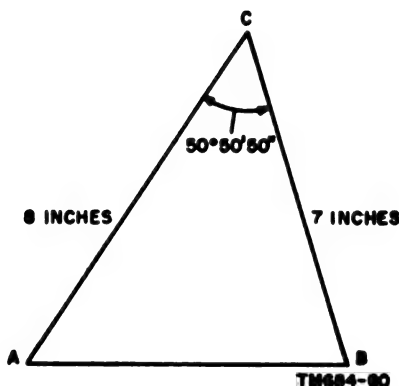


Figure 58. Solving for the area of an oblique triangle when two sides and the included angle are given.

171. Finding the Area of a Triangle When Two Angles and a Side Are Given

The formula for finding the area of a triangle when two angles and a side are given is $S = \frac{a^2 \sin B \sin C}{2 \sin A}$ where S is the area of the triangle, B and C are the given angles, and a is the given side.

Example: Find the area of oblique triangle ABC (fig. 53) when the two angles are $38^\circ 42' 48''$ and $68^\circ 52' 42''$ and the side is 10 inches.

$$\begin{aligned}
 \angle A &= 180^\circ - \angle B - \angle C \\
 \angle A &= 180^\circ - 38^\circ 42' 48'' - 68^\circ 52' 42'' \\
 \angle A &= 180^\circ - 107^\circ 35' 30'' \\
 \angle A &= 72^\circ 24' 30'' \\
 S &= \frac{a^2 \sin B \sin C}{2 \sin A} \\
 S &= \frac{10^2 \sin 38^\circ 42' 48'' \sin 68^\circ 52' 42''}{2 \sin 72^\circ 24' 30''} \\
 \sin 38^\circ 42' &= .62524 \\
 \sin 38^\circ 42' 48'' &= .62524 + x \\
 \sin 38^\circ 43' &= .62547 \\
 \frac{48}{60} \text{ or } \frac{4}{5} &= \frac{x}{.00023} \\
 5x &= .00092 \\
 x &= .00018 \\
 \sin 38^\circ 42' 48'' &= .62524 + .00018 = .62542 \\
 \sin 68^\circ 52' &= .93274 \\
 \sin 68^\circ 52' 42'' &= .93274 + x
 \end{aligned}$$

$$\begin{aligned}
\sin 68^\circ 53' &= .93285 \\
\frac{42}{60} \text{ or } \frac{7}{10} &= \frac{x}{.00011} \\
10x &= .00077 \\
x &= .000077 \text{ or } .00008 \\
\sin 68^\circ 53' 42'' &= .93274 + .00008 = .93282 \\
\sin 72^\circ 24' &= .95319 \\
\sin 72^\circ 24' 30'' &= .95319 + x \\
\sin 72^\circ 25' &= .95328 \\
\frac{30}{60} \text{ or } \frac{1}{2} &= \frac{x}{.00009} \\
2x &= .00009 \\
x &= .000045 \text{ or } .00005 \\
\sin 72^\circ 24' 30'' &= .95319 + .00005 = .95324 \\
S &= \frac{100 \times .62542 \times .93282}{2 \times .95324} \\
S &= \frac{50 \times .62542 \times .93282}{.95324} \\
S &= \log 50 + \log .62542 + \log .93282 - \log .95324 \\
\log 50 &= 1.6990 \\
\log .62500 &= 9.7959-10 \\
\log .62542 &= 9.7959-10 + x \\
\log .62600 &= 9.7966-10 \\
\frac{42}{100} &= \frac{x}{.0007} \\
100x &= .0294 \\
x &= .000294 \text{ or } .0003 \\
\log .62542 &= 9.7959-10 + .0003 = 9.7962-10 \\
\log .93200 &= 9.9694-10 \\
\log .93282 &= 9.9694-10 + x \\
\log .93300 &= 9.9699-10 \\
\frac{82}{100} &= \frac{x}{.0005} \\
100x &= .0410 \\
x &= .00041 \text{ or } .0004 \\
\log .93282 &= 9.9694-10 + .0004 = 9.9698-10 \\
\log .95300 &= 9.9791-10 \\
\log .95324 &= 9.9791-10 + x \\
\log .95400 &= 9.9795-10 \\
\frac{24}{100} &= \frac{x}{.0004} \\
100x &= .0096 \\
x &= .000096 \text{ or } .0001 \\
\log .95324 &= 9.9791-10 + .0001 = 9.9792-10 \\
S &= 1.6990 + 9.7962-10 + 9.9698-10 - 9.9792-10 \\
&\quad 1.6990 \\
&\quad 9.7962-10 \\
&+ 9.9698-10 \\
&\quad 21.4650-20 \\
&- 9.9792-10 \\
&\quad 11.4858-10 \text{ or } 1.4858
\end{aligned}$$

$$\begin{aligned}
\text{antilog } 1.4857 &= 30.6 \\
\text{antilog } 1.4858 &= 30.6 + x \\
\text{antilog } 1.4871 &= 30.7 \\
\frac{1}{14} &= \frac{x}{.1} \\
14x &= .1 \\
x &= .007 \\
\text{antilog } 1.4858 &= 30.6 + .007 = 30.607 \\
S &= 30.607 \text{ square inches}
\end{aligned}$$

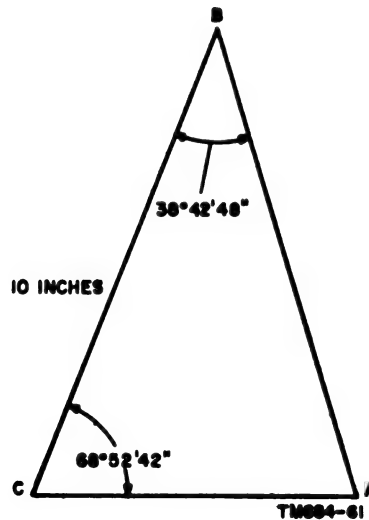


Figure 59. Solving for the area of an oblique triangle when two angles and a side are given.

172. Finding the Area of Triangle When Three Sides Are Given

To find the area of triangle when three sides are given, use the formula

$$S = \sqrt{s(s-a)(s-b)(s-c)}$$

where a , b , and c are the sides of the triangle and $s = \frac{1}{2}(a + b + c)$.

Example: Find the area of an oblique triangle when the sides are 8, 11, and 15 inches, respectively.

$$\begin{aligned}
s &= \frac{1}{2}(a + b + c) \\
s &= \frac{1}{2}(8 + 11 + 15) \\
s &= \frac{1}{2}(34) \\
s &= 17 \\
S &= \sqrt{s(s-a)(s-b)(s-c)} \\
S &= \sqrt{17(17-8)(17-11)(17-15)} \\
S &= \sqrt{17(9)(6)(2)} \\
S &= \sqrt{1836} \\
S &= 42.84 \text{ square inches}
\end{aligned}$$

173. Review Problems—Trigonometric Laws

a. In an oblique triangle ABC , angle $A = 42^\circ 15' 12''$, angle $B = 75^\circ 28' 10''$, and side b measures 21 inches. Solve the triangle for angle C and side a .

b. In an oblique triangle ABC , angle $C = 52^\circ 30'$, side $b = 45$ inches, and side $c = 38$ inches. Solve for angle B .

c. In an oblique triangle ABC , sides a , b , and c opposite angles A , B , and C have lengths of 9, 16, and 21 inches, respectively. Find the three angles of the triangle.

d. In an oblique triangle where a and b are any two sides and A and B are the angles opposite these sides, angle $C = 57^\circ 20' 45''$, $a =$

9.78 inches, and $b = 6.47$ inches. Find angles A and B .

e. The three sides of a triangle are 40, 37, and 13 inches, respectively. Find the area of the triangle.

f. Two sides of an oblique triangle measure 12 and 18 feet, respectively. The angle between the two sides is 115° . Find the area of the triangle.

g. In a triangle ABC , angle $A = 30^\circ$ and angle $B = 60^\circ$. The side opposite angle $C = 16$ inches. Find the area of the triangle.

h. In an oblique triangle ABC , angle $C = 62^\circ 50'$. The side opposite angle A measures 9.65 inches, and the side opposite angle B measures 17.85 inches. Find angles A and B and the length of the side opposite angle C .

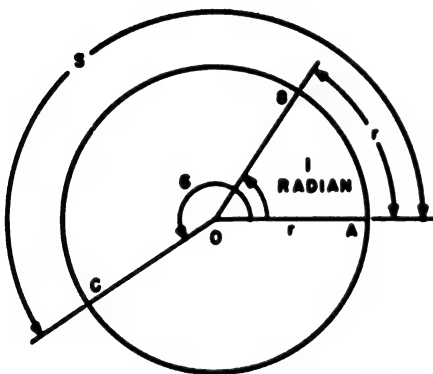
CHAPTER 11

RADIANS

174. Angular Measurement Using Radians

a. *Definition.* A radian is a unit of angular measurement equal to that angle which, when its vertex is upon the center of a circle, intercepts an arc that is equal in length to the radius of the circle. Thus, in figure 54, central angle AOB is equal to 1 radian because arc AB is equal to radius OA .

- (1) The system that makes use of the radian is called the *natural system* of angular measurement because it has no arbitrary unit, such as the degree, but is founded upon the observation that the absolute size of any angle is the ratio of its arc to the radius of that arc. Where the arc and radius are equal, the ratio is 1, and this unit is the radian.
- (2) The natural system of angular measurement—also called the circular system and the radian system—is used



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Figure 54. The radian or circular system of measurement.

extensively in electrical formulas (part II).

b. *Finding Any Angle.* To find any angle, such as angle AOC in figure 54, when the length of arc AB is known, determine the number of times that radius r will go into arc length ABC , thus determining the number of radians in the angle.

Thus,

$$\text{Angle} = \frac{\text{arc}}{\text{radius}}$$

or, if angle AOC is denoted by the Greek letter θ (Theta) and arc ABC by s ,

$$\theta = \frac{s}{r} \text{ radians}$$

Example: A circle has a radius of 6 inches. Find the angle subtended at the center of the circle by an arc 9 inches in length.

$$\begin{aligned} \theta &= \frac{s}{r} \\ &= \frac{9}{6} \\ &= 1.5 \text{ radians} \end{aligned}$$

c. *Finding Length of Arc.* To find the length of an arc intercepted by a central angle when the radius of the circle and the number of radians in the angle are known, use the formula in b above in the form—

$$s = r\theta$$

Example: A circle has a radius of 5 feet. How long is the arc intercepted by a central angle of 1.5 radians?

$$\begin{aligned} s &= r\theta \\ &= 5 \times 1.5 \\ &= 7.5 \text{ feet} \end{aligned}$$

175. The Relation Between Degrees and Radians

a. General. It is often necessary to convert an angle from degrees to radians or from radians to degrees. If the angle is one complete revolution, the arc is one complete circumference of a circle; thus, it is 2π times the radius. Therefore, the angle is equal to $2\pi r$ divided by r —that is, 2π radians ($\pi = 3.1416$).

Therefore,

$$1 \text{ revolution} = 2\pi \text{ radians}$$

also

$$1 \text{ revolution} = 360^\circ$$

Thus,

$$2\pi \text{ radians} = 360^\circ$$

$$1 \text{ radian} = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi} = 57.29578^\circ$$

and since

$$360^\circ = 2\pi \text{ radians}$$

$$1^\circ = \frac{2\pi}{360} = \frac{\pi}{180} = 0.017453 \text{ radians}$$

To change radians to degrees, accurate to seconds, use figures accurate to at least five decimal places.

b. Changing Degrees to Radians and Radians to Degrees.

Example 1: Change 2.74 radians to degrees, minutes, and seconds.

$$1 \text{ radian} = 57.29578^\circ$$

$$2.74 \text{ radians} = 2.74(57.29578)$$

$$= 156.99044^\circ$$

$$1^\circ = 60'$$

$$.99044^\circ = .99044(60)'$$

$$= 59.4264'$$

$$.4264' = .4264(60)''$$

$$= 25.5''$$

$$2.74 \text{ radians} = 156^\circ 59' 25.5''$$

Example 2: Change $57^\circ 15' 18''$ to radians.

Step 1. Change the minutes and seconds to decimals of a degree:

$$1' = 60''$$

$$18'' = \frac{18}{60}$$

$$= .3'$$

$$15.3' = \frac{15.3}{60}$$

$$= .255^\circ$$

$$57^\circ 15' 18'' = 57.255^\circ$$

Step 2. Change to radians:

$$1^\circ = .017453 \text{ radian}$$

$$57.255^\circ = 57.255(.017453)$$

$$= .99927 \text{ radian}$$

c. Expressing Angles in Radians as Multiples of π . It is often convenient to express angles in radians as multiples of π . Since $360^\circ = 2\pi$ radians, $90^\circ = \frac{1}{2}\pi$ radians, $40^\circ = \frac{1}{5}\pi$ radians, etc. It is necessary only to multiply the degrees by $\frac{\pi}{180}$ to change to radians.

Example: Express 135° in radians as a multiple of π .

$$\begin{aligned} 135^\circ &= 135\left(\frac{\pi}{180}\right) \\ &= \frac{3}{4}\pi \text{ radians} \end{aligned}$$

176. Review Problems—Radians

a. Find the angle θ for the following arc lengths and radii:

- (1) $r = 5$ inches, $s = 2$ inches.
- (2) $r = 3$ feet, $s = 12$ feet.
- (3) $r = .8$ miles, $s = 6.4$ miles.
- (4) $r = 27$ meters, $s = 75$ meters

b. Find the arc lengths for the following angles and radii:

- (1) $\theta = 5$ radians, $r = 7$ inches
- (2) $\theta = 8$ radians, $r = 2.2$ feet
- (3) $\theta = 2.1$ radians, $r = 9$ miles
- (4) $\theta = .03$ radians, $r = .066$ inch

c. Express the following angles in radians:

- (1) 30°
- (2) $263^\circ 12'$
- (3) $158^\circ 33'$
- (4) $336^\circ 24' 22''$

d. Express the following angles in degrees:

- (1) π radians
- (2) 2π radians
- (3) 3.45 radians
- (4) 3π radians

e. Express the following angles as multiples of π :

- (1) 30°
- (2) 60°
- (3) 225°
- (4) 720°

CHAPTER 12

VECTORS

177. Plane Vectors

a. A line segment used to represent a quantity that has direction as well as magnitude is called a vector. The length of a vector is proportionate to the magnitude, and the arrow, or head, of the vector indicates the direction of the quantity represented.

b. The quantity represented by a vector is called a vector quantity. This is the directed magnitude itself. Electrical quantities, such as current and voltage, are vector quantities in ac circuits (par. 194).

Example: An airplane is flying northeast at 120 miles per hour. Its speed is represented on figure 55 by line OA . The direction in which the airplane is traveling is represented by the direction of the line.

178. Vector Notation

Because a vector quantity has direction as well as magnitude, the methods of denoting a vector are different from the methods of de-

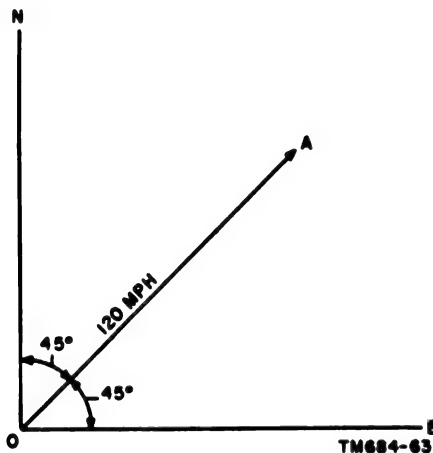


Figure 55. The velocity of an airplane described by a vector.

noting a scalar quantity. A vector may be denoted by two letters, the first indicating the origin, or initial point, and the other indicating the head or terminal point. For example, a vector may be represented by the letters AB , indicating that the quantity went from A to B . A small arrow sometimes is placed over the

letters for emphasis; for example, $\vec{AB}$. Another method of notation is A/θ , where A represents the magnitude of the quantity, and θ represents the angle the vector makes with some reference line. For example, if line OE in figure 55 were used as the reference line, vector OA could be represented by the notation $120/45^\circ$, where 120 represents the magnitude of the quantity, and 45° represents the direction with respect to line OE . With respect to line ON , vector OA , would be represented by the notation $120/-45^\circ$.

179. Addition of Vectors, Parallelogram Method

The addition of vectors by the parallelogram method is shown in figure 56. To add vector OA to OC , draw a vector OC with its initial point located at the initial point of vector OA , and complete the parallelogram with these vectors forming two sides. The diagonal vector OB , with its initial point at the same initial point of OA and OC and its terminal point at the opposite vertex of the parallelogram, is the sum of OA and OC . Thus, two vectors (OA and OC) acting simultaneously on a point or object may be replaced by a single vector called the *resultant* (OB). The resultant vector will pro-

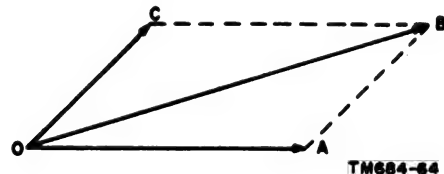
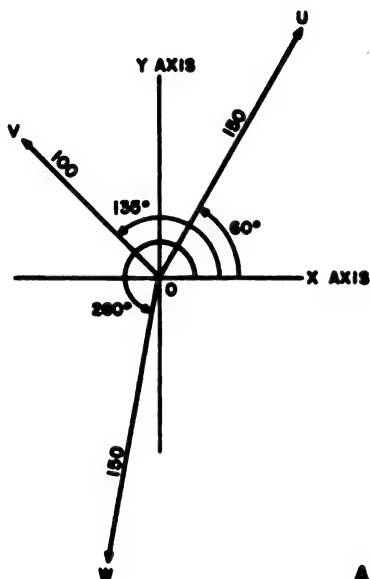
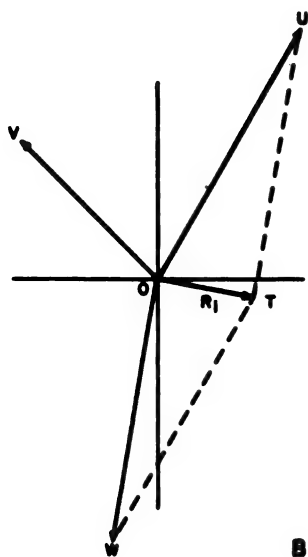


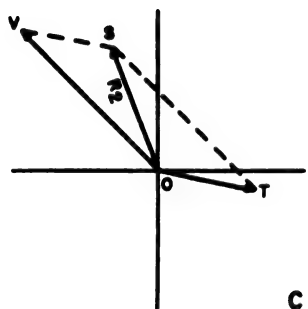
Figure 56. Adding vectors, parallelogram method.



A



B



C

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Figure 57. Resolution of three vectors.

duce the same effect on the object as the joint action of the two vectors.

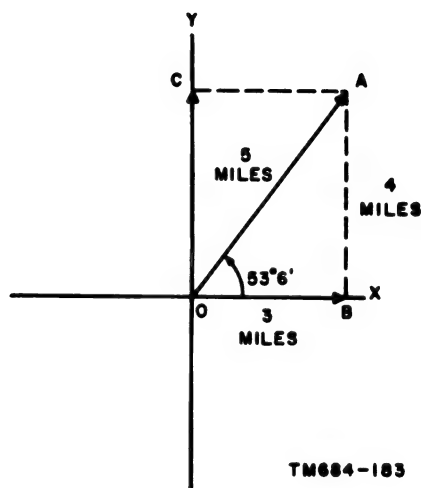
180. Addition of More Than Two Vectors

a. In determining the resultant (par. 179) of vectors when more than two quantities are represented, proceed as follows:

- (1) Find the resultant of two of the vector quantities,
- (2) Determine the final resultant between the third quantity and the resultant obtained from (1), above.

b. Assume three forces U, V, and W are acting on point O as shown in A, figure 57. Force U exerts 150 pounds at an angle of 60° , V exerts 100 pounds at an angle of 135° , and W exerts 150 pounds at an angle of 260° . Find the resultant of forces on point O.

- (1) The resultant of any two vectors, such as U and W, are determined graphically by the line R_1 (B, fig. 57). To solve this problem first draw the vectors to scale at the designated angles; then construct the parallelogram OUTW with adjacent sides WT and UT. The resultant R_1 of OW and OU will be the diagonal OT.
- (2) Combine the resultant R_1 with force V, then construct another parallelogram to scale as in (1), above. The final resultant R_2 is similarly determined by the line SO (C, fig. 57).



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Figure 58. Horizontal and vertical components of vector.

This, then, is the resolution of all three forces U, V, and W acting on point O.

181. Components of a Vector

a. A vector may be resolved into components along any two specified directions. If the directions of the components are chosen so that they are at right angles to each other, the components are called *rectangular components*.

b. By placing the initial point of a vector at the origin of the X and Y axes, the rectangular components are readily obtained either graphically or by computation. In figure 58, a vector with a magnitude of 5 and a direction of $53^{\circ} 6'$ is shown broken down into a horizontal compo-

nent of 3 and a vertical component of 4. This is done by using the sine and cosine function as follows:

$$\sin 53^{\circ} 6' = \frac{BA}{5}$$

$$.79968 = \frac{BA}{5}$$

$$BA = 5 \times .79968 \\ = 4 \text{ (approx)}$$

$$\cos 53^{\circ} 6' = \frac{OB}{5}$$

$$.60042 = \frac{OB}{5}$$

$$OB = 5 \times .60042 \\ = 3 \text{ (approx)}$$

PART II

APPLICATIONS OF MATHEMATICAL PRINCIPLES TO COMMON COMMUNICATIONS-ELECTRONICS PROBLEMS

CHAPTER 13 INTRODUCTION

182. Series Circuits

In a *series circuit*, electrical energy is supplied to a number of devices in series; that is the same current passes through each device in completing its path to the source of supply. Figure 59 shows a resistance, an inductor, and a capacitor connected in series with a voltage source.

a. The current is the same in all parts of a series circuit.

b. The total voltage drop (E_t) in a series circuit is equal to the sum of the voltage drops across individual loads:

$$E_t = E_1 + E_2 + E_3 + \dots$$

c. The total resistance (R_t) of a series circuit is equal to the sum of all individual resistance:

$$R_t = R_1 + R_2 + R_3 + \dots$$

d. The total inductance L_t of a series circuit is equal to the sum of the individual inductances:

$$L_t = L_1 + L_2 + L_3 + \dots$$

e. The reciprocal of the total capacitance (C_t) is equal to the sum of the reciprocals of the separate capacitances. The total capacitance is also less than the capacitance of any one of the capacitors, and is expressed as follows:

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

If only two capacitances are in series, a simplified formula can be derived by combining fractions over an LCD, and taking the reciprocal:

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_t} = \frac{C_2}{C_1 C_2} + \frac{C_1}{C_1 C_2}$$

$$\frac{1}{C_t} = \frac{C_1 + C_2}{C_1 C_2}$$

$$C_t = \frac{C_1 C_2}{C_1 + C_2}$$

If two or more capacitors of equal value are placed in series, the total capacitance is equal to the value of one capacitor (c) divided by the number of capacitors used (n):

$$C_t = \frac{C}{n}$$

This equation can be derived as follows (assuming 3 equal-value capacitors):

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_t} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$\frac{1}{C_t} = \frac{3}{C}$$

$$C_t = \frac{C}{3}$$

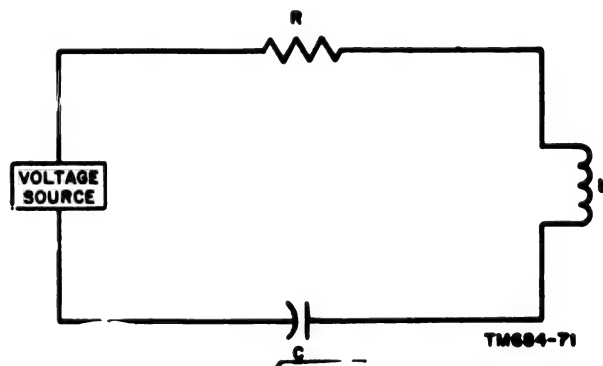


Figure 59. Example of a series circuit.

183. Parallel Circuits

Figure 60 is an example of a simple parallel circuit, with two resistors connected in parallel across a generator. As indicated by the arrows, the current from the generator separates into two parts, each resistor receiving a part of the total current. The larger fraction of current flows through the branch of less resistance, and the smaller fraction of current flows through the branch of greater resistance. The two parts of the current join again upon leaving the resistors.

a. The total current (I_t) in a parallel circuit is the sum of the currents in the separate branches:

$$I_t = I_1 + I_2 + I_3 + \dots$$

b. The voltage (E) across each branch of a parallel circuit is the same:

$$E_1 = E_2 = E_3 = \dots$$

c. The reciprocal of the total resistance (R_t) of all resistors in a parallel circuit is equal to the sum of the reciprocals of the separate resistance. The total resistance is also less than the resistance of any one of the resistors, and is expressed as follows:

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

If only two resistors are in parallel, a simplified formula can be derived for the total resistance as for total capacitance in a series circuit (par. 182e):

$$R_t = \frac{R_1 R_2}{R_1 + R_2}$$

If two or more resistors of the same value are placed in parallel, the total resistance is equal to the value of one resistor (R) divided by the number of resistors used (n), as for capacitances, in series (par. 182e):

$$R_t = \frac{R}{n}$$

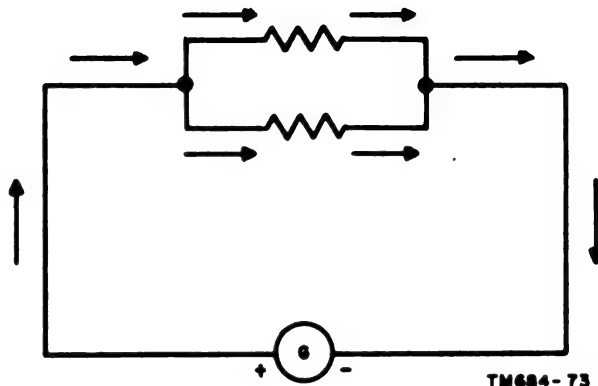


Figure 60. Example of a parallel circuit.

d. The reciprocal of the total inductance (L_t) in a parallel circuit is equal to the sum of the reciprocals of the separate inductances, as with resistances (c above):

$$\frac{1}{L_t} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots$$

The rules covering the calculation of resistances in parallel (c above) also apply to inductances in parallel.

e. The total capacitance in a parallel circuit is equal to the sum of the individual capacitances, as for resistances and inductances in series (par. 182c and d):

$$C_t = C_1 + C_2 + C_3 + \dots$$

The rules covering the calculation of resistances and inductances in series also apply to capacitances in parallel.

184. Series-Parallel Circuit

A series-parallel circuit is simply a combination of a series circuit and a parallel circuit. The rules covering series circuits (par. 182) apply to the series portion of the circuit, and the rules covering parallel circuits (par. 183) apply to the parallel portion of the circuit. The examples given in chapters 14 through 18 more clearly illustrate the various types of circuits.

CHAPTER 14

PROBLEMS IN DC ELECTRICITY

185. General

In circuits using constant-value dc electricity, only the effects of the resistance in the circuit are significant, because inductance and capacitance depend on varying current or voltage. Consequently, the examples given in this chapter involve only resistances.

186. Ohm's Law

a. An important relationship between current (I), voltage (E), and resistance (R) in a circuit is given by Ohm's law which states that the current in an electrical circuit varies directly as the voltage and inversely as the resistance. Expressed in a formula, the relationship is:

$$I = \frac{E}{R}$$

The formula may also appear in the following forms:

$$E = IR$$

$$R = \frac{E}{I}$$

b. The following example illustrates Ohm's law:

Example: Solve the following problem:
A voltmeter (voltage measuring device) connected directly

across a resistance reads 65 volts (fig. 61). An ammeter (current measuring device) connected in series reads 5.8 amperes. What is the value of the resistance in ohms?

$$E = 65, I = 5.8, R = ?$$

$$R = \frac{E}{I}$$

$$R = \frac{65}{5.8} \\ = 12.26 \text{ or } 12.3 \text{ ohms.}$$

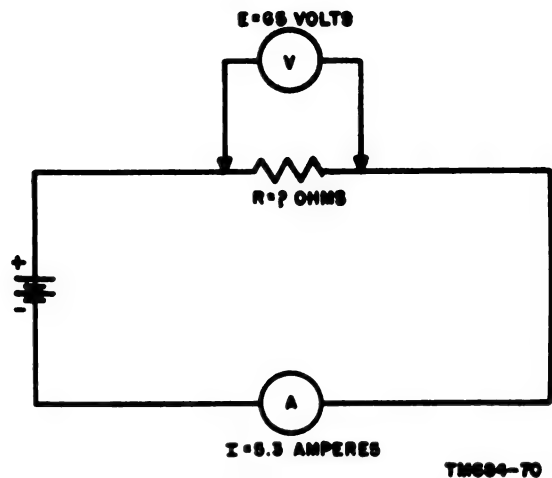


Figure 61. Simple circuit with unknown resistance.

187. Solving Series Circuits

The following example illustrates the method of using Ohm's law and the principles of series circuits (par. 182) to solve series dc circuits.

Example: Solve the following problem:

Resistors R_1 , R_2 , and R_3 are connected in series across a 110-volt generator (fig. 62). If resistor $R_1 = 6.5$ ohms, resistor $R_2 = 10.3$ ohms, and resistor $R_3 = 7.6$ ohms, what is the total current in the circuit? What is the voltage drop across each resistance?

- Step 1.** Find the total resistance in the circuit.
 $R_t = R_1 + R_2 + R_3$
 $= 6.5 + 10.3 + 7.6$
 $= 24.4 \text{ ohms total resistance}$
- Step 2.** Find the total current in the circuit.
 $E = IR$
 $110 = I(24.4)$
 $24.4I = 110$
 $I = 4.508 \text{ amperes total current}$
- Step 3.** Find the voltage drop across R_1 .
 $E = IR$
 $= 4.508(6.5)$
 $= 29.302 \text{ volts across } R_1$
- Step 4.** Find the voltage drop across R_2 .
 $E = IR$
 $= 4.508(10.3)$
 $= 46.432 \text{ volts across } R_2$
- Step 5.** Find the voltage drop across R_3 .
 $E = IR$
 $= 4.508(7.6)$
 $= 34.261 \text{ volts across } R_3$
- Check:** $34.261 + 46.432 + 29.303 = 109.996 \text{ or } 110 \text{ volts.}$

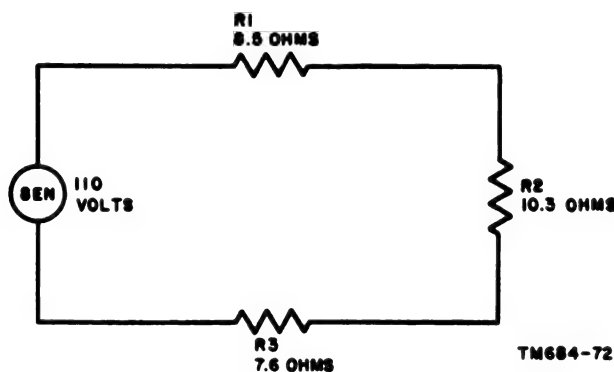


Figure 62. Series circuit with unknown current.

188. Solving Parallel Circuits

The following example illustrates the method of using Ohms' law and the principles of parallel circuits (par. 183) to solve parallel dc circuits.

Example: Solve the following problem:

In figure 63, a resistor of 200 ohms (R_1), a resistor of 600 ohms (R_2), and an unknown resistor (R_3) are connected in parallel across a source of emf. The voltage across R_1 is 40 volts. The current through the resistor of unknown value (R_3) is 0.40 ampere. Find (a) the value of R_3 , (b) the total resistance of the circuit, and (c) the total current, in the circuit.

Step 1. Find the voltage across R_3 .

$$E_1 = E_2 = E_3$$

Since the voltage across R_1 is 40 volts, the voltage across R_3 is also 40 volts.

Step 2. Find the resistance of R_3 .

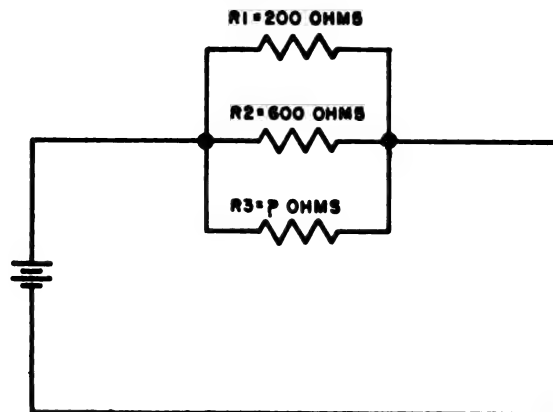
$$\begin{aligned} R_3 &= \frac{E_3}{I_3} \\ &= \frac{40}{.4} \\ &= 100 \text{ ohms} \end{aligned}$$

Step 3. Find the total resistance of the three resistors.

$$\begin{aligned} \frac{1}{R_t} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\ \frac{1}{R_t} &= \frac{1}{200} + \frac{1}{600} + \frac{1}{100} \\ \frac{1}{R_t} &= \frac{3}{600} + \frac{1}{600} + \frac{6}{600} \\ \frac{1}{R_t} &= \frac{10}{600} \\ 10R_t &= 600 \\ R_t &= \frac{600}{10} \\ &= 60 \text{ ohms} \end{aligned}$$

Step 4. Find the line current in the circuit.

$$\begin{aligned} I_t &= \frac{E_t}{R_t} \\ &= \frac{40}{60} \\ &= 0.667 \text{ ampere} \end{aligned}$$



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Figure 63. Parallel circuit with three resistances, one unknown.

189. Solving Series-Parallel Circuits

A simple series-parallel circuit, with series-connected resistors R_2 and R_3 connected in parallel with resistor R_1 , and the combination connected in series with resistors R_4 and R_5 , is shown in A, figure 64. The following example uses B through D, figure 64, to illustrate the method of solving series-parallel dc circuits.

Example: Find the current through each resistance and the voltage drop across each resistance in A, figure 64.

Step 1. Since R_2 and R_3 are in series, their total resistance is the sum (B, fig. 64) of the two resistances.

$$\begin{aligned} R_{2,3} &= R_2 + R_3 \\ &= 5 + 15 \\ &= 20 \text{ ohms} \end{aligned}$$

Step 2. $R_{2,3}$ is in parallel with R_4 . Find the total resistance of the combination (C, fig. 64).

$$\begin{aligned} \frac{1}{R_{2,3,4}} &= \frac{1}{R_{2,3}} + \frac{1}{R_4} \\ \frac{1}{R_{2,3,4}} &= \frac{1}{20} + \frac{1}{30} \\ \frac{1}{R_{2,3,4}} &= \frac{3}{60} + \frac{2}{60} \\ \frac{1}{R_{2,3,4}} &= \frac{5}{60} \\ 5R_{2,3,4} &= 60 \\ R_{2,3,4} &= \frac{60}{5} \\ R_{2,3,4} &= 12 \text{ ohms} \end{aligned}$$

Step 3. $R_1, R_{2,3,4}$ and R_5 are in series. Their total resistance is the sum (D, fig. 64) of the resistances.

$$\begin{aligned} R_{1,2,3,4,5} &= R_1 + R_{2,3,4} + R_5 \\ &= 3 + 12 + 10 \\ &= 25 \text{ ohms} \end{aligned}$$

Step 4. Find the total current sent through these resistances by a voltage of 100 volts.

$$\begin{aligned} I_t &= \frac{E_t}{R_t} \\ &= \frac{100}{25} \\ &= 4 \text{ amperes} \end{aligned}$$

Step 5. Find the voltage drop across $R_{2,3,4}$.

$$\begin{aligned} E_{2,3,4} &= IR_{2,3,4} \\ &= 4 \times 12 \\ &= 48 \text{ volts} \end{aligned}$$

Step 6. Analyze the parallel circuit.

The voltage across R_4 is 48 volts. Find the current.

$$\begin{aligned} I_4 &= \frac{E_4}{R_4} \\ &= \frac{48}{30} \\ &= 1.6 \text{ amperes} \end{aligned}$$

The voltage across R_2 and R_3 also is 48 volts, and the resistance $R_{2,3}$ is 20 ohms. Find the current.

$$\begin{aligned} I_{2,3} &= \frac{E_{2,3}}{R_{2,3}} \\ &= \frac{48}{20} \\ &= 2.4 \text{ amperes } (I_2 = I_3) \end{aligned}$$

Step 7. Find all voltage drops.

$$\begin{aligned} E_1 &= I_1 R_1 \\ &= 4 \times 3 \\ &= 12 \text{ volts} \\ E_2 &= I_2 R_2 \\ &= 2.4 \times 5 \\ &= 12 \text{ volts} \\ E_3 &= I_3 R_3 \\ &= 2.4 \times 15 \\ &= 36 \text{ volts} \\ E_4 &= I_4 R_4 \\ &= 1.6 \times 30 \\ &= 48 \text{ volts} \\ E_5 &= I_5 R_5 \\ &= 4 \times 10 \\ &= 40 \text{ volts} \end{aligned}$$

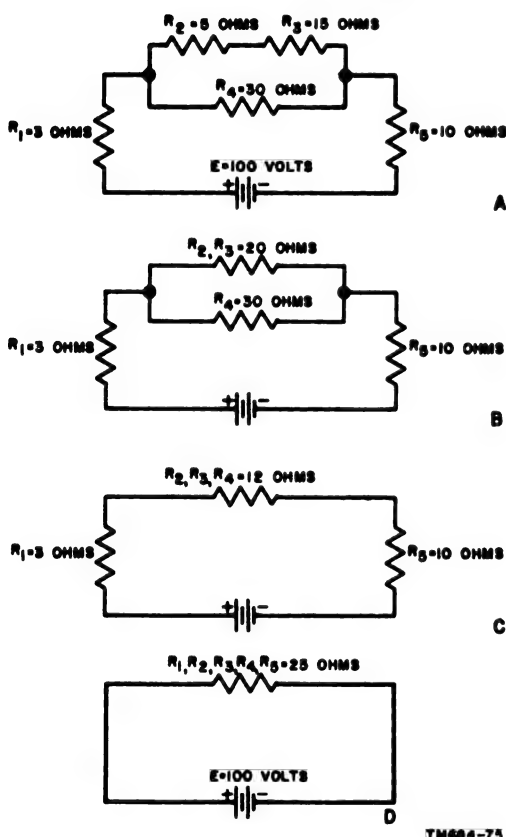


Figure 64. Solving a series-parallel circuit.

190. Solving More Complex Electrical Problems by Using Kirchhoff's Laws

a. General. The more complex series-parallel problems are often more readily solved by using Kirchhoff's laws. A full treatment of the electrical phenomena embodied in Kirchhoff's laws is not within the scope of

this manual. For a complete treatment of electrical theory on this subject, see TM 11-661. The basic principles of Kirchhoff's laws are as follows:

- (1) The algebraic sum of the currents at any junction of conductors is zero.
- (2) The algebraic sum of the electromotive forces and voltage drops around a closed circuit is zero.

b. Understanding Kirchhoff's Laws. The first of Kirchhoff's laws simply means that there is just as much current flowing away from a point as there is flowing to it. The second law simply means that the voltage source is equal to the sum of the voltage drops around any closed circuit. For example, starting at point X (fig. 65) and going around the circuit clockwise, the following equation is obtained:

$$E - IR_1 - IR_2 - IR_3 = 0$$

Substituting the values of resistance as indicated in the figure, the equation becomes:

$$37 - 13I - 9I - 11I = 0$$

Collecting like terms and solving for I gives:

$$37 - 33I = 0$$

$$33I = 37$$

$$I = 1.121 \text{ amperes}$$

To prove that this is correct, use Ohm's law as follows:

$$E_1 = IR_1 = 1.121 \times 13 = 14.58 \text{ volts}$$

$$E_2 = IR_2 = 1.121 \times 9 = 10.09 \text{ volts}$$

$$E_3 = IR_3 = 1.121 \times 11 = 12.33 \text{ volts}$$

$$E_t = IR_t = 1.121 \times 33 = 37.00 \text{ volts}$$

Thus, the sum of the voltage drops equals the applied voltage and the second law is verified.

c. Solving Series-Parallel Circuits Using Kirchhoff's Laws. Problems involving series-parallel circuits are readily solved by using Kirchhoff's laws and simultaneous equations (par. 84). The example below illustrates such a problem.

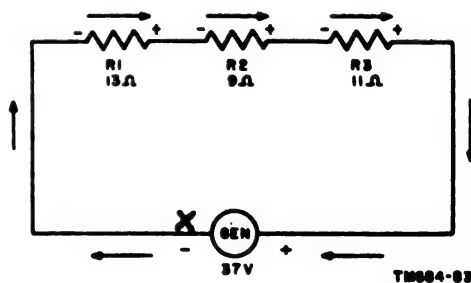


Figure 65. Example of Kirchhoff's second law.

Example: Solve for the current in each branch of the circuit shown in figure 66.

Step 1. Assume a direction for the current flow in each branch, as shown in the figure. (It will be shown that the direction assumed does not affect the accuracy of the result.) According to Kirchhoff's first law, the current I_1 flowing through the 6-ohm resistor plus the current I_2 flowing through the 7- and 8-ohm resistors equals the current $I_1 + I_2$ flowing through the remainder of the circuit, which includes the 5-ohm resistor.

Step 2. Considering the first part of the circuit, from point B through the generator and around the circuit back to point B through the 6-ohm resistor, the application of Kirchhoff's second law yields the following equation:

$$\begin{aligned} 10 - 5(I_1 + I_2) - 6I_1 &= 0 \\ 10 - 5I_1 - 5I_2 - 6I_1 &= 0 \\ 10 - 11I_1 - 5I_2 &= 0 \quad (\text{equation 1}). \end{aligned}$$

Step 3. Considering the path from point B through the generator and through points A, X, and Y back to B, the application of Kirchhoff's second law yields the following equation:

$$\begin{aligned} 10 - 5(I_1 + I_2) - 7I_2 - 8I_2 &= 0 \\ 10 - 5I_1 - 5I_2 - 7I_2 - 8I_2 &= 0 \\ 10 - 5I_1 - 20I_2 &= 0 \quad (\text{equation 2}). \end{aligned}$$

Step 4. Using the methods of solving simultaneous equations described in paragraph 116, solve for I_1 by multiplying equation 1 by 4 and subtracting equation 2 from the new equation:

$$\begin{array}{r} 40 - 44I_1 - 20I_2 = 0 \\ 10 - 5I_1 - 20I_2 = 0 \\ \hline 30 - 39I_1 = 0 \\ - 39I_1 = -30 \\ I_1 = 0.769 \text{ ampere} \end{array}$$

Step 5. Solve for I_2 by substituting the value of I_1 in either equation 1 or equation 2, or by eliminating I_1 in solving the simultaneous equations. Substituting I_1 in equation 2 yields the following:

$$\begin{aligned} 10 - 5(0.769) - 20I_2 &= 0 \\ 10 - 3.845 - 20I_2 &= 0 \\ - 20I_2 &= -6.155 \\ I_2 &= 0.308 \text{ ampere} \end{aligned}$$

Step 6. The current in the left-hand side of the circuit is $I_1 + I_2$ or 1.077 amperes.

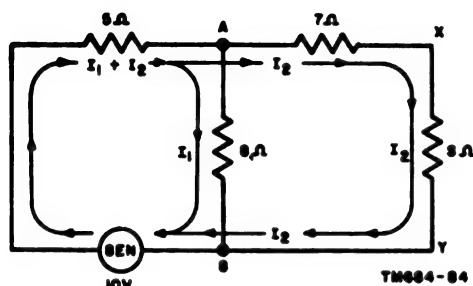


Figure 66. Solving series-parallel circuits, using Kirchhoff's laws.

d. Direction of Current Flow. If the direction of current flow is assumed incorrectly, the computed value for the current will have a negative sign; however, the magnitude of the current will be the same. Therefore, to correct the error, simply reverse the assumed direction of current flow on the diagram.

e. Facts to Remember When Working Problems. The solution of problems involving series-parallel circuits by the above method normally is relatively simple. The important facts to remember when working such problems are:

- (1) Assume any direction of current flow in the beginning.
- (2) Take any path around any portion of the circuit, as long as the path is a complete circuit.
- (3) Observe the polarities of the circuit, both voltage sources and voltage drops.
- (4) Be sure to have as many equations as there are unknowns.

191. Dc Power

In dc circuits, the amount of power absorbed by a resistor or the resistance of a circuit is easily determined by Joule's law:

$P = I^2R$, where:

P = power absorbed in watts

I = total current in amperes

R = total resistance of the circuit in ohms

Since the voltage drop (E) across a resistor (R) is equal to IR , the formula above may also be written: $P = IR \times I = EI$.

Example 1: Find the power consumed in a 50-ohm resistor when a current of 5 amperes flows through it.

$$\begin{aligned} P &= I^2R \\ &= 5^2 \times 50 \\ &= 1,250 \text{ watts} \end{aligned}$$

Example 2: Find the power delivered by a 12-volt battery when the current drain is 6 amperes.

$$\begin{aligned} P &= EI \\ &= 12 \times 6 \\ &= 72 \text{ watts} \end{aligned}$$

192. Review Problems—Dc Electricity

a. (1) The resistance of a tungsten lamp is 20 ohms when the lamp is cold. What current will the lamp draw the instant it is placed across a 110-volt line? (2) When the lamp is glowing at full brilliancy, its resistance rises to 84 ohms. What is the final steady current of the lamp?

b. An adjustable resistor has a minimum setting of 14 ohms and a maximum setting of 50 ohms. (1) What ranges of resistance can be covered with two of these resistors connected in series? (2) What ranges of resistance can be covered with two of these resistors connected in parallel?

c. When a 6,500-ohm resistance is connected into the plate circuit of a radio tube, the plate current is 34 milliamperes. (1) What is the voltage drop across the 6,500-ohm resistance? (2) How much power is consumed by the resistor?

d. Three resistors of 20 ohms, 30 ohms, and 50 ohms, respectively, are connected in series. The current through R_1 (20 ohms) is 0.8 ampere. (1) What is the current through R_2 (50 ohms)? (2) What is the voltage across R_2 (30 ohms)? (3) What is the total voltage drop across the three resistors?

e. A divided circuit has three branches of 5, 10, and 20 ohms resistance, respectively. (1) What is the joint conductance of the three branches? (Conductance is the reciprocal of resistance.) (2) What is the joint resistance? (3) A current of 20 amperes flows in the 5-

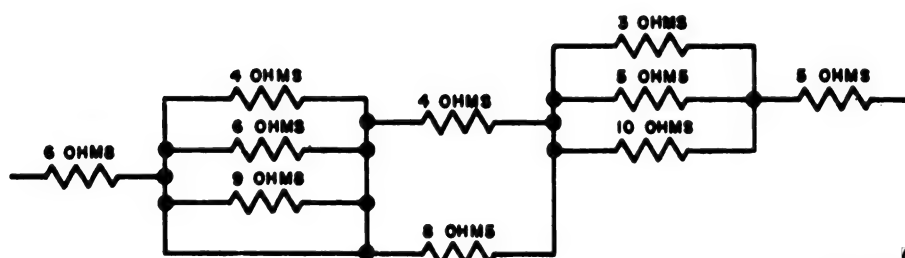
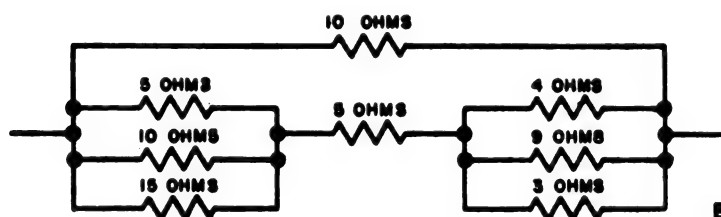
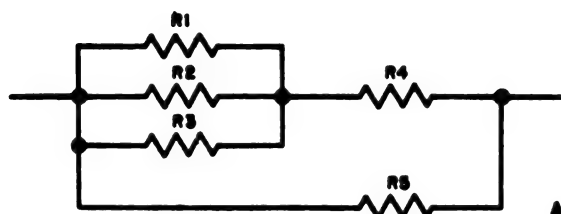
ohm branch; find the current in each of the other branches. (4) Find the combined current.

f. A parallel circuit has branches with resistances of 1, 3, 10, 20, and 50 ohms, respectively. (1) What is the conductance of each branch? (2) What is the conductance of the combination? (3) What is the resistance of the combination?

g. Three resistors R_1 (36 ohms), R_2 (42 ohms) and R_3 are connected in series with a generator. An ammeter inserted in the circuit

between R_1 and R_2 reads 2.4 amperes, and a voltmeter across R_3 reads 41 volts. (1) What is the resistance of R_3 ? (2) What is the voltage across R_1 ? (3) What is the voltage across R_2 ? (4) What is the voltage across the generator?

h. Find the total resistance of: (1) circuit A when $R_1 = 6$ ohms, $R_2 = 9$ ohms, $R_3 = 17$ ohms, $R_4 = 5$ ohms, $R_5 = 11$ ohms; (2) circuit a when $R_1 = 12$ ohms, $R_2 = 25$ ohms, $R_3 = 19$ ohms, $R_4 = 8$ ohms, $R_5 = 12$ ohms. (3) circuit B; (4) circuit C.



C
TM554-76

i. A 10-ohm resistor is connected in series with a 15-ohm resistor. (1) What voltage must be placed across the two resistors to send a current of 5 amperes through it? (2) What would the voltage be across each resistor?

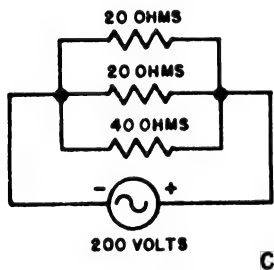
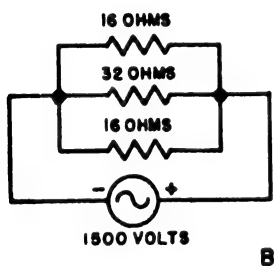
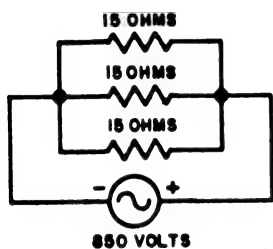
j. (1) What voltage is required to force a current of 10 amperes through a parallel combination of three branches having resistances of 15.3 ohms, 1.3 ohms, and 10.5 ohms, re-

spectively? (2) What will the current be in each branch? (3) What is the voltage drop across each branch?

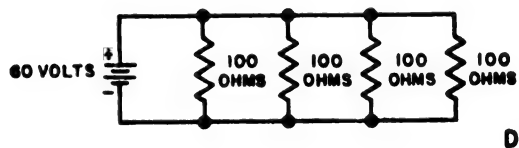
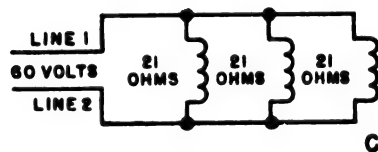
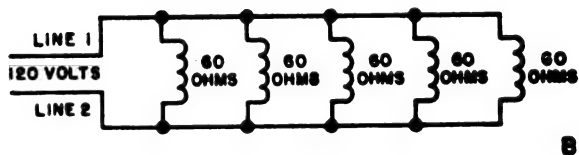
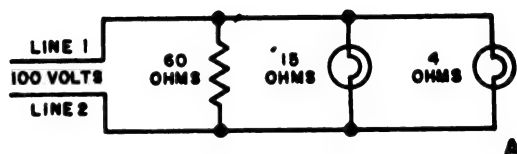
k. A generator has an output voltage of 110 volts. (1) What current is flowing in a wire of 0.02 ohm connected across the terminals? (2) What current will flow if an incandescent lamp of 484 ohms is also connected across the generator?

l. Find the total resistance of each of the parallel circuits A, B, and C.

m. Find the total resistance of each of the parallel circuits A, B, C, and D.



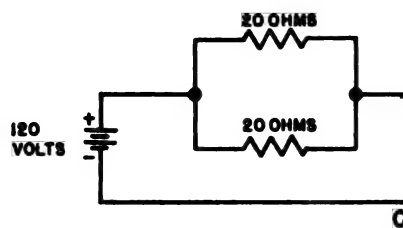
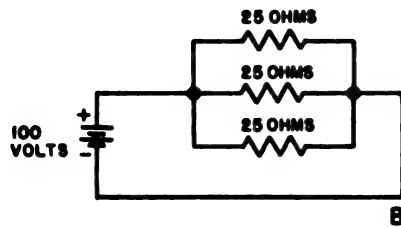
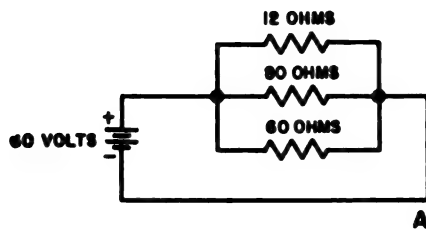
NOTE:
 INDICATES GENERATOR
 TM684-77



NOTE:
 INDICATES BATTERY

TM684-78

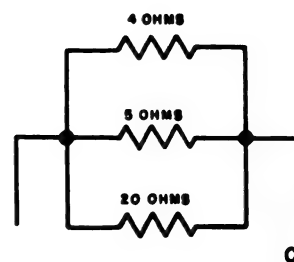
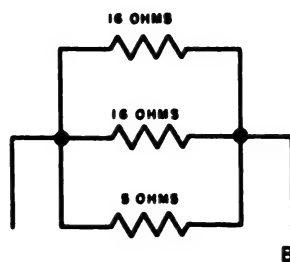
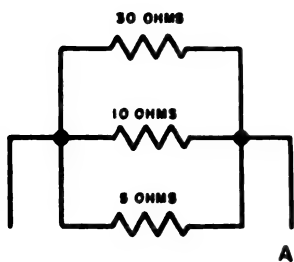
8. Find the total resistance of each of the parallel circuits A, B, and C.



NOTE:
 INDICATES BATTERY

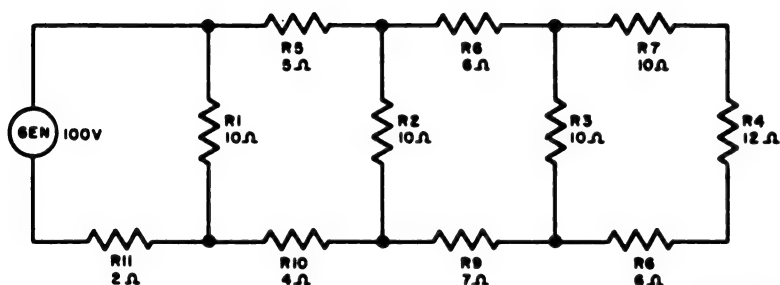
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9. Find the total resistance of each of the parallel circuits A, B, and C.

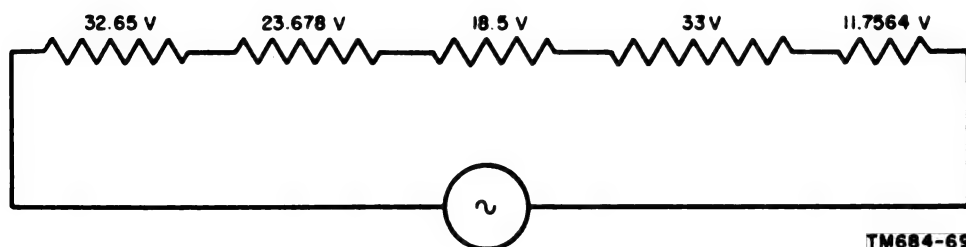


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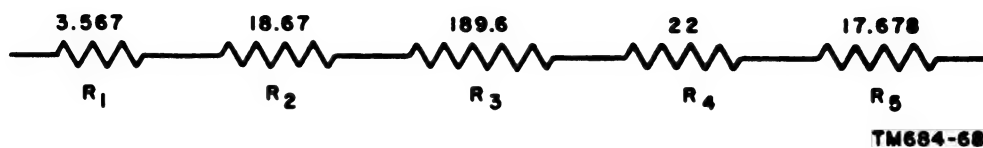
p. Find the current through each resistor in the circuit below.



q. Find the total resistance in the circuit below when a current of .5 amperes flows through it.



r. Find the current through the resistors in the circuit below when 115 volts is applied across the circuit.



CHAPTER 15

PROBLEMS IN AC ELECTRICITY

193. General

In circuits using ac electricity¹, the current is affected by inductance and capacitance as well as resistance. In addition, certain combinations of these loads will produce unusual effects, such as resonance (par. 202), not experienced in dc circuits. These phenomena are used extensively in electrical and electronic circuits. Consequently, problems in ac electricity are more complex than corresponding problems in dc electricity.

194. Application of Vectors and Trigonometry in Solving Ac Circuit Problems

a. As discussed in chapter 12, a vector is a line whose length and direction represent accurately a given quantity; the quantity thus represented is a *vector quantity*. Because the magnitude of ac currents and voltages varies from instant to instant, the magnitude is a function of time, and the current and voltage can be expressed as vectors: The length of the vector represents the magnitude of the current or voltage, and the direction represents its relationship in time to another vector (b below).

b. When a circuit contains inductance or capacitance, the current in the circuit is not in phase with the voltage that produces it. In other words, the instant the voltage is zero, the current that it produces has a value other than zero, or when the voltage is at its maximum, the current has a value different from its maximum value. The current is said to *lead* the voltage if the current reaches its maximum *before* the voltage maximum occurs; the current is said to *lag* the voltage if the current

¹ This chapter is limited to the application of mathematics to single-phase, sinusoidal ac. The electrical phenomena of this type of ac are treated briefly. See TM 11-681 for a complete treatment of single-phase, sinusoidal ac.

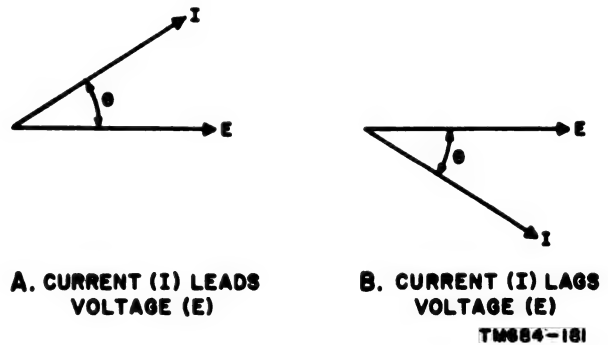


Figure 67. Vector representation of leading and lagging current.

reaches its maximum *after* the voltage maximum occurs. The relationship between current and voltage can be represented by vectors, with one vector representing current, another voltage, and with the angle between them indicating the amount of lag or lead. Figure 67 shows a vector representation of leading and lagging current. The angle is called the phase angle.

c. The voltage drop across a resistor also may be represented by a vector having the same direction as the vector representing the current flowing through the resistor. In other words, the voltage across the resistor and the current flowing through it are in phase.

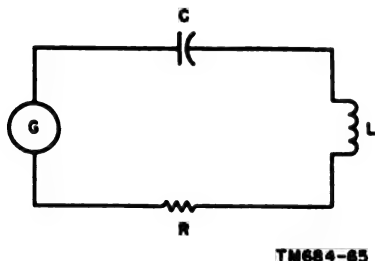
d. The voltage drop across a capacitor may be represented by a vector making an angle of 90° with the vector representing the current flowing through the capacitor. In a purely capacitive circuit, the current will lead the applied voltage by an angle of 90° .

e. The voltage drop across an inductor may be represented by a vector making an angle of 90° with the vector representing the current flowing through the inductance. In a purely inductive circuit, the current will lag the applied voltage by an angle of 90° .

f. In a circuit that contains inductance, capacitance, and resistance, the current will lead or lag the applied voltage by a phase angle of less than 90° .

g. The example below illustrates the use of vectors in the solution of a typical ac circuit problem. Paragraphs 199 through 201 give a more detailed coverage of problems of this type.

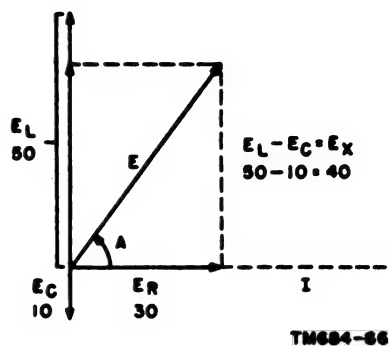
Example: In a series circuit (fig. 68), the voltage drop across the capacitor (E_C) is 10 volts, the voltage drop across the inductance (E_L) is 50 volts, and the voltage drop across the resistance (E_R) is 30 volts. Determine the magnitude of the applied voltage. By what phase angle (A) does the current lead or lag the applied voltage in the circuit?



TM684-65

Figure 68. An ac series circuit containing inductance, capacitance, and resistance.

- Step 1.** The vector diagram for this circuit is shown in figure 69. In a series circuit, the same current flows through each element. Draw the vector representing the current (I) in a horizontal position. The angles of all vectors representing voltage drops are given with respect to the current.
- Step 2.** Draw the vector E_L , representing the voltage drop across the inductance, at an angle of 90° with the vector I .
- Step 3.** Draw the vector E_C , representing the voltage drop across the capacitor, at a angle of -90° with the vector I .



TM684-66

Figure 69. A vector diagram of an ac series circuit containing inductance, capacitance, and resistance.

- Step 4.** The vector E_R , representing the voltage drop across the resistor, has the same direction as the vector I .
- Step 5.** The vector sum of these voltage drops is equal to the applied voltage.
- Along the horizontal:
- $$E_L = 0, E_C = 0, E_R = 30$$
- Along the vertical:
- $$E_L = 50, E_C = -10, E_R = 0$$
- Step 6.** Adding the horizontal and vertical voltage drops, respectively:
- $$E_L + E_C + E_R = 0 + 0 + 30 = 30$$
- $$E_L + E_C + E_R = 50 + (-10) + 0 = 40$$
- Step 7.** Because the vectors form a right triangle, with the applied voltage E as the hypotenuse and E_R and E_X as the sides (fig. 69), the law of right triangles (par. 183) can be used to solve for one of the quantities when the other two are known. From this law, the relationship between E , E_R , and E_X is expressed by the formula
- $$E = \sqrt{E_R^2 + E_X^2}$$
- $$E = \sqrt{E_R^2 + E_C^2}$$
- $$= \sqrt{(30)^2 + (40)^2}$$
- $$= \sqrt{900 + 1600}$$
- $$= \sqrt{2500}$$
- $$= 50 \text{ volts}$$

Step 8. The formula for determining angle A which the vector representing the applied voltage makes with the vector I (fig.

$$69) \text{ is } \tan A = \frac{E_X}{E_R}.$$

$$\begin{aligned}\tan A &= \frac{E_X}{E_R} \\ &= \frac{40}{30} \text{ or } \frac{4}{3} \\ &= 1.33333\end{aligned}$$

$$A = 53^\circ 7' 48''$$

Step 9. The circuit is predominately inductive; therefore, the current lags the applied voltage by a phase angle of $53^\circ 7' 48''$.

195. Ohm's Law Applied to Ac Circuits

Because of the effects of inductance and capacitance in ac circuits, Ohm's law (par. 186) must be modified to take these added effects into consideration.

a. If the circuit contains a combination of resistance and inductive reactance (par. 196) or capacitive reactance (par. 197), or both, the overall effect is called impedance (par. 198), and Ohm's law is modified to read:

$$I = \frac{E}{Z}$$

where I is the current in amperes, E the ac voltage in volts, and Z the impedance in ohms. This formula may also be written:

$$E = IZ$$

$$Z = \frac{E}{I}$$

b. If the circuit contains reactances only, the formulas become:

$$I = \frac{E}{X}$$

$$E = IX$$

$$X = \frac{E}{I}$$

where X is the total reactance (par. 198a) of the circuit in ohms.

c. If the circuit contains resistance only, the formula is the same as in a dc circuit (par. 186).

d. The application of these formulas in solving ac circuit problems is covered in paragraphs 196 through 203.

196. Inductive Reactance

Inductance enables an electric circuit to build up a voltage by electromagnetic induction whenever the current strength changes. The induced voltage always opposes the applied voltage and thus retards the change in the current. *Inductive reactance* is the effect of inductance expressed in ohms. The formula for finding inductive reactance is:

$$X_L = 2\pi fL$$

where X_L is the inductive reactance in ohms, L is the inductance in henrys, and f is the frequency in cps.

Example 1: Determine the inductive reactance of a coil if the ac in the circuit has a frequency of 100 cps, and the inductance of the coil is 0.036 henry.

$$\begin{aligned}X_L &= 2\pi fL \\ &= 2 \times 3.14 \times 100 \times .036 \\ &= 628 \times .036 \\ &= 22.608 \text{ ohms}\end{aligned}$$

Example 2: If a coil with an inductance of 0.2 henry and negligible resistance is connected across the terminals of a 220-volt, 60-cycle ac generator, how much current will flow through the coil?

Step 1. Find the inductive reactance of the coil.

$$\begin{aligned}X_L &= 2\pi fL \\ &= 2 \times 3.14 \times 60 \times .2 \\ &= 376.8 \times .2 \\ &= 75.36 \text{ ohms}\end{aligned}$$

Step 2. Find the amount of current that will flow through the coil.

$$\begin{aligned}I &= \frac{E}{X_L} \\ &= \frac{220}{75.36} \\ &= 2.92 \text{ amperes}\end{aligned}$$

197. Capacitive Reactance

Capacitance enables a capacitor to retain an electric charge which opposes any changes in the voltage of the circuit in which the capacitor is connected. *Capacitive reactance* is the effect of the capacitance expressed in ohms.

The formula for finding capacitive reactance is:

$$X_c = \frac{1}{2\pi fC}$$

where X_c is the capacitive reactance, C is the capacitance expressed in farads, and f is the frequency in cycles per second.

Example: A 110-volt, 60-cycle ac generator is connected in series with a 1-microfarad (10^{-6} farad) capacitance. What is the capacitive reactance of the circuit?

$$\begin{aligned} X_c &= \frac{1}{2\pi fC} \\ &= \frac{1}{2 \times 3.14 \times 60 \times 10^{-6}} \\ &= \frac{10^6}{6.28 \times 60} \\ &= \frac{1,000,000}{376.8} \\ &= 2,653 \text{ ohms} \end{aligned}$$

198. Impedance

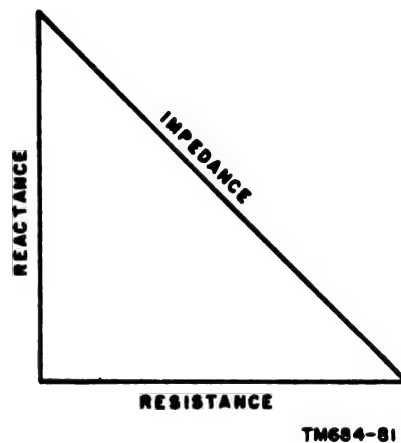
a. The *impedance* of a circuit is the circuit's total opposition to the flow of current. In a dc circuit, the opposition consists of resistance alone. In an ac circuit, the opposition consists of resistance and reactance (X). Inductive and capacitive reactances can be combined, but because their effects in the circuit are exactly opposite—inductive reactance causes the current to lag the voltage by 90° and capacitive reactance causes the current to lead the voltage by 90° —they are combined by subtraction:

$$X = X_L - X_c \text{ or}$$

$$X = X_c - X_L \quad (\text{subtracting the smaller from the larger})$$

b. Resistance and reactance cannot be added directly, but they can be considered as two vectors acting at right angles to each other. Thus, the relation between resistance, reactance, and impedance may be illustrated by a right triangle (fig. 70). Since these quantities may be related to the sides of a right triangle, the formula for finding the impedance of a circuit is:

$$Z^2 = R^2 + X^2 \text{ or } Z = \sqrt{R^2 + X^2}$$



TM684-81

Figure 70. The resistance-reactance-impedance triangle.

where Z is the impedance in ohms, R is the resistance in ohms, and X is the reactance in ohms.

Example 1: A 110-volt, 60-cycle ac generator is connected in series with a 1-microfarad capacitance and a 1,000-ohm resistance. The capacitive reactance of the circuit is 2,650 ohms. What is the impedance of the circuit?

$$\begin{aligned} Z &= \sqrt{R^2 + X^2} \\ &= \sqrt{(1000)^2 + (2650)^2} \\ &= \sqrt{(10^3)^2 + (2.65 \times 10^3)^2} \\ &= \sqrt{10^6 + 7.023 \times 10^6} \\ &= \sqrt{8.023 \times 10^6} \\ &= 2.83 \times 10^3 \\ &= 2,830 \text{ ohms} \end{aligned}$$

Example 2: A 300-volt, variable-frequency ac generator is connected in series with an inductive reactance of 300 ohms, a capacitive reactance of 100 ohms, and a resistance of 100 ohms. What is the impedance of the circuit?

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_c)^2} \\ &= \sqrt{(100)^2 + (300 - 100)^2} \\ &= \sqrt{(100)^2 + (200)^2} \\ &= \sqrt{(10^2)^2 + (2 \times 10^2)^2} \\ &= \sqrt{10^4 + 4 \times 10^4} \\ &= \sqrt{5 \times 10^4} \\ &= 2.236 \times 10^2 \\ &= 223.6 \text{ ohms} \end{aligned}$$

199. Solving Ac Circuits Having Resistance and Inductance

a. *Series Circuits.* The following examples illustrate the method of solving series ac circuits having resistance and inductance (called series RL circuits) by using the principles described in paragraphs 198 through 198.

Example 1: An ac circuit with a resistance of 1,000 ohms and an inductance of 5 henrys is connected in series with a generator (fig. 71). The voltage drop across the resistance is 51.5 volts, and the voltage drop across the inductance is 97 volts. Find the applied voltage in the circuit. If the impedance of the circuit is 2,182 ohms, what is the phase angle by which the current lags the applied voltage?

Step 1. The vector diagram for this circuit is shown in figure 77. In an ac series circuit, the same current flows through all parts of the circuit—in this case, 0.051 ampere. Draw the vector E_R to represent the voltage drop across the resistance. Draw the vector E_L to represent the voltage drop across the inductance.

Step 2. The vector sum of these voltage drops is equal to the applied voltage. Adding the horizontal and vertical voltage drops, respectively:

$$E_L + E_R = 0 + 51.5 = 51.5$$

$$E_L + E_R = 97 + 0 = 97$$

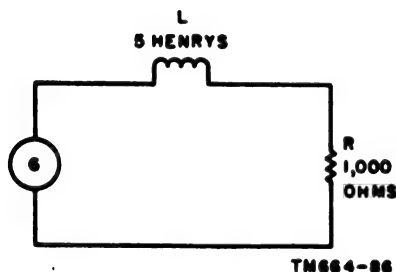
Step 3. Find the applied voltage as follows:

$$\begin{aligned} E^2 &= E_L^2 + E_R^2 \\ &= (97)^2 + (51.5)^2 \\ &= 9409 + 2652.25 \\ &= 12061.25 \end{aligned}$$

$$\begin{aligned} E &= \sqrt{12061.25} \\ &= 109.8 \text{ or approx } 110 \text{ volts} \end{aligned}$$

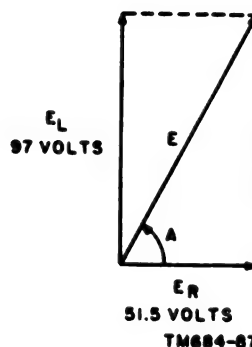
Step 4. Find the phase angle by which the current lags the applied voltage.

$$\begin{aligned} \cos A &= \frac{R}{Z} \text{ (for series circuit)} \\ &= \frac{1000}{2182} \\ &= 0.46904 \\ A &= 62^\circ 1' 19'' \end{aligned}$$



TM664-86

Figure 71. An ac series circuit containing inductance and resistance.



TM664-87

Figure 72. Ac series circuit containing inductance and resistance, vector diagram.

Step 5. Therefore, the current lags the applied voltage by a phase angle of $62^{\circ} 1' 19''$.

Example 2: A 110-volt, 60-cycle ac generator is connected in a series circuit to a load consisting of an inductance of 3 henrys and a resistance of 10,000 ohms (A, fig. 73).

Step 1. Find the inductive reactance of the circuit.

$$\begin{aligned} X_L &= 2\pi fL \\ &= 2 \times 3.14 \times 60 \times 3 \\ &= 6.28 \times 180 \\ &= 1130.4 \\ &= 1,130 \text{ ohms (approx)} \end{aligned}$$

Step 2. Find the impedance of the circuit.

$$\begin{aligned} Z &= \sqrt{R^2 + X_L^2} \\ &= \sqrt{(10,000)^2 + (1130)^2} \\ &= \sqrt{100,000,000 + 1,276,900} \\ &= \sqrt{101,276,900} \\ &= 10,063.64 \\ &= 10,064 \text{ ohms (approx)} \end{aligned}$$

Step 3. Find the effective current in the circuit. (The effective value is the equivalent heating value of an alternating current as compared to a direct current. It is also called the root-mean-square (rms) value.)

$$\begin{aligned} I &= \frac{E}{Z} \\ &= \frac{110}{10,065} \\ &= 0.0109 \text{ ampere} \end{aligned}$$

Step 4. In a series circuit, the same current flows through all parts of the circuit. Therefore, the current through both the inductance and the resistance is 0.0109 ampere.

Step 5. Find the voltage drop across the inductance.

$$\begin{aligned} E_L &= IX_L \\ &= 0.0109 \times 1130 \\ &= 12.317 \\ &= 12 \text{ volts (approx)} \end{aligned}$$

Step 6. Find the voltage drop across the resistance.

$$\begin{aligned} E_R &= IR \\ &= 0.0109 \times 10,000 \\ &= 109 \text{ volts} \end{aligned}$$

Step 7. Find the total voltage in the circuit. In an ac series circuit, voltage drops are added vectorially (B, fig. 73).

$$\begin{aligned} E_t^2 &= E_L^2 + E_R^2 \\ &= (12)^2 + (109)^2 \\ &= 144 + 11,881 \\ &= 12,025 \\ E_t &= \sqrt{12,025} \\ &= 109.6 \\ &= 110 \text{ volts (approx)} \end{aligned}$$

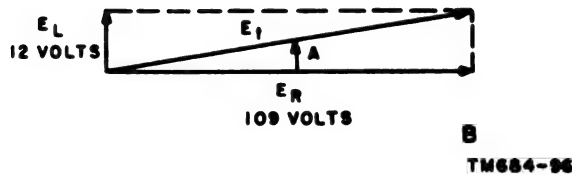
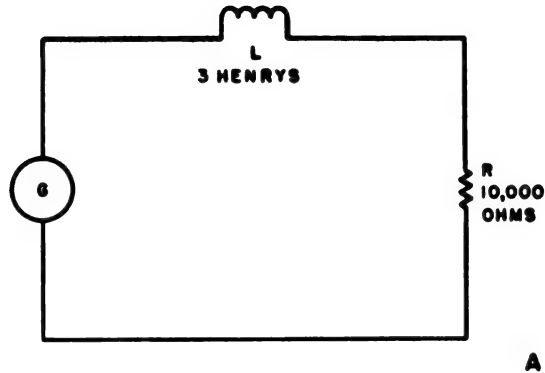


Figure 73. Ac series circuit having inductance and resistance, schematic and vector diagrams.

Step 8. Find the phase angle by which the current lags the applied voltage.

$$\begin{aligned}\cos A &= \frac{R}{Z} \\ &= \frac{10,000}{10,065} \\ &= 0.99354 \\ A &= 6^\circ 31'\end{aligned}$$

b. *Parallel Circuits.* The following examples illustrate the method of solving parallel RL circuits by using the principles described in paragraphs 193 through 198.

Example 1: An ac circuit has an inductance and resistance connected in parallel (fig. 74). The current flowing through the inductance is 0.0584 ampere, and the current flowing through the resistance is 0.11 ampere. What is the total current in the circuit? If the impedance of the circuit is 884 ohms, what is the phase angle by which the line current lags the applied voltage?

Step 1. The vector diagram for this circuit is shown in figure 75. In a parallel circuit the voltage drop across each inductance or resistance is the same—in this circuit, 110 volts. Draw the vector I_R to represent the current through the resistor. Draw the vector I_L to represent the current through the inductance.

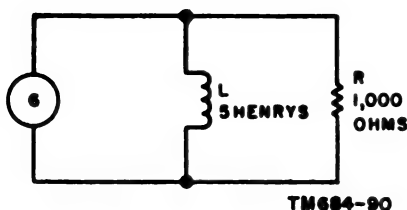


Figure 74. An ac parallel circuit containing inductance and resistance.

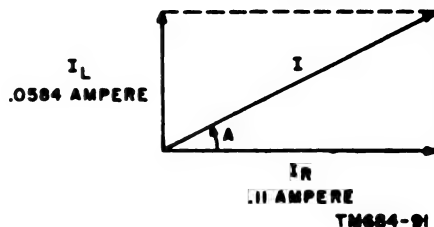


Figure 75. Ac parallel circuit containing inductance and resistance, vector diagram.

Step 2. The horizontal and vertical currents, respectively are:

$$I_R = 0.11 \text{ ampere}$$

$$I_L = 0.0584 \text{ ampere}$$

Step 3. Find the total current as follows:

$$I^2 = I_L^2 + I_R^2$$

$$= (0.0584)^2 + (0.11)^2$$

$$= 0.0034 + .0121$$

$$= 0.0155$$

$$I = \sqrt{0.0155}$$

$$= 0.1245 \text{ ampere}$$

Step 4. Find the phase angle by which the line current lags the applied voltage.

$$\cos A = \frac{Z}{R} \text{ (for parallel circuit)}$$

$$= \frac{884}{1,000}$$

$$= 0.88400$$

$$A = 27^\circ 52' 43''$$

Step 5. Thus, the line current lags the applied voltage by a phase angle of $27^\circ 52' 43''$.

Example 2: A 110-volt, 60-cycle ac generator is connected in a parallel circuit to a load consisting of an inductance of 3 henrys and a resistance of 10,000 ohms (A, fig. 76).

Step 1. Find the inductive reactance of the circuit.

$$X_L = 2\pi fL$$

$$= 2 \times 3.14 \times 60 \times 3$$

$$= 6.28 \times 180$$

$$= 1130.4$$

$$= 1130 \text{ ohms (approx)}$$

Step 2. Find the impedance of the circuit.

$$Z = \frac{RX_L}{\sqrt{R^2 + X_L^2}}$$

$$= \frac{10,000 \times 1130}{\sqrt{(10,000)^2 + (1130)^2}}$$

$$= \frac{10^4 \times 1.13 \times 10^3}{\sqrt{(10^4)^2 + (1.13 \times 10^3)^2}}$$

$$= \frac{1.13 \times 10^7}{\sqrt{10^8 + 1.277 \times 10^6}}$$

$$\begin{aligned}
&= \frac{1.13 \times 10^7}{\sqrt{100 \times 10^3 + 1.277 \times 10^3}} \\
&= \frac{1.13 \times 10^7}{\sqrt{101.277 \times 10^3}} \\
&= \frac{1.13 \times 10^7}{10.07 \times 10^3} \\
&= .1123 \times 10^4 \\
&= 1123 \text{ ohms (approx)}
\end{aligned}$$

Step 3. Find the line current in the circuit.

$$\begin{aligned}
I &= \frac{E}{Z} \\
&= \frac{110}{1123} \\
&= 0.09795 \text{ ampere}
\end{aligned}$$

Step 4. Find the current flowing through the inductance.

$$\begin{aligned}
I_L &= \frac{E}{X_L} \\
&= \frac{110}{1130} \\
&= .09734 \\
&= 0.0973 \text{ ampere (approx)}
\end{aligned}$$

Step 5. Find the current flowing through the resistance.

$$\begin{aligned}
I_R &= \frac{E}{R} \\
&= \frac{110}{10,000} \\
&= 0.011 \text{ ampere}
\end{aligned}$$

Step 6. Find the total current in the circuit. In an ac parallel circuit, the currents through the separate parts of the circuit are added vectorially (B, fig. 76).

$$\begin{aligned}
I_t^2 &= I_L^2 + I_R^2 \\
&= (0.097)^2 + (0.011)^2 \\
&= (9.7 \times 10^{-2})^2 + (1.1 \times 10^{-2})^2 \\
&= 94.09 \times 10^{-4} + 1.21 \times 10^{-4} \\
&= 95.3 \times 10^{-4} \\
I_t &= \sqrt{95.3 \times 10^{-4}} \\
&= 9.8 \times 10^{-2} \\
&= .098 \text{ ampere (approx)}
\end{aligned}$$

Step 7. In a parallel circuit, the voltage drop across one element would be the same as the voltage drop across another element in parallel with it. Thus, the voltage drop across both the inductance and the resistance is 110 volts.

Step 8. Find the phase angle by which the line current lags the applied voltage.

$$\begin{aligned}
\cos A &= \frac{Z}{R} \\
&= \frac{1123}{10,000} \\
&= 0.11230 \\
A &= 83^\circ 33' 52''
\end{aligned}$$

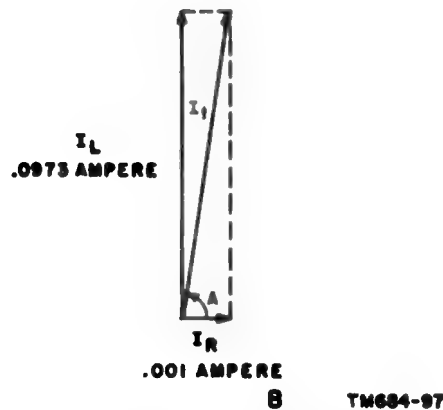
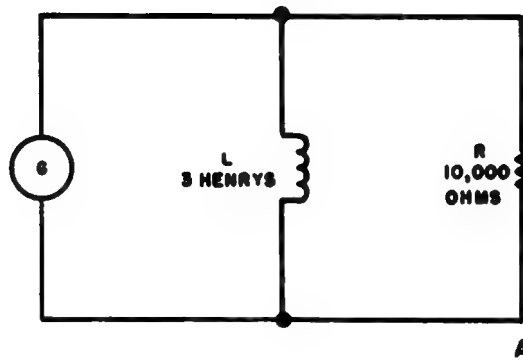


Figure 76. Ac parallel circuit having inductance and resistance, schematic and vector diagrams.

200. Solving Ac Circuits Having Resistance and Capacitance

a. *Series Circuits.* The following examples illustrate the method of solving series ac circuits having resistance and capacitance (called series RC circuits) by using the principles described in paragraphs 193 through 198.

Example 1: An ac generator in a series circuit is connected to a load consisting of a capacitance and a resistance (fig. 77). The voltage drop across the capacitance is 103 volts, and the voltage drop across the resistance is 39 volts. What is the applied voltage in the circuit? If the impedance of the circuit is 2,840 ohms, what is the phase angle by which the current leads the applied voltage?

Step 1. The vector diagram for this circuit is shown in figure 78. In a series circuit, the same current flows through all parts of the circuit—in this case, 0.039 ampere. Draw the vector E_R to represent the voltage drop across the resistance. Draw the vector E_C to represent the voltage drop across the capacitance.

Step 2. The vector sum of these voltage drops is equal to the applied voltage. Adding the horizontal and vertical voltage drops, respectively:

$$E_C + E_R = 0 + 39 = 39$$

$$E_C + E_R = 103 + 0 = 103$$

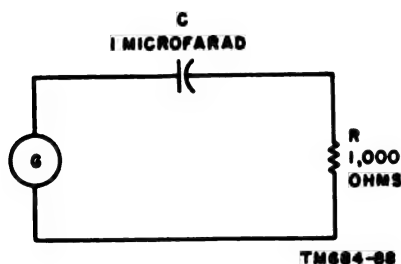


Figure 77. An ac series circuit containing capacitance and resistance.

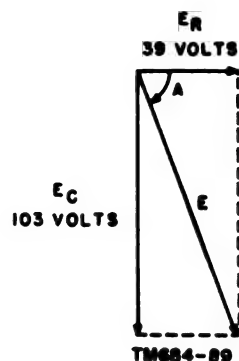


Figure 78. Ac series circuit containing capacitance and resistance, vector diagram.

Step 3. Find the applied voltage as follows:

$$\begin{aligned}
 E^2 &= E_C^2 + E_R^2 \\
 &= (103)^2 + (39)^2 \\
 &= (1.03 \times 10^3)^2 + (3.9 \times 10^2)^2 \\
 &= 1.061 \times 10^4 + 15.2 \times 10^3 \\
 &= 106.1 \times 10^3 + 15.2 \times 10^3 \\
 &= 121.3 \times 10^3 \\
 E &= \sqrt{121.3 \times 10^3} \\
 &= 11.01 \times 10 \\
 &= 110.1 \text{ volts}
 \end{aligned}$$

Step 4. Find the phase angle by which the current leads the applied voltage.

$$\begin{aligned}
 \cos A &= \frac{R}{Z} \\
 &= \frac{1000}{2840} \\
 &= 0.35211 \\
 &= 69^\circ 24'
 \end{aligned}$$

Step 5. Thus, the current leads the applied voltage by a phase angle of $69^\circ 24'$.

Example 2: A 110-volt, 60-cycle ac generator is connected in a series circuit to a load consisting of a 2-microfarad capacitor and a 10,000-ohm resistor (A, fig. 79).

Step 1. Find the capacitive reactance of the circuit.

$$\begin{aligned}
 X_C &= \frac{1}{2\pi fC} \\
 &= \frac{1}{2 \times 3.14 \times 60 \times 2 \times 10^{-6}} \\
 &= \frac{1}{753.6 \times 10^{-6}} \\
 &= \frac{1}{7.536 \times 10^{-4}} \\
 &= \frac{10^4}{7.536} \\
 &= \frac{10,000}{7.536} \\
 &= 1,327 \text{ ohms (approx)}
 \end{aligned}$$

Step 2. Find the impedance of the circuit.

$$\begin{aligned}
 Z &= \sqrt{R^2 + X_c^2} \\
 &= \sqrt{(10,000)^2 + (1327)^2} \\
 &= \sqrt{(10^4)^2 + (1.327 \times 10^3)^2} \\
 &= \sqrt{10^8 + 1.761 \times 10^6} \\
 &= \sqrt{100 \times 10^6 + 1.761 \times 10^6} \\
 &= \sqrt{101.761 \times 10^6} \\
 &= 10.088 \times 10^3 \\
 &= 10,088 \text{ ohms (approx)}
 \end{aligned}$$

Step 3. Find the current in the circuit.

$$\begin{aligned}
 I &= \frac{E}{Z} \\
 &= \frac{110}{10,088} \\
 &= 0.0109 \text{ ampere (approx)}
 \end{aligned}$$

Step 4. In a series circuit, the same current flows through all parts of the circuit; therefore, the current through both the capacitance and the resistance is 0.0109 ampere.

Step 5. Find the voltage drop across the capacitance.

$$\begin{aligned}
 E_c &= IX_c \\
 &= 0.0109 \times 1327 \\
 &= 14.46 \\
 &= 14 \text{ volts}
 \end{aligned}$$

Step 6. Find the voltage drop across the resistance.

$$\begin{aligned}
 E_R &= IR \\
 &= 0.0109 \times 10,000 \\
 &= 109 \text{ volts}
 \end{aligned}$$

Step 7. Find the total voltage in the circuit (B, fig. 79).

$$\begin{aligned}
 E_t^2 &= E_R^2 + E_c^2 \\
 &= (109)^2 + (14)^2 \\
 &= (1.09 \times 10^2)^2 + (1.4 \times 10)^2 \\
 &= 1.1881 \times 10^4 + 1.96 \times 10^2 \\
 &= 118.81 \times 10^2 + 1.96 \times 10^2 \\
 &= 120.77 \times 10^2 \\
 E &= \sqrt{120.77 \times 10^2} \\
 &= 10.99 \times 10 \\
 &= 109.9 \text{ or } 110 \text{ volts}
 \end{aligned}$$

Step 8. Find the phase angle by which the current leads the applied voltage.

$$\begin{aligned}
 \cos A &= \frac{R}{Z} \\
 &= \frac{10,000}{10,088} \\
 &= 0.991178 \\
 &= 0.99118 \\
 A &= 7^\circ 37'
 \end{aligned}$$

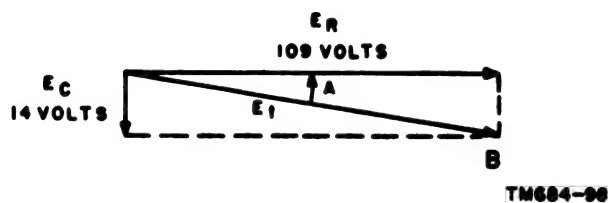
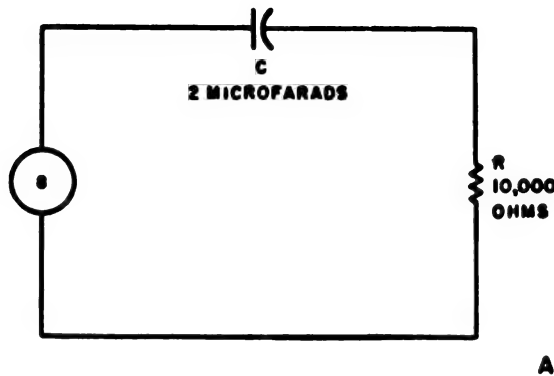


Figure 79. Ac series circuit having capacitance and resistance, schematic and vector diagrams.

b. Parallel Circuits. The following examples illustrate the method of solving parallel RC circuits by using the principles described in paragraphs 193 through 198.

Example 1: An ac circuit has a capacitance and resistance connected in parallel (fig. 80). The current flowing through the capacitance is 0.0415 ampere, and the current flowing through the resistance is 0.11 ampere. What is the total current in the circuit? If the impedance of the circuit is 938 ohms, what is the phase angle by which the current leads the applied voltage?

Step 1. The vector diagram for this circuit is shown in figure 81. In a parallel circuit, the voltage drop across each capacitance or resistance is the same—in this case, 110 volts. Draw the vector I_R to represent the current through the resistor. Draw the vector I_C to represent the current through the capacitance.

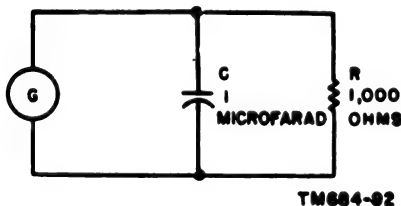


Figure 80. An ac parallel circuit containing capacitance and resistance.

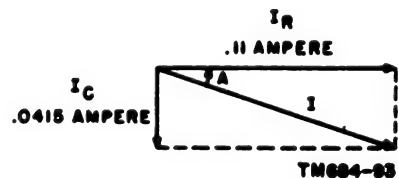


Figure 81. Ac parallel circuit containing capacitance and resistance, vector diagram.

Step 2. The vector sum of the currents through the separate parts of the circuit will be equal to the total current. Adding the horizontal and vertical currents, respectively:

$$I_C + I_R = 0 + .11 = .11$$

$$I_C + I_R = .0415 + 0 = .0415$$

Step 3. Find the total current as follows:

$$\begin{aligned} I^2 &= I_C^2 + I_R^2 \\ &= (.0415)^2 + (.11)^2 \\ &= (1.45 \times 10^{-3})^2 + (11 \times 10^{-2})^2 \\ &= 2.1 \times 10^{-4} + 121 \times 10^{-4} \\ &= 123.1 \times 10^{-4} \\ I &= \sqrt{123.1 \times 10^{-4}} \\ &= 11.1 \times 10^{-2} \\ &= .111 \text{ ampere} \end{aligned}$$

Step 4. Find the phase angle by which the current leads the applied voltage.

$$\begin{aligned} \cos A &= \frac{Z}{R} \\ &= \frac{938}{1,000} \\ &= .93800 \\ A &= 20^\circ 17' 6'' \end{aligned}$$

Step 5. Thus, the current leads the applied voltage by a phase angle of $20^\circ 17' 6''$.

Example 2: A 110-volt, 60-cycle ac generator is connected to a load consisting of a 2-microfarad capacitance and a 10,000-ohm resistance in parallel (A, fig. 82).

Step 1. Find the capacitance reactance of the circuit.

$$\begin{aligned} X_C &= \frac{1}{2\pi fC} \\ &= \frac{1}{2 \times 3.14 \times 60 \times 2 \times 10^{-6}} \\ &= \frac{1}{7.536 \times 10^{-4}} \\ &= \frac{10^4}{7.536} \\ &= \frac{10,000}{7.536} \\ &= 1,327 \text{ ohms} \end{aligned}$$

Step 2. Find the impedance of the circuit.

$$\begin{aligned} Z &= \frac{RX_C}{\sqrt{R^2 + X_C^2}} \\ &= \frac{10,000 \times 1327}{\sqrt{(10,000)^2 + (1327)^2}} \\ &= \frac{10^4 \times 1.327 \times 10^3}{\sqrt{(10^4)^2 + (1.327 \times 10^3)^2}} \\ &= \frac{1.327 \times 10^7}{\sqrt{10^8 + 1.76 \times 10^6}} \\ &= \frac{1.327 \times 10^7}{\sqrt{100 \times 10^6 + 1.76 \times 10^6}} \end{aligned}$$

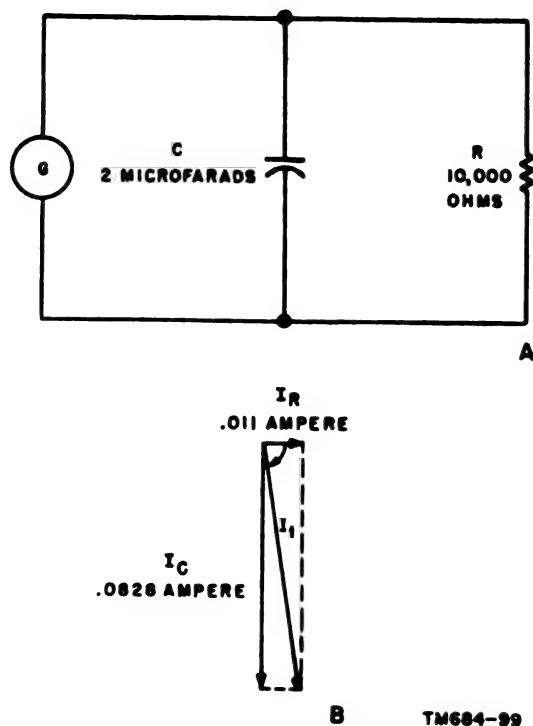


Figure 82. Ac parallel circuit having resistance and capacitance, schematic and vector diagrams.

$$\begin{aligned}
 &= \frac{1.327 \times 10^7}{\sqrt{101.76 \times 10^4}} \\
 &= \frac{1.327 \times 10^7}{10.088 \times 10^3} \\
 &= .1315 \times 10^4 \\
 &= .1315 \text{ ohms (approx)}
 \end{aligned}$$

Step 3. Find the current flowing through the capacitance.

$$\begin{aligned}
 I_C &= \frac{E}{X_C} \\
 &= \frac{110}{1327} \\
 &= 0.08289 \\
 &= 0.0829 \text{ ampere}
 \end{aligned}$$

Step 4. Find the current flowing through the resistance.

$$\begin{aligned}
 I_R &= \frac{E}{R} \\
 &= \frac{110}{10,000} \\
 &= 0.011 \text{ ampere}
 \end{aligned}$$

Step 5. Find the total current in the circuit.

$$\begin{aligned}
 I_T^2 &= I_R^2 + I_C^2 \\
 &= (.011)^2 + (.0829)^2 \\
 &= (1.1 \times 10^{-2})^2 + (8.29 \times 10^{-2})^2 \\
 &= 1.21 \times 10^{-4} + 68.72 \times 10^{-4} \\
 &= 69.93 \times 10^{-4}
 \end{aligned}$$

$$\begin{aligned}
 I_t &= \sqrt{69.93 \times 10^{-4}} \\
 &= 8.36 \times 10^{-2} \\
 &= .0836 \text{ ampere (approx)}
 \end{aligned}$$

- Step 6. In a parallel circuit, the voltage drop across each capacitance or resistance in parallel is the same. Thus, the voltage drop across both the capacitance and the resistance is 110 volts.
- Step 7. Find the phase angle by which the current leads the applied voltage.

$$\begin{aligned}
 \cos A &= \frac{Z}{R} \\
 &= \frac{1315}{10,000} \\
 &= 0.13150 \\
 A &= 82^\circ 26' 37''
 \end{aligned}$$

201. Solving Ac Circuits Having Resistance, Inductance, and Capacitance

a. *Series Circuits.* The following examples illustrate the method of solving series ac circuits having resistance, inductance, and capacitance (called series RLC circuits) by using the principles described in paragraphs 193 through 198.

Example 1: A 300-volt, 60-cycle ac generator is connected in series with a 6-ohm resistance, an 8-ohm inductive reactance, and a 16-ohm capacitive reactance (fig. 83). Find (1) the resultant reactive voltage, (2) the current flowing in the circuit, and (3) the voltage drops across the resistance, the inductance, and the capacitance. (4) Check the solution by vectorially adding E_L , E_C , and E_R . The result should equal the applied voltage. (5) Find the phase angle by which the current leads or lags the applied voltage.

- Step 1. The vector diagram for this circuit is shown in figure 84. Since E_C and E_L are 180° out of phase, their vector sum

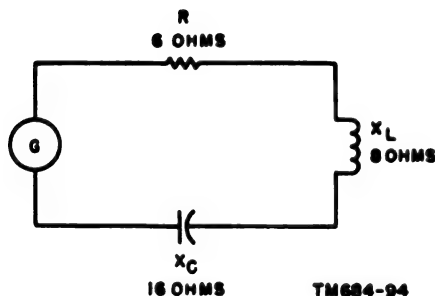


Figure 83. An ac series circuit containing resistance, inductive reactance, and capacitance reactance.

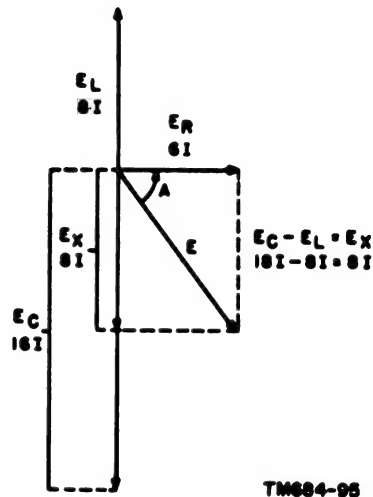


Figure 84. Ac series circuit containing resistance, inductive reactance, and capacitive reactance vector diagram.

is the difference between the two. E_C is greater than E_L ; thus, the resultant reactive voltage, E_X , is $16I - 8I = 8I$.

Step 2. Find the current flowing in the circuit.

$$\begin{aligned} E^2 &= E_R^2 + E_X^2 \\ E^2 &= (IR)^2 + (IX)^2 \\ (300)^2 &= (6I)^2 + (8I)^2 \\ (3 \times 10^2)^2 &= (6I)^2 + (8I)^2 \\ 9 \times 10^4 &= 36I^2 + 64I^2 \\ 9 \times 10^4 &= 100I^2 \\ 9 \times 10^4 &= 10^2 I^2 \\ \sqrt{9 \times 10^4} &= \sqrt{10^2 I^2} \\ 3 \times 10^2 &= 10I \\ 300 &= 10I \\ I &= 30 \text{ amperes} \end{aligned}$$

Step 3. Find the voltage drop across the resistance.

$$\begin{aligned} E_R &= IR \\ &= 30 \times 6 \\ &= 180 \text{ volts} \end{aligned}$$

Step 4. Find the voltage drop across the inductance.

$$\begin{aligned} E_L &= IX_L \\ &= 30 \times 8 \\ &= 240 \text{ volts} \end{aligned}$$

Step 5. Find the voltage drop across the capacitance.

$$\begin{aligned} E_C &= IX_C \\ &= 30 \times 16 \\ &= 480 \text{ volts} \end{aligned}$$

Step 6. Find the resultant reactive voltage.

$$\begin{aligned} E_X &= E_C - E_L \\ &= 480 - 240 \\ &= 240 \text{ volts} \end{aligned}$$

Step 7. Vectorially add the voltages in the circuit. The result should equal the applied voltage.

$$\begin{aligned} E^2 &= E_R^2 + E_X^2 \\ &= (180)^2 + (240)^2 \\ &= (1.8 \times 10^2)^2 + (2.4 \times 10^2)^2 \\ &= 3.24 \times 10^4 + 5.76 \times 10^4 \\ &= 9 \times 10^4 \\ E &= \sqrt{9 \times 10^4} \\ &= 3 \times 10^2 \\ &= 300 \text{ volts} \end{aligned}$$

Step 8. Find the phase angle by which the current leads or lags the applied voltage in the circuit.

$$\begin{aligned} \tan A &= \frac{X_L - X_C}{R} \\ &= \frac{8}{6} \\ &= 1.33333 \\ A &= 53^\circ 7' 48''. \end{aligned}$$

Step 9. The circuit is predominantly capacitive; therefore, the current *leads* the applied voltage by a phase angle of $53^{\circ} 7' 48''$.

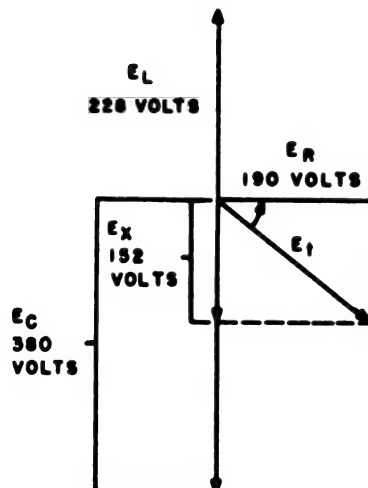
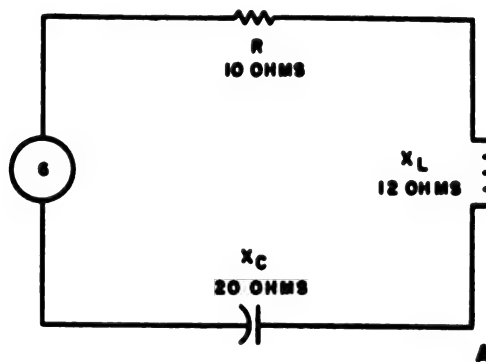
Example 2: A 60-cycle ac generator is connected in series with a 10-ohm resistance, a 12-ohm inductive reactance, and a 20-ohm capacitive reactance (A, fig. 85). The current flowing through the circuit is 19 amperes. (1) Find the voltage drop across each circuit element. (2) Find the total voltage. (3) Find the phase angle between the current and the applied voltage.

Step 1. Find the voltage drop across the resistance.

$$\begin{aligned} E_R &= IR \\ &= 19 \times 10 \\ &= 190 \text{ volts} \end{aligned}$$

Step 2. Find the voltage drop across the inductance.

$$\begin{aligned} E_L &= IX_L \\ &= 19 \times 12 \\ &= 228 \text{ volts} \end{aligned}$$



B

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Figure 85. An ac series circuit having inductances, capacitances, and resistance, schematic and vector diagrams.

Step 3. Find the voltage drop across the capacitance.

$$\begin{aligned} E_c &= IX_c \\ &= 19 \times 20 \\ &= 380 \text{ volts} \end{aligned}$$

Step 4. Find the resultant reactive voltage.

$$\begin{aligned} E_x &= E_c - E_L \\ &= 380 - 228 \\ &= 152 \text{ volts} \end{aligned}$$

Step 5. Find the total voltage in the circuit.

$$\begin{aligned} E^2 &= E_R^2 + E_x^2 \\ &= (190)^2 + (152)^2 \\ &= (1.9 \times 10^2)^2 + (1.52 \times 10^2)^2 \\ &= 3.61 \times 10^4 + 2.31 \times 10^4 \\ &= 5.92 \times 10^4 \\ E &= \sqrt{5.92 \times 10^4} \\ &= 2.43 \times 10^2 \\ &= 243 \text{ volts} \end{aligned}$$

Step 6. Find the phase angle by which the current leads the applied voltage in the circuit. Since the capacitive reactance is greater and cancels the inductive reactance, the circuit is capacitive and the current leads the applied voltage by the phase angle A .

$$\begin{aligned} \tan A &= \frac{X_c - X_L}{R} \\ &= \frac{20 - 12}{10} \\ &= \frac{8}{10} \\ &= .80000 \\ A &= 38^\circ 39' 35'' \end{aligned}$$

b. Parallel Circuits. The following example illustrates the method of solving parallel ac circuits having resistance, inductance, and capacitance (called parallel RLC circuits) by using the principles described in paragraphs 193 through 198.

Example: A parallel circuit has a 300-volt input, a 150-ohm resistance, a 125-ohm inductive reactance, and a 100-ohm capacitive reactance (A, fig. 86).

Step 1. Since this is a parallel circuit, the same voltage is impressed across the inductance, the resistance, and the capacitance. Thus, the voltage across each of them is 300 volts.

Step 2. Find the current flowing through the resistor.

$$\begin{aligned} I_R &= \frac{E}{R} \\ &= \frac{300}{150} \\ &= 2 \text{ amperes} \end{aligned}$$

Step 3. Find the current flowing through the inductance.

$$\begin{aligned} I_L &= \frac{E}{X_L} \\ &= \frac{300}{125} \\ &= 2.4 \text{ amperes} \end{aligned}$$

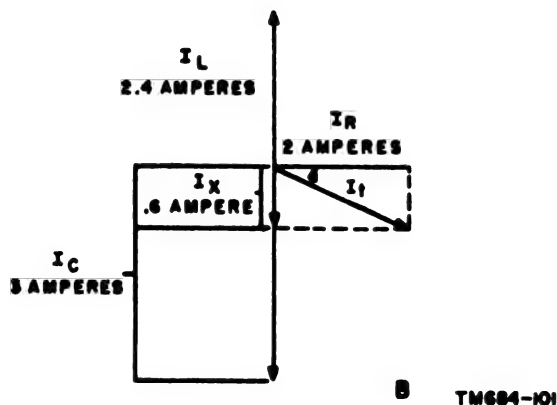
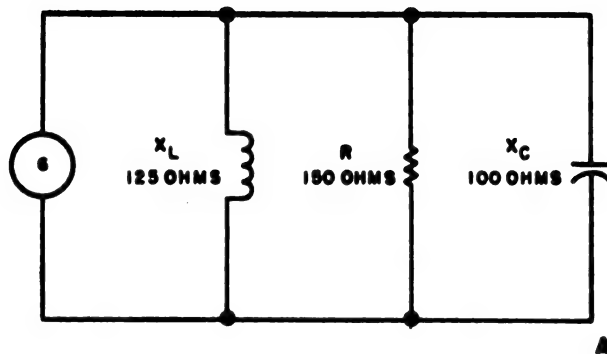


Figure 86. A parallel circuit having inductance, capacitance, and resistance, schematic and vector diagrams.

Step 4. Find the current flowing through the capacitor.

$$\begin{aligned}
 I_C &= \frac{E}{X_C} \\
 &= \frac{300}{100} \\
 &= 3 \text{ amperes}
 \end{aligned}$$

Step 5. Find the total current in the circuit (B, fig. 86).

$$\begin{aligned}
 I_X &= I_C - I_L \\
 &= 3 - 2.4 \\
 &= .6 \text{ ampere} \\
 I_T^2 &= I_R^2 + I_X^2 \\
 &= (2)^2 + (.6)^2 \\
 &= 4 + .36 \\
 &= 4.36 \\
 I_T &= \sqrt{4.36} \\
 &= 2.0889 \\
 &= 2.089 \text{ amperes}
 \end{aligned}$$

Step 6. Find the impedance of the circuit.

$$\begin{aligned} Z &= \frac{E}{I_t} \\ &= \frac{300}{2.089} \\ &= 143.6 \\ &= 144 \text{ ohms (approx)} \end{aligned}$$

Step 7. Find the phase angle by which the current leads the applied voltage. Since this is a parallel circuit in which the inductive reactance is greater than the capacitive reactance, the circuit is capacitive and the current leads the applied voltage.

$$\begin{aligned} \cos A &= \frac{Z}{R} \\ &= \frac{144}{150} \\ &= .96000 \\ A &= 16^\circ 15' 38'' \end{aligned}$$

202. Resonance

In a series or parallel ac circuit containing inductance and capacitance, a condition known as *resonance* exists when the inductive reactance equals the capacitive reactance. This condition occurs at a specific frequency called the *resonant frequency*. A formula for finding the resonant frequency is derived by equating the formulas for inductive reactance and capacitive reactance, as follows:

$$\begin{aligned} X_L &= X_C \\ 2\pi f L &= \frac{1}{2\pi f C} \\ 4\pi^2 f^2 LC &= 1 \\ f^2 &= \frac{1}{4\pi^2 LC} \\ f_r &= \frac{1}{2\pi\sqrt{LC}} \end{aligned}$$

where f_r is the resonant frequency.

Example: Find the resonant frequency of a circuit containing a 4-millihenry inductance and a 40-micromicrofarad capacitor in series with a variable frequency ac source.

$$\begin{aligned} f &= \frac{1}{2\pi\sqrt{LC}} \\ &= \frac{1}{6.28\sqrt{4 \times 10^{-3} \times 4 \times 10^{-11}}} \\ &= \frac{1}{6.28 \times 4 \times 10^{-7}} \\ &= \frac{10^7}{25.12} \\ &= \frac{10,000,000}{25.12} \\ &= 398,000 \text{ cps or } 398 \text{ kilocycles (kc).} \end{aligned}$$

203. Ac Power

a. In an ac circuit containing both resistance and reactance the only power actually dissipated is the power absorbed by the resistance of the circuit (*b* below). However, if the circuit contained reactance only, large amounts of power would still *appear* to be consumed because of the phase difference between voltage and current. Consequently, in either case an ac generator supplying power to the circuit would receive less power from the circuit than it delivers to the circuit. The power which the generator delivers to the circuit is called the *apparent* power and is equal to the product of the effective value of the voltage ($E_{\max}/\sqrt{2}$) and the effective value of the current ($I_{\max}/\sqrt{2}$). Therefore,

$$\begin{aligned} P \text{ (apparent power)} &= \frac{E_{\max}}{\sqrt{2}} \cdot \frac{I_{\max}}{\sqrt{2}} \\ &= \frac{E_{\max} I_{\max}}{2} \end{aligned}$$

b. Apparent power is different from the actual power consumed by the load, which is called the average or true power and is the energy absorbed by the resistance of the circuit. The average or true power is expressed by the formula $P = EI \cos \theta$, where

E = effective value of the voltage across the circuit

I = effective value of the current in the circuit

θ = phase angle between current and voltage

c. Apparent power may also be expressed by the following formulas:

$$P = EI$$

$$P = I^2 Z$$

$$P = \frac{E^2}{Z}$$

d. In a purely resistive circuit, average or true ac power also may be expressed by Joule's law ($P = I^2 R$) as in the dc case (par. 191).

e. The following examples illustrate some of the above principles.

Example 1: Find the power that an ac generator must deliver to a circuit if the peak voltage is 230 volts and the peak current is 5 amperes.

$$\begin{aligned} P &= \frac{E_{\max} I_{\max}}{2} \\ &= \frac{230 \times 5}{2} \\ &= 575 \text{ watts.} \end{aligned}$$

Example 2: Find the average power consumed in a circuit if the effective ac voltage is 115 volts, the effective current is 7 amperes, and the current leads the voltage by 60° .

$$\begin{aligned} P &= EI \cos \theta \\ &= 115 \times 7 \times \cos 60^\circ \\ &= 115 \times 7 \times .5 \\ &= 402.5 \text{ watts.} \end{aligned}$$

204. Review Problems—Ac Electricity

a. An alternator is connected to a 520-volt, 60-cycle ac parallel circuit having a resistance of 96 ohms, an inductance of 249 millihenrys, and a capacity of 19.8 microfarads. (1) Find the inductive reactance of the circuit. (2) Find the capacitive reactance of the circuit. (3) Determine whether the current leads or lags the voltage. (4) Find the impedance of the circuit. (5) Determine the value of the current in the circuit.

b. Determine the inductive reactance of a coil if the ac in the circuit has a frequency of 60 cps, and the inductance of the coil is 0.025 henry.

c. A 110-volt, 25-cycle ac generator is connected in series with a 0.1-microfarad capacitance and a 2,000-ohm resistance. What is the capacitive reactance of the circuit?

d. What is the value of the reactance of a circuit if the impedance $Z = 100$ ohms and the resistance $R = 60$ ohms?

e. Find the resonant frequency of a series RLC circuit if the inductance is 0.478 millihenry and the capacitance is 256 micromicrofarads.

f. A series RLC circuit consists of 6 ohms resistance, 8 ohms inductive reactance, and 16 ohms capacitive reactance. (1) Find the current in the circuit. (2) Find the voltage drop across the resistance. (3) Find the voltage drop across the capacitance. (4) Find the voltage drop across the inductance.

g. A parallel RLC circuit has an input voltage of 300 volts, an inductive reactance of 75 ohms, a capacitive reactance of 50 ohms, and a resistance of 100 ohms. (1) Find the current through the resistance. (2) Find the current through the inductance. (3) Find the

current through the capacitance. (4) Find the total impedance of the circuit. (5) Find the phase angle between the line or total current and the applied voltage. (6) Find the average power. (7) Find the apparent power.

CHAPTER 16

APPLICATIONS OF LOGARITHMS TO TRANSMISSION PROBLEMS

205. The Transmission Unit

When signal power is transmitted along a transmission line, there is a power loss or attenuation; if an amplifier is used in the circuit, there may be a power gain. This loss or gain of power, resulting in a decrease or increase in the intensity of the signal, is measured in terms of the decibel (db). The decibel is a measure of power *ratio* and is probably the most widely used unit in communications. The formula for measuring transmission loss or gain is:

$$\text{db} = 10 \log_{10} \frac{P_1}{P_2}$$

where $\frac{P_1}{P_2}$ is the ratio of the two powers being compared (par. 206).

206. Converting Power Ratio to Decibels

When converting a power ratio into its decibel expression, represent the larger power as P_1 and the smaller power as P_2 , regardless of whether the larger power is the input or output. Thus, the power ratio will always be greater than 1, and its logarithm will be a positive number. Prefix a plus sign to the answer if the power change is a gain (the power output greater than the power input); prefix a minus sign if the power change is a loss.

Example 1: The input power to a transmission line is 10 milliwatts, and the output power is 2.46 milliwatts. Express the power change in db.

$$\begin{aligned} \text{db} &= 10 \log \frac{P_1}{P_2} \\ &= 10 \log \frac{10}{2.46} \\ &= 10 \times \log 4.07 \\ &= 10 \times .6096 \\ &= +6.096 \end{aligned}$$

Thus, the loss of the transmission line is 6.096 db, since input is greater than output.

Example 2: A repeater amplifier has an input power of 2 milliwatts and an output power of 400 watts. Calculate the power change.

$$\begin{aligned} \text{db} &= 10 \log \frac{P_1}{P_2} \\ &= 10 \log \frac{400}{.002} \\ &= 10 \times \log 200,000 \\ &= 10 \times 5.3010 \\ &= +53.01 \end{aligned}$$

The gain of the repeater amplifier is 53.01 db, since output is greater than input.

207. Converting Decibels to Power Ratio

To find the power ratio when the gain or loss is expressed in decibels, reverse the procedure given in paragraph 206. If the number of decibels is positive, the circuit has a power gain and the output power is greater than the input power. If the number of decibels is negative, the circuit has a power loss and the output power is less than the input power. Insert the power change in decibels in the formula given in paragraph 200 and divide by 10; then find the antilog of both sides of the equation (par. 118) to obtain the power ratio.

Example 1: A circuit is known to have a power change of +12 db. Find the power ratio.

$$\begin{aligned} \text{db} &= 10 \log \frac{P_1}{P_2} \\ 12 \text{ db} &= 10 \log \frac{P_1}{P_2} \\ 1.2 &= \log \frac{P_1}{P_2} \end{aligned}$$

Find the antilog of both sides of the equation.

$$\frac{P_1}{P_2} = 15.85 \text{ of } 15.9$$

Since the number of decibels is given as positive, the circuit has a gain and its output power is 15.9 times its input power.

Example 2: A certain wire transmission circuit has a power change of -25 db. Calculate the power ratio.

$$\text{db} = 10 \log \frac{P_1}{P_2}$$

$$25 \text{ db} = 10 \log \frac{P_1}{P_2}$$

$$2.5 = \log \frac{P_1}{P_2}$$

$$\frac{P_1}{P_2} = 316.2$$

Since the number of decibels has a minus sign, the circuit attenuates power. The output power is less than the input power by a ratio of 1 to 316.2.

208. Review Problems—Transmission Problems

a. A network has a loss of 16 decibels. What power ratio correspond to this loss?

b. The input to a powerline 50 miles long is 210 milliwatts. The power delivered at the end of the line is 40 microwatts. What is the attenuation in decibels per mile?

c. A power of 10 milliwatts is required to drive an audiofrequency (af) amplifier. The output of the amplifier is 120 milliwatts. What is the gain in decibels?

d. What is the ratio of the output power to the input power if there is a power gain of 14 decibels?

CHAPTER 17

MISCELLANEOUS ELECTRICAL PROBLEMS

209. Efficiency

Efficiency is the ratio of output to input and usually is expressed in percent (ch. 2). Generators, motors, and other electrical devices often are rated according to their efficiency. To express efficiency in percent, write the ratio of output to input as a fraction, convert to a decimal, and then convert the decimal to a percent (par. 4).

Example: What is the efficiency of a generator that has an output of 60 kilowatts (kw) and an input of 75 kilowatts?

$$\begin{aligned}\text{Efficiency} &= \frac{\text{Output}}{\text{Input}} \\ &= \frac{60}{75} \\ &= 0.80 \\ &= 80\%\end{aligned}$$

210. Percent Overload

Another application of percent is the overload rating of motors, generators, etc. In this application, the amount of power, that can be applied to or taken from an electrical device, above the rated output, is expressed as a percent of the rated output.

Example 1: What is the percent of overload capacity of a generator that has a rated output of 500 watts and can provide a maximum of 550 watts?

$$\begin{aligned}\text{Overload} &= \text{maximum power} - \text{rated power} \\ &= 550 - 500 \\ &= 50 \text{ watts}\end{aligned}$$

$$\begin{aligned}\text{Percent overload} &= \frac{\text{Overload}}{\text{Rated power}} \\ &= \frac{50}{500} \\ &= 10\%\end{aligned}$$

Example 2: Find the maximum output of a generator that is rated at 1,500 watts, and has a 10 percent overload capacity.

$$0.10 \times 1,500 = 150 \text{ watts}$$

$$1,500 + 150 = 1,650 \text{ watts maximum output.}$$

211. Tolerances

A tolerance is an allowance for variations from the standard or specified value. In the manufacture of resistors, for example, the resistance is permitted to be within a specified percentage of the standard value. This percentage is indicated in the color code of the resistors.

Example: Find the possible low and high values of a 20,000-ohm resistor with a tolerance of ± 5 percent.

$$0.05 \times 20,000 = 1,000 \text{ ohms}$$

$$20,000 + 1,000 = 21,000 \text{ ohms (high value)}$$

$$20,000 - 1,000 = 19,000 \text{ ohms (low value)}$$

Therefore, since the tolerance is plus or minus 5%, the value of the resistor should be between 21,000 and 19,000 ohms.

212. Transformer Relationships

a. *General.* In a transformer, relationships exist between the currents, voltages, impedances, and number of turns of wire in the windings. These relationships are expressed by equations containing ratios involving these quantities.

b. *Relationship Between Voltage and Number of Turns.* This relationship is expressed by the following equation:

$$\frac{E_2}{N_2} = \frac{E_1}{N_1}$$

where E_1 is the voltage across the primary winding, N_1 is the number of turns on the primary winding, E_2 is the voltage across the secondary winding, and N_2 is the number of turns on the secondary winding (fig. 87). The equation may also be written:

$$\begin{aligned}\frac{E_2}{E_1} &= \frac{N_2}{N_1} \\ E_2 N_1 &= E_1 N_2 \\ E_2 &= \frac{E_1 N_2}{N_1} \text{ or } E_1 \left(\frac{N_2}{N_1} \right) \\ E_1 &= \frac{E_2 N_1}{N_2} \text{ or } E_2 \left(\frac{N_1}{N_2} \right)\end{aligned}$$

The ratios N_2/N_1 and N_1/N_2 are called the *turns ratios* and may be expressed as a single factor.

Example: Find the voltage across the secondary winding of a transformer if the primary voltage is 100 volts and the turns ratio from primary to secondary is 1 to 4.

$$\begin{aligned}\frac{N_2}{N_1} &= \frac{1}{4} \text{ or } \frac{N_1}{N_2} = 4 \\ E_2 &= E_1 \left(\frac{N_2}{N_1} \right) \\ E_2 &= 100 (4) \\ &= 400 \text{ volts}\end{aligned}$$

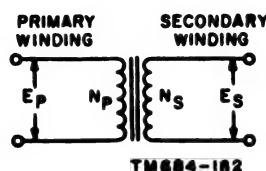


Figure 87. Simple Transformer.

c. *Relationship Between Current and Number of Turns.* This relationship is expressed by the following equation:

$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$

where I_1 is the primary current, I_2 is the secondary current, and N_1 and N_2 the number of turns on the primary and secondary as before. The equation may be written:

$$\begin{aligned}I_1 N_1 &= I_2 N_2 \\ I_1 &= \frac{I_2 N_2}{N_1} \text{ or } I_2 \left(\frac{N_2}{N_1} \right) \\ I_2 &= \frac{I_1 N_1}{N_2} \text{ or } I_1 \left(\frac{N_1}{N_2} \right)\end{aligned}$$

Example: Find the primary current in a transformer if the secondary current is 5 milliamperes and the turns ratio from primary to secondary is 20 to 1.

$$\frac{N_s}{N_p} = \frac{20}{1} \text{ or, } \frac{N_s}{N_p} = \frac{1}{20}$$

$$I_p = I_s \left(\frac{N_s}{N_p} \right)$$

$$= 5 \times 10^{-3} \left(\frac{1}{20} \right)$$

$$I_p = \frac{10^{-4}}{4}$$

$$= \frac{0.00100}{4}$$

$$= 0.00025 \text{ amperes, or, } 0.25 \text{ milliamperes}$$

d. Relationship Between Current and Voltage. By combining the relationships given in b and c above, a relationship can be derived between primary and secondary currents as follows:

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} \text{ (from b above)}$$

$$\frac{I_s}{I_p} = \frac{N_p}{N_s} \text{ (from c above)}$$

$$\text{Therefore, } \frac{E_s}{E_p} = \frac{I_p}{I_s} \text{ (because both are equal to } \frac{N_p}{N_s} \text{)}$$

The equation may also be written:

$$E_p = \frac{E_s I_p}{I_s}$$

$$I_p = \frac{E_s I_s}{E_p}$$

$$E_p = \frac{E_s I_s}{I_p}$$

$$I_p = \frac{E_s I_s}{E_p}$$

$$E_p I_p = E_s I_s$$

Since voltage multiplied by current equals power, the last form of the equation states that the power absorbed by the primary winding is equal to the power delivered to the secondary winding. This is true in an ideal transformer which has no loss, and is essentially true in an actual transformer which has very little loss; efficiencies of 98 percent are common in actual transformers.

Example: Find the voltage across the secondary winding of a transformer if the primary voltage is 150 volts, the primary current is 5 amperes, and the secondary current is 25 amperes.

$$E_p = \frac{E_s I_s}{I_p}$$

$$= \frac{150 \times 5}{25}$$

$$= 30 \text{ volts}$$

e. Relationship Between Impedance and Number of Turns. This relationship also can be derived from the relationships given in *b* and *c* above by dividing one by the other, as follows:

$$\begin{aligned}\frac{E_2}{E_1} &= \frac{N_2}{N_1} \text{ (from } b \text{ above)} \\ \frac{I_2}{I_1} &= \frac{N_1}{N_2} \text{ (from } c \text{ above)} \\ \frac{\frac{E_2}{E_1}}{\frac{I_2}{I_1}} &= \frac{\frac{N_2}{N_1}}{\frac{N_1}{N_2}} \text{ (dividing the first by the second)} \\ \frac{I_2}{I_1} \cdot \frac{E_2}{E_1} &= \frac{N_2}{N_1} \cdot \frac{N_2}{N_1} \\ \frac{E_2}{I_1} \cdot \frac{I_2}{E_1} &= \frac{N_2^2}{N_1^2} \\ Z_1 \cdot \frac{1}{Z_2} &= \frac{N_2^2}{N_1^2} \text{ (substituting } Z \text{ for } \frac{E}{I}) \\ \frac{Z_1}{Z_2} &= \frac{N_2^2}{N_1^2} \text{ or, } \frac{Z_2}{Z_1} = \left(\frac{N_2}{N_1}\right)^2\end{aligned}$$

where Z_1 is the impedance of the primary winding and Z_2 is the impedance of the secondary winding in ohms. The equation may also be written:

$$\begin{aligned}Z_2 &= Z_1 \left(\frac{N_2}{N_1}\right)^2 \\ Z_1 &= Z_2 \left(\frac{N_1}{N_2}\right)^2\end{aligned}$$

Example: Find the impedance of the secondary winding of a transformer if the impedance of the primary winding is 200 ohms and the turns ratio from primary to secondary is 5 to 1.

$$\begin{aligned}\frac{N_2}{N_1} &= \frac{5}{1} \text{ or, } \frac{N_1}{N_2} = \frac{1}{5} \\ Z_1 &= Z_2 \left(\frac{N_1}{N_2}\right)^2 \\ &= 200 \left(\frac{1}{5}\right)^2 \\ &= 200 \times \frac{1}{25} \\ &= 8 \text{ ohms}\end{aligned}$$

213. Conductance

Conductance is a measure of the ease with which current flows in a circuit. It is given the symbol G and is equal to the reciprocal of resistance: $G = 1/R$. The unit of conductance is the *mho*, which is the word ohm spelled backwards.

Example: Find the conductance of a circuit consisting of a 4-ohm resistor in parallel with a 5-ohm resistor. In a parallel circuit, the

reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances:

$$\begin{aligned}\frac{1}{R_t} &= \frac{1}{R_1} + \frac{1}{R_2} \\ \text{or } G &= \frac{1}{R_1} + \frac{1}{R_2} \\ G &= \frac{1}{4} + \frac{1}{5} \\ &= .25 + .20 \\ &= .45 \text{ mhos}\end{aligned}$$

214. Energy Stored in an Inductance

The amount of energy stored in an inductance is determined from the formula

$$P = \frac{LI^2}{2}, \text{ where}$$

L = inductance in henrys

I = current in amperes

P = energy in joules.

Example: Find the energy stored in a coil if the inductance is 7 millihenrys and the current is 3 milliamperes. Using scientific notation (par. 106), the energy in joules is:

$$\begin{aligned} P &= \frac{LI^2}{2} \\ &= \frac{7 \times 10^{-3} (3 \times 10^{-3})^2}{2} \\ &= \frac{7 \times 10^{-3} 9 \times 10^{-6}}{2} \\ &= \frac{63 \times 10^{-9}}{2} \\ &= 31.5 \times 10^{-9} \text{ joules.} \end{aligned}$$

215. Delta-Wye Transformations

a. A delta circuit consists of three resistors or other circuit components connected together to form the Greek letter *delta* (Δ). In a wye circuit, the resistors or other circuit components are connected together to form the letter *Y*. Figure 88 shows an example of each type of circuit constructed of resistors.

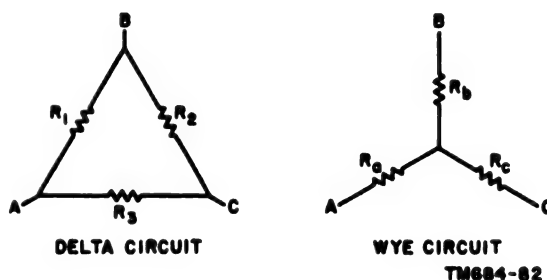


Figure 88. Delta and wye circuits.

b. If the resistances are known for a delta circuit, they can be found for an equivalent wye circuit from the following equations:

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

Example: Find the equivalent resistances for a wye circuit if the resistances of a delta circuit are 10, 20, and 70 ohms.
Let $R_1 = 10$ ohms, $R_2 = 20$ ohms, and $R_3 = 70$ ohms.

$$\begin{aligned} R_a &= \frac{R_1 R_3}{R_1 + R_2 + R_3} \\ &= \frac{10 \times 70}{10 + 20 + 70} \\ &= \frac{700}{100} \\ &= 7 \text{ ohms} \end{aligned}$$

$$\begin{aligned} R_b &= \frac{R_1 R_2}{R_1 + R_2 + R_3} \\ &= \frac{10 \times 20}{100} \\ &= 2 \text{ ohms} \end{aligned}$$

$$\begin{aligned} R_c &= \frac{R_2 R_3}{R_1 + R_2 + R_3} \\ &= \frac{20 \times 70}{100} \\ &= 14 \text{ ohms} \end{aligned}$$

c. If the resistances are known for a wye circuit, they can be found for an equivalent delta circuit from the following equations:

$$\begin{aligned} R_1 &= \frac{R_a R_b + R_a R_c + R_b R_c}{R_a} \\ R_2 &= \frac{R_a R_b + R_a R_c + R_b R_c}{R_b} \\ R_3 &= \frac{R_a R_b + R_a R_c + R_b R_c}{R_c} \end{aligned}$$

Example: Find the equivalent resistance for a delta circuit if the resistances of a wye circuit are 10, 20, and 30 ohms.
Let $R_a = 10$ ohms, $R_b = 20$ ohms, and $R_c = 30$ ohms.

$$\begin{aligned} R_1 &= \frac{R_a R_b + R_a R_c + R_b R_c}{R_a} \\ &= \frac{10 \times 20 + 10 \times 30 + 20 \times 30}{10} \\ &= \frac{200 + 300 + 600}{10} \\ &= \frac{1,100}{10} \\ &= 110 \text{ ohms} \\ R_2 &= \frac{R_a R_b + R_a R_c + R_b R_c}{R_b} \\ &= \frac{1,100}{20} \\ &= 55 \text{ ohms} \\ R_3 &= \frac{R_a R_b + R_a R_c + R_b R_c}{R_c} \\ &= \frac{1,100}{30} \\ &= 36.666 \\ &= 36.67 \text{ ohms} \end{aligned}$$

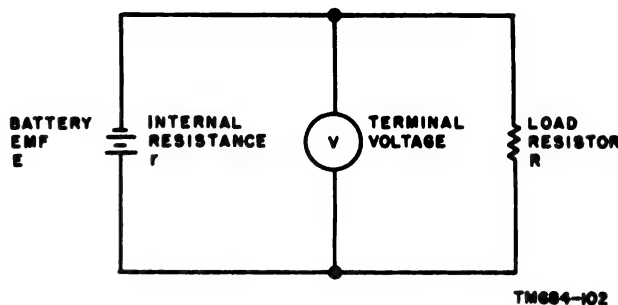


Figure 89. Maximum power transfer.

216. Maximum Power Transfer

a. Quadratic equations (par. 87-94) are used in problems involving the transfer of power from a source to a load. Such a problem can be illustrated by referring to figure 89. In the figure, the battery voltage is given as E , the internal resistance of the battery as r , the terminal voltage as V , and the load resistance as R . The total resistance of the circuit is $(R + r)$. From Ohm's law, $E = I(R + r)$, and the power delivered to the load is $P = VI$ where $V = E - Ir$.

b. The current through the circuit passes through the battery and drops the battery voltage to what is called the *terminal voltage*. Substituting for V in the power equation, $P = (E - Ir)I$, or $P = EI - I^2r$. A quadratic equation in I is obtained when the terms are rearranged. Thus, $-I^2r + EI - P = 0$. This equation can be solved for maximum current by using the method for finding the minimum value as a quadratic (par. 91). In the equation $-rI^2 + EI - P = f(I)$, $a = -r$, $b = E$, and $c = -P$. Substituting in the equation $I = \frac{-b}{2a}$, $I_{\text{max power}} = \frac{-E}{2(-r)} = \frac{E}{2r}$. This equation will give the current through the circuit when maximum power is delivered.

Example: If a 12-volt battery has an internal resistance of 3 ohms, find the current flowing in the circuit when maximum power is being delivered to the load.

$$\begin{aligned} I &= \frac{E}{2r} \\ &= \frac{12}{2 \times 3} \\ &= 2 \text{ amperes} \end{aligned}$$

c. If the value for current at maximum power transfer $\left(\frac{E}{2r}\right)$ is substituted in the original equation $I(R + r) = E$, a relationship between the load resistance and the internal resistance of the battery for maximum power transfer can be derived as follows:

$$\begin{aligned} I(R + r) &= E \\ \frac{E}{2r}(R + r) &= E \\ (R + r) &= 2r \quad (\text{dividing by } E \text{ and multiplying by } 2r) \\ R &= 2r - r \\ R &= r \end{aligned}$$

Consequently, to obtain the maximum power transfer from the source to the load, the value of the load resistance must be equal to the internal resistance of the source.

217. Review Problems—Miscellaneous Electrical Problems

a. A generator is rated at 2,000 watts with a maximum output of 2,100 watts. What is the percent of overload capacity?

b. If the power input of a rotary converter is 48,000 watts and the power output is 37,300 watts, what is the efficiency?

c. The output of a generator is increased from 2,560 watts to 2,944 watts. How much is the increase when expressed in percent?

d. A 12,000-ohm resistance has a tolerance of plus or minus 5%. What is the maximum possible resistance?

e. If the input of an electric motor is 860 watts and the output is 746 watts, what is the efficiency of the motor?

f. A generator is rated at 2,000 watts and has a 10% overload capacity. What is the maximum output of the generator?

g. If the inductance L is 80 henrys and the energy P stored in the circuit is 100 joules, find the current I in amperes.

CHAPTER 18

GRAPHICAL REPRESENTATION AND SOLUTION OF ELECTRICAL PROBLEMS

218. Constructing and Reading Engineering Graphs

a. Constructing Graphs.

- (1) Engineering graphs of operational or experimental data are constructed in the same manner as graphs of equations (pars. 100–102). First a chart is compiled of the available data, and then the data is plotted on an axis. The *independent variable* (the variable to which values are assigned) usually is plotted on the x axis, and the *dependent variable* on the y axis. The scales on the axes should be as large as practicable and, at the same time, keep the graph within the space available. Sometimes it may be convenient to choose a unit length for

the ordinate different from that of the abscissa. Before selecting the units on the axes, examine the table for the maximum and minimum values of the variables and then choose the units on the axes to fit these values in the space available for the graph. Number the points at uniform intervals along the length of each axis, and label each scale.

- (2) As an example, an experiment is conducted to determine the plate current (I_p) of a 6J5 electron tube at various values of plate voltage (E_p) when the tube has a grid bias of -6 volts. The plate voltages applied range from 120 to 235 volts. The information is tabulated as follows:

E_p	120	140	158	170	180	190	200	210	218	225	235
I_p	.5	1	2	3	4	5	6	7	8	9	10

- (3) The plate current is then plotted against the plate voltage and the points joined by a smooth curve. The resulting graph (fig. 90) is a picture of the plate current-voltage characteristic of a 6J5 electron tube with a grid bias of -6 volts.

b. *Reading Graphs.* The process of finding properties of a function by inspection of the graph representing it is called *reading the graph*. From a study of the graph in figure 90, certain information is evident and additional information can be easily obtained.

- (1) There is a gradual increase in the plate current of the 6J5 tube when the plate voltage is increased from 120 to 158 volts. From 158 to 210 volts, the current increase is fairly

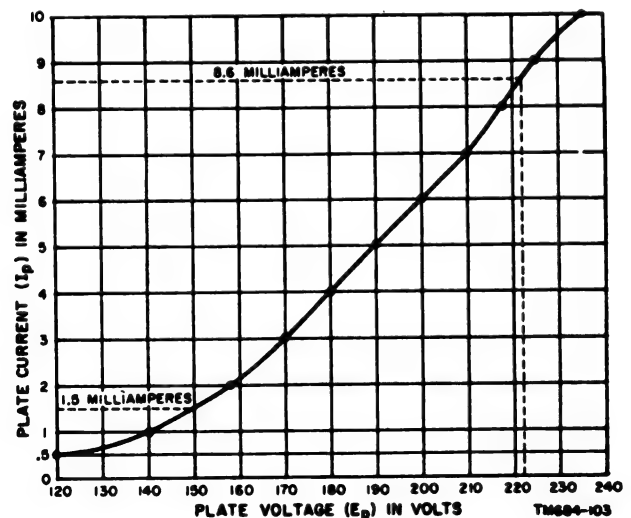


Figure 90. Graph showing plate current versus plate voltage characteristics of 6J5 electron tube with grid bias of -6 volts.

steady. From 210 to 225 volts, however, the increase is sharper, but flattens out slightly from 225 to 235 volts.

- (2) To determine the plate voltage that must be applied to result in a plate current of 8.6 milliamperes, draw a horizontal line from the 8.6 point on the y axis to the curve (fig. 90). At the point where this horizontal line intercepts the curve, drop a vertical line to the x axis. The required plate voltage is 222 volts. Similarly, to obtain a plate current of 1.5 milliamperes, a plate voltage of 150 volts must be applied.

R	0	2	4	6	8	10	12	14	16	18	20
I	∞	60	30	20	15	12	10	8.5	7.5	6.6	6

- (2) After a study of the table, it will be found that it is more convenient to use a much smaller unit of measurement on the x axis than on the y axis. Also, the entire graph falls in the first quadrant as all values are positive. The resulting graph (fig. 92) is the current-resistance characteristic of the circuit. Note that the current decreases as the resistance increases. The current for any value of the variable resistance can be found by reading the graph.

b. Example 2.

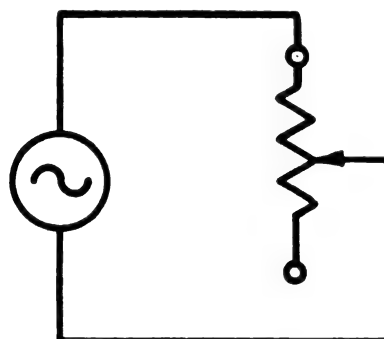
- (1) Figure 93 shows an ac series circuit with a coil having a fixed inductance connected across an ac generator that can be varied in frequency from 100 to 1,000 cps in steps of 100 cycles. It is assumed that the effect of the inductance L is so much greater than the resistance of the coil that the effect of the resistance can be neglected. The problem is to plot induc-

f	100	200	300	400	500	600	700	800	900	1,000
X_L	25.1	50.2	75.4	100.5	125.5	150.7	175.8	201.0	226.1	251.2

219. Application of Graphs to Electrical Laws

a. Example 1.

- (1) A variable resistance is connected across a generator that maintains a potential of 120 volts (fig. 91). The problem is to plot the current as the resistance is varied in 2-ohm steps from 0 to 20 ohms. Ohm's law, $I = \frac{E}{R}$, is used to obtain the coordinates. The voltage E is constant, the resistance R is the independent variable, and the current I is the dependent variable; thus, current will be plotted against resistance, and the independent variable, resistance, will be plotted along the x axis. The following chart is compiled:

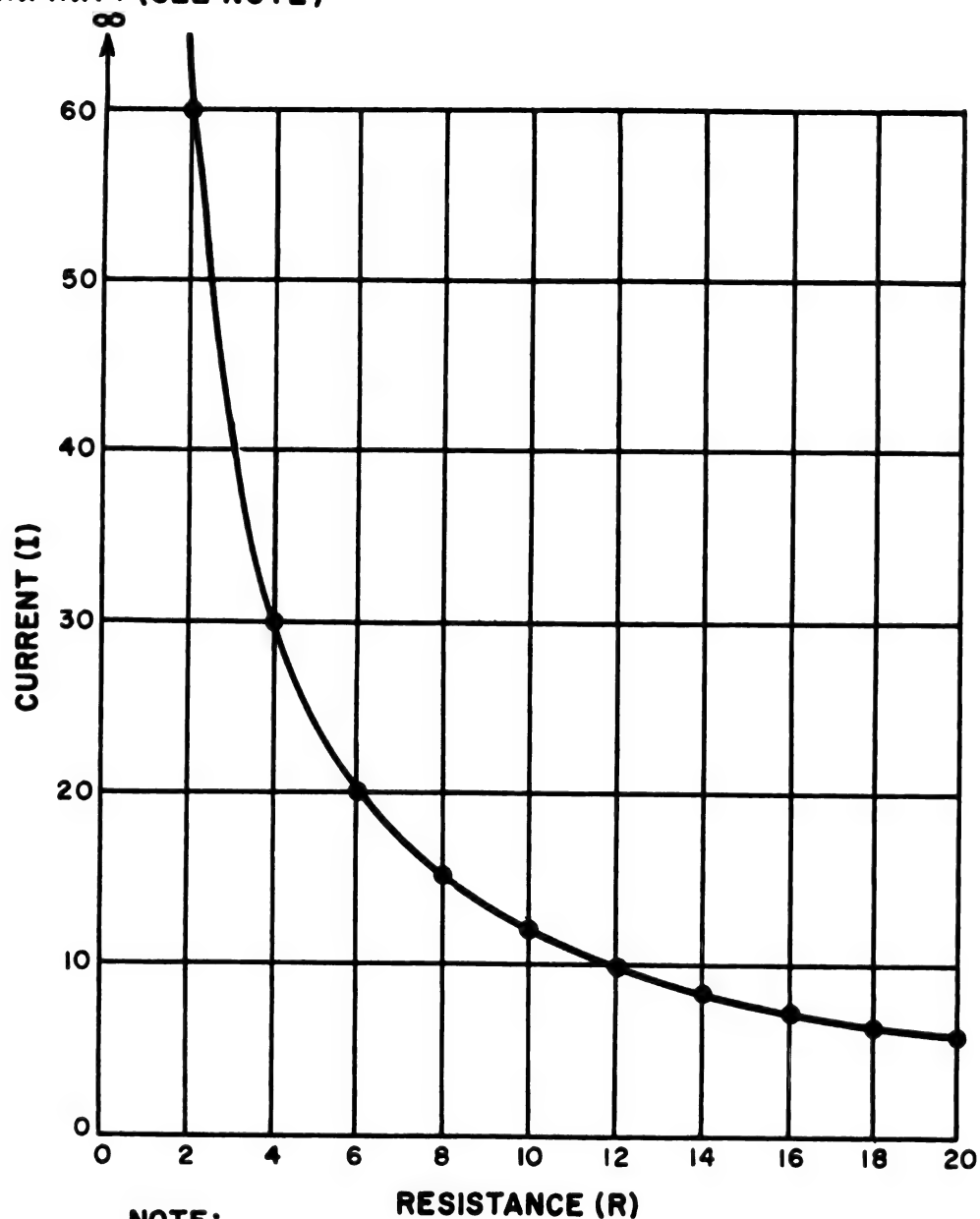


TM684-104

Figure 91. Series circuit showing variable resistance connected across generator.

tive reactance X_L in the formula $X_L = 2\pi fL$. The frequency f is varied to determine the effect upon the inductive reactance. L is constant at 0.04 henry, and 2π equals 6.28; thus, inductive reactance will be plotted against frequency, with the frequency plotted along the x axis. The following chart is compiled:

INFINITY (SEE NOTE)

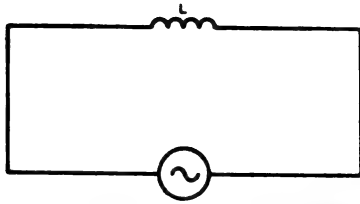


NOTE:

ZERO DIVIDED INTO ANY NUMBER (EXCEPT ZERO)
IS REPRESENTED BY THE INFINITY SYMBOL (∞).

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Figure 92. Graph showing current versus resistance curve for series circuit with 120-volt potential.



TM684-106

Figure 93. Series circuit showing inductance connected across ac generator.

- (2) Since all values are positive, the entire graph will lie in the first quadrant. The resulting graph (fig. 94)

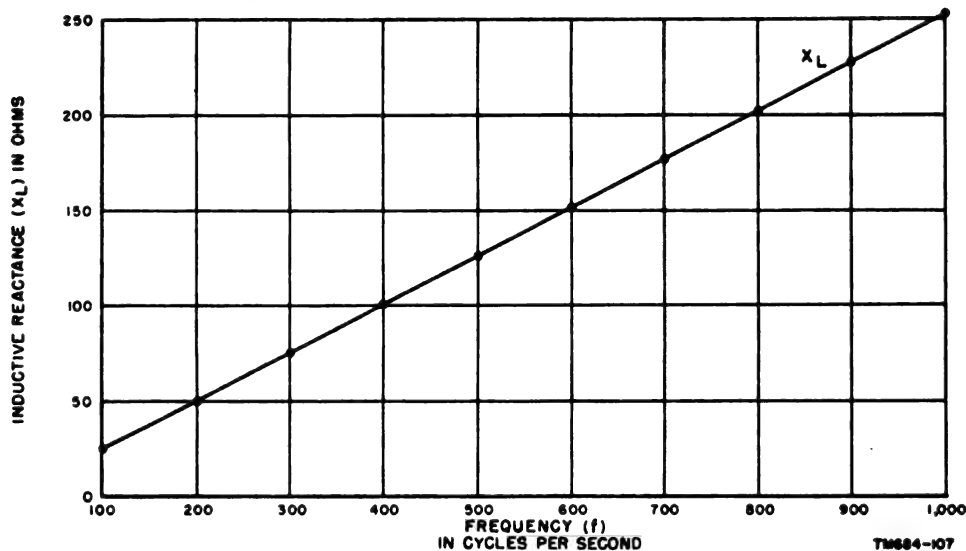


Figure 94. Graph showing reactance of 0.4-henry inductor at frequencies from 100 to 1,000 cps.

H	0	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
R	0	30	87	95	68	60	77	85	68

b. As the output current I_o in milliamperes is varied by a full-wave rectifier voltage quadrupler, the output voltage E_o in volts changes

in accordance with the following data. Plot the curve and determine the current at a voltage of 380 volts.

I_o	45.5	42.0	39.5	36.0	32.5	28.5	24.0	19.5	14.0	8.0	4.0
E_o	292	305	317	330	350	370	390	415	448	488	515

c. When two coils are arranged so that a change in current in one coil causes a voltage to be induced in the other, the coils are said to possess mutual inductance. Given the mutual inductance M in henrys for two coils S centi-

meters apart, plot the curve of the mutual inductance against the separation between the coils. What is the mutual inductance when the coils are separated by a distance of 7 centimeters?

<i>S</i>	0	2	4	6	8	10	12	14
<i>M</i>	0.051	0.049	0.041	0.033	0.025	0.017	0.011	0.007

d. The vertical sag *S* in a powerline depends on the temperature *T*. With the sag being measured in feet and the temperature in °F. the following data is available for a 400-foot

span. Plot the vertical sag against the temperature. If the sag is not to exceed 8.1 feet, what is the maximum permissible temperature?

<i>T</i>	—40	—20	0	20	40	60	80	100
<i>S</i>	6.8	7.0	7.2	7.4	7.6	7.8	8.0	8.2

e. The values of current *I* in milliamperes obtained by applying *E* volts to a selenium rectifier plate is shown in the following chart.

Plot the current against the voltage and determine the current when the voltage is 0.8 volt.

<i>E</i>	1.5	1.3	1.1	0.9	0.7	0	—2	—4	—6	—8	—10
<i>I</i>	100	80	60	40	20	0	—0.05	—1	—2	—3	—4

f. Using the formula $XC = \frac{1,000,000}{2\pi fC}$ to determine the values of variables, plot a graph showing reactance *XC* of a circuit having a capacitance of 2 microfarads at frequencies *f* variable from 1,000 to 10,000 cps in 1,000-cycle steps. ($2\pi = 6.28$.)

potential. Using Ohm's law, plot the current through the resistance against the voltage across the resistance as the voltage is varied from 0 to 120 volts in 10-volt steps.

g. A circuit consists of a resistance of 5 ohms connected across a source of variable

h. A variable resistance is connected across a generator that maintains a potential of 220 volts. Plot the current through the resistance as the resistance is varied in 5-ohm steps from 5 to 60 ohms.

CHAPTER 19

BINARY NUMBERS

221. Scope and Background

a. This chapter serves as an introduction to the theory and arithmetic of binary numbers. It explains the difference between binary numbers and the more conventional decimal numbering system.

b. Binary numbers are of primary interest to the electronic technician because of their use in digital computers and similar devices. These computers fundamentally depend on either a conducting or nonconducting state of vacuum tubes or transistors, or they may depend on the storage states of magnetic cores. Hence they are bistable; that is, they are in one of two stable conditions.

c. The decimal numbering system uses 10 digits, 0 through 9. A digital computer using the decimal system would be large and complex; hence the binary system was adapted for digital computer use—the two digits, 0

and 1, of the binary system correspond to the bistable states discussed in *b* above. These two digits are called *bits*, a contraction of *Binary digITS*.

222. Comparison Between Decimal and Binary Systems

a. *Decimal System.* In the decimal system, the value of a number depends on the position of its digits. For example, in the decimal number 63, the digit 3 represents 3; however, in 63,444, the digit 3 represents 3,000; thus, changing the place of a digit in a number changes the value of the digit.

(1) As another example, decimal number 825 means $8 \times 10^2 + 2 \times 10^1 + 5 \times 10^0$. Ten (10) is considered the radix or base of the decimal system. Positional values for the equivalent powers of 10 are given in the following chart:

Position	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
Value	1,000,000	100,000	10,000	1,000	100	10	1
Power	10^6	10^5	10^4	10^3	10^2	10^1	10^0

(2) Again, using 63,444 as an example, the number can be analyzed as follows:

Position digit	6	3	4	4	4
Power	10^4	10^3	10^2	10^1	10^0

(3) Now multiply the position digit by the power (value) and add:

$$\begin{array}{rcl}
 6 \times 10^4 & = & 60,000 \\
 3 \times 10^3 & = & 3,000 \\
 4 \times 10^2 & = & 400 \\
 4 \times 10^1 & = & 40 \\
 4 \times 10^0 \text{ (or 1)} & = & + 4 \\
 & & \hline
 & & 63,444
 \end{array}$$

b. *Binary System.* In place of 10, the binary system uses 2 as the base or radix. All powers are powers of 2. An expansion (multiplication) of some of the powers of 2 follows:

$$\begin{array}{rcl}
 2^0 & = & 1 \\
 2^1 & = & 2 \\
 2^2 & = & 4 \\
 2^3 & = & 8 \\
 2^4 & = & 16 \\
 2^5 & = & 32 \\
 2^6 & = & 64 \\
 2^7 & = & 128 \\
 2^8 & = & 256, \text{ etc.}
 \end{array}$$

- (1) The binary system operates like the decimal system. Use the number 11111 as an example:

(a) In the decimal system:

$$11111 = 10^5 + 10^4 + 10^3 + 10^2 + 10^1 + 10^0 = 11,111.$$

- (b) In the binary system:

$$11111 = 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 63.$$

- (2) A portion of the positional values and equivalent powers used in the binary system is now given.

Position	Sixty-four	Thirty-two	Sixteen	Eight	Four	Two	Units
Value	64	32	16	8	4	2	1
Power	2^6	2^5	2^4	2^3	2^2	2^1	2^0

223. Tabular Conversion of Decimal Numbers to Binary Numbers

a. The following chart expresses the decimal numbers 0 through 10 in the binary system:

Decimal numbers	Binary numbers					
	2^5	2^4	2^3	2^2	2^1	2^0
	32	16	8	4	2	1
0	0	0	0	0	0	0
1	0	0	0	0	0	1
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	0	0	1	0	0
5	0	0	0	1	0	1
6	0	0	0	1	1	0
7	0	0	0	1	1	1
8	0	0	1	0	0	0
9	0	0	1	0	0	1
10	0	0	1	0	1	0

- (1) If a power of 2 appears in the decimal number in the left column, place a 1 in the column in which the power of two appears. If a power of 2 is not used, place a 0 in that column.
- (2) The decimal number 0 is equivalent to the binary number 0. Thus, a 0 is required in the extreme right-hand position of the binary system.
- (3) The decimal number 2 equals 2^1 —place a 1 under 2^1 and a 0 under all other powers of 2.
- (4) The decimal number 3 equals $2^1 + 2^0$. Place a 1 in each of these columns and a 0 under all other powers of 2.

- (5) The decimal number 4 equals 2^2 . Place a 1 under 2^2 and a 0 under all other powers of 2.
- (6) The decimal number 5 equals $2^2 + 2^0$. Place a 1 under each of these powers of 2 and a 0 under all of the remaining powers.
- (7) Use the procedures outlined above to check the remaining values in the chart.

b. Additional tabular conversions follow:

Decimal numbers	Binary numbers					
	2^5	2^4	2^3	2^2	2^1	2^0
	32	16	8	4	2	1
20	0	1	0	1	0	0
30	0	1	1	1	1	0
40	1	0	1	0	0	0
45	1	0	1	1	0	1
50	1	1	0	0	1	0
57	1	1	1	0	0	1

224. Nontabular Conversion of Decimal Numbers to Binary Numbers

The tabular conversion of decimal numbers to binary numbers is tedious and somewhat awkward. An easier method is to divide the decimal number by 2, and the answer again by two, continuing until you have a remainder of 1. In the example below, 37 will be converted to its binary equivalent. Notice that throughout the operation all numbers will be either exactly divisible by 2 or will be divisible with a remainder of 1. If 2 divides evenly, place a

0 to the right of that quotient; if 2 does not divide evenly, place a 1 to the right of that quotient; repeat until further division by 2 is impossible.

Example: $2/37$
 $2/18$ 1
 $2/9$ 0
 $2/4$ 1
 $2/2$ 0
 $2/1$ 0
 0 1

Bit position	1	0	1	1	0	1	0
Power value	2 ⁶	2 ⁵	2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
Multiply bit position times power value	64	0	16	8	0	2	0
Add horizontally	64 +	0 +	16 +	8 +	0 +	2 +	0
Total	90,						
The decimal equivalent, therefore, is 90.							

226. Nontabular Conversion of Binary Numbers to Decimal Numbers

The following procedure illustrates an alternative method of converting from binary numbers to decimal numbers—the same binary number, 1011010 is used:

a. Start with the bit at the extreme left.

- (1) If the next bit to the right is a 0, double the leftmost bit.
- (2) However, if this next bit is a 1, double the leftmost bit and add 1.

b. The complete conversion of 1011010 follows:

- (1) At the first left bit, double 1 to get 2 since the second bit from left is a 0.
- (2) At the second left bit, double 2 and add 1 to get 5 since the third bit from left is a 1.
- (3) At the third left bit, double 5 and add 1 to get 11 since the fourth bit from the left is a 1.
- (4) At the fourth bit, double 11 to get 22 since the fifth bit from the left is a 0.
- (5) At the fifth bit, double 22 and add 1 to get 45 since the sixth bit is a 1.
- (6) At the sixth bit, double 45 to get 90

The binary number, 100101, is obtained by reading from *bottom to top*. This result may be checked against the tabular system of conversion (par. 257).

225. Tabular Conversion of Binary Numbers to Decimal Numbers

Using the binary number 1011010, the following procedure illustrates one method of converting from binary numbers to decimal numbers:

since the seventh bit is a 0. This is the answer and the end of the operation.

- (7) Since the seventh bit is the last bit, no further operations are required. Remember that no mathematical operation is required for the extreme right-hand bit when converting by the nontabular method.

227. Addition of Binary Numbers

a. Addition in the binary system is similar to addition in the decimal system. The rules for binary addition follow:

- (1) $0 + 0 = 0$
- (2) $0 + 1 = 1$
- (3) $1 + 1 = 10$, 0 with 1 to carry into the next place. This rule may be expanded further to include: $1 + 1 = 11$, or 1 with 1 to carry to the next place. $1 + 1 + 1 + 1 = 100$, or 0 with 10 to carry in the next place.

b. The following example illustrates binary addition.

Binary	Decimal
1101	13
+ 1111	15
11100	28

- (1) Begin at the extreme right bit:
 $1 + 1 = 10$. Write 0, carry 1.
- (2) $0 + 1 + 1$ (carried over) = 10. Write 0, carry 1.
- (3) $1 + 1 + 1$ (carried over) = 11. Write 1, carry 1.
- (4) $1 + 1 + 1$ (carried over) = 11. Write 11.
- (5) The answer is 11100. Check answer by converting to decimal numbers and then adding (as shown).

c. Binary addition is further illustrated by the next example:

Binary	Decimal
101 1101 1101	1501
+ 111 0010 1101	+ 1837
1101 0000 1010	3338

228. Subtraction of Binary Numbers

a. Subtraction in the binary system is similar to subtraction in the decimal system. The rules for binary subtraction follow:

- (1) $0 - 0 = 0$
- (2) $1 - 1 = 0$
- (3) $1 - 0 = 1$
- (4) $0 - 1 = 1$, and then proceed to change all numbers in the top row until you change a 1 to a 0.

b. The following example illustrates binary subtraction:

Binary	Decimal
1011 (minuend)	11
— 0101 (subtrahend)	5
0110 (remainder)	6

- (1) Begin at the extreme right bit:
 $1 - 1 = 0$. Bring down 0, none to carry.
- (2) $1 - 0 = 1$. Bring down 1, none to carry.
- (3) $0 - 1 = 1$. The bit 1 to the extreme left in the minuend is changed to 0.
- (4) $0 - 0 = 0$. End of operation.
- (5) The complete calculation may be

checked by adding the subtrahend and the remainder.

Binary	Decimal
0101	5
+ 0110	+ 6
1011	11

c. Binary subtraction is further illustrated in the next example:

Binary	Decimal
1101 0000 1010 (minuend)	3338
— 111 0010 1101 (subtrahend)	— 1837
101 1101 1101 (remainder)	1501

Proof:

Binary	Decimal
101 1101 1101	1501
+ 111 0010 1101	+ 1837
1101 0000 1010	3338

229. Complementary Addition of Binary Numbers

The direct subtraction of binary numbers is not used in some data equipments. Instead, the subtraction processes are carried out by complement addition. To subtract two binary numbers using this system, proceed as follows:

a. Use the following problem as an example:

1101101 (minuend)
— 10010 (subtrahend)

b. First determine the complement of the subtrahend.

- (1) Add zeros to the left until the subtrahend has the same number of bits as the minuend; for example, 0010010.
- (2) Note the first 1 counting from the right and bring down this 1 as well as any of the zeros to its right; then reverse all other bits proceeding toward the left. The subtrahend of (1) above becomes 1101110. This number is the complement.
- (3) Now add the minuend to the complement:

110 1101
+ 110 1110
1101 1011

- (4) Delete the 1 to the extreme left and the remainder becomes 1011011.

(5) Check the answer by binary subtraction:

$$\begin{array}{r} 110\ 1101\ (\text{minuend}) \\ - 1\ 0010\ (\text{subtrahend}) \\ \hline 101\ 1011\ (\text{remainder}) \end{array}$$

c. The next problem is solved by using the principles explained in a and b above.

(1) 11101101 (minuend)
 111100 (subtrahend)

(2) 00111100 becomes the complement 11000100 .

(3) $1110\ 1101$ (minuend)
 $+ 1100\ 0100$ (complement)
 $\hline 11011\ 0001$ (remainder)

(4) Proof by subtraction
 11101101 (minuend)
 $- 111100$ (subtrahend)
 $\hline 10110001$ (remainder)

230. Multiplication of Binary Numbers

a. Multiplication is the simplest of all the binary processes. The rules are:

- (1) $0 \times 0 = 0$
- (2) $0 \times 1 = 0$
- (3) $1 \times 1 = 1$

b. Remember that binary addition is important to binary multiplication. Two examples of multiplication are given below.

<i>Example 1:</i>	<i>Binary</i>	<i>Decimal</i>
	1011	11
	$\times 10$	$\times 2$
	$\hline 0000$	
	1011	
	$\hline 10110$	22

<i>Example 2:</i>	<i>Binary</i>	<i>Decimal</i>
	111011	59
	$\times 101$	$\times 5$
	$\hline 111011$	
	000000	
	111011	
	$\hline 10010011$	295

231. Division of Binary Numbers

a. Division of binary numbers is similar to division in the decimal system. The simple rules are:

- (1) $0 \div 0 = 0$
- (2) $0 \div 1 = 0$
- (3) $1 \div 1 = 1$

b. Remember that binary subtraction is important to binary division. Two examples of binary division are given below.

<i>Example 1:</i>	<i>Binary</i>	<i>Decimal</i>
	111	7
	$10 \overline{)1110}$	$2 \overline{)14}$
	$\underline{10}$	$\underline{14}$
	11	
	$\underline{10}$	
	10	
	$\underline{10}$	

<i>Proof:</i>	<i>Binary</i>	<i>Decimal</i>
	111	7
	$\times 10$	$\times 2$
	$\hline 1110$	$\hline 14$

<i>Example 2:</i>	<i>Binary</i>	<i>Decimal</i>
	$1001 \frac{100}{110}$	$9 \frac{4}{6}$
	$110 \overline{)111010}$	$6 \overline{)58}$
	$\underline{110}$	$\underline{54}$
	1010	$4/6$
	$\underline{110}$	
	100	

Note the remainder of $100/110$.

<i>Proof:</i>	
1001	(partial quotient)
$\times 110$	(divisor)
$\hline 10010$	
1001	
$\hline 110110$	(partial dividend)
$+ 100$	(add remainder)
$\hline 111010$	(total dividend)

232. Fractions in the Binary System

a. The system of expressing fractions with binary numbers is similar to the decimal numbering methods. For example, the common fraction $\frac{3}{5}$ may be expressed in binary numbers as

$\frac{11}{101}$ Also, binary fractions may be expressed as decimal fractions when the powers of 2 are used with negative exponents. The binary fraction 0.011 is equivalent to the decimal fraction 0.375 and may be written as:

$$0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} = 0 + \frac{1}{4} + \frac{1}{8} = \frac{3}{8} \text{ or } .375.$$

b. The following table lists some of the fractional values and their equivalents in both systems:

<i>Decimal equivalents</i>	<i>Power of 2</i>	<i>Binary equivalent</i>
$\frac{1}{2}$ or .5	2^{-1}	.1
$\frac{1}{4}$ or .25	2^{-2}	.01
$\frac{1}{8}$ or .125	2^{-3}	.001
$\frac{1}{16}$ or .0625	2^{-4}	.0001
$\frac{1}{32}$ or .03125	2^{-5}	.00001
$\frac{1}{64}$ or .015625	2^{-6}	.000001
$\frac{1}{128}$ or .0078125	2^{-7}	.0000001
$\frac{1}{256}$ or .00390625	2^{-8}	.00000001
$\frac{1}{512}$ or .001953125	2^{-9}	.000000001

c. Using values from the table, the decimal fraction 0.375 is equal to .25 + .125 and hence has the binary equivalency of .01 + .001 = .011.

233. Conversion of Decimal Fractions to Binary Numbers

Usually, the decimal fractions are converted to binary fractions by performing a series of multiplications by 2. This method is directly opposite to the method explained in paragraph 224. As a rule, decimal fractions cannot be converted to exact binary equivalents. The extent of error must be tolerable for a given application and the number of bits used must be reasonable.

a. To convert 0.375 to a binary number, proceed as follows:

- (1) Multiply the decimal 0.375 by 2 to obtain a new integer (whole number) and a new decimal, 0.75. Since in 0.75 the integer to the left of the decimal point is 0, place a 0 in the binary equivalent as .0.

<u>Decimal $\times$ 2</u>	<u>New integer and decimal</u>	<u>Partial binary equivalent</u>
0.375×2	0.75	.0

- (2) Multiply the decimal 0.75 by 2 to obtain a new integer and decimal. Since the integer to the left of the decimal point is a 1, place a 1 in the binary equivalent as .01.

<u>Decimal $\times$ 2</u>	<u>New integer and decimal</u>	<u>Partial binary equivalent</u>
0.75×2	1.50	.01

- (3) Drop the integer 1 and multiply the decimal 0.50 by 2 to obtain a new integer and decimal. Since the new integer to the left of the decimal is a 1, place another 1 in the binary equivalent as .011.

<u>Decimal $\times 2$</u>	<u>New integer and decimal</u>	<u>Partial binary equivalent</u>
0.50×2	1.00	.011

- (4) Note that the operation ends when the decimal part has been expanded to 0.00. The decimal fraction, 0.375, is equivalent to the binary fraction, .011. In this instance, the binary and decimal fractions have exactly the same value.

b. The next example illustrates the conversion of 0.3465 to its binary equivalent. Note that the partial binary equivalents are added at the end of the operation to obtain the complete equivalent:

<i>Multiplication $\times 2$</i>	<i>Binary equivalent</i>	<i>Decimal value of binary equivalent</i>
A	B	C
$0.3465 \times 2 = .6930$	.0	.0
$0.693 \times 2 = 1.386$	.01	.25
$0.386 \times 2 = 0.772$	.000	.00
$0.772 \times 2 = 1.544$	.0001	.0625
$0.554 \times 2 = 1.108$	<u>.00001</u>	<u>.03125</u>
Add up all entries under B and C.	.01011	.34375

c. In b above, the binary and decimal fractions differ in value and the amount of error may be determined by subtraction:

$$0.3465 - 0.34375 = 0.00275 \text{ (fraction of error)}$$

234. Mixed Binary Numbers

a. A mixed binary number is a combination of whole numbers and binary fractions. Examples of this are:

<i>Binary number</i>	<i>Decimal equivalent</i>
1011.1	11.5
1110.011	14.375
10.000001	1.015625

b. The fundamental operations (addition, subtraction, multiplication, and division) for mixed binary numbers or binary fractional numbers alone are in accordance with the principles already explained in this chapter.

APPENDIX I

BASIC SLIDE RULE OPERATIONS

1. General

This appendix describes the basic slide rule and covers the operations of multiplication, division, squaring, and square root.

2. Description of Slide Rule

a. Slide rules are made in several different sizes and styles, and in an assortment of scales. However, they all contain the same basic scales and use them in the same manner.

b. The most common type of slide rule is about 10 inches long and generally has scales on both sides. The most frequently used scales, and the ones covered here, are the A, B, C, and D. Figure 95 is a simplified drawing of a slide rule of this type, showing these scales and the other essential parts of the rule. Note that these scales have indexes (the number 1) on both ends. Also note that the A and B scales have an additional index in the center that divides these scales into two equal parts. The left-hand part of the scales is called A-left or B-left, and the right-hand part, A-right or B-right.

3. Basic Principles of Operation

The slide rule is based on the principle of the logarithm; that is, the segments on the rule represent exponents, or logarithms, but are indicated by the antilogs, or numbers corresponding to those logarithms. Consequently, when the slide rule is used so that two

line segments are added, the logarithms of the numbers shown are actually being added, and the sum of the two line segments is represented by the antilog of the sum of the logarithms. Since the sum of the logarithms of two numbers is equal to the logarithm of the product of the two numbers (par. 121), adding two line segments on a slide rule will give the product of the two numbers represented by the line segments. This is the technique used in multiplication with a slide rule (par. 6 of this app). In the division process, the reverse procedure is used; that is the two line segments are used so that one is subtracted from the other.

4. Accuracy

The accuracy of a slide rule depends on the length of the rule and on the portion of the rule being used. With the 10-inch rule shown in figure 95, numbers can be approximated to four significant figures on the left-hand end of the C or D scales, but only to three significant figures on the right-hand end of these scales. Despite this fact, the results obtained with the slide rule are sufficiently accurate for many practical purposes; in any case, the results serve as a rapid and efficient check of more complex computations.

5. Reading the Scales

a. Since the scales on a slide rule do not have uniform increments along their lengths,

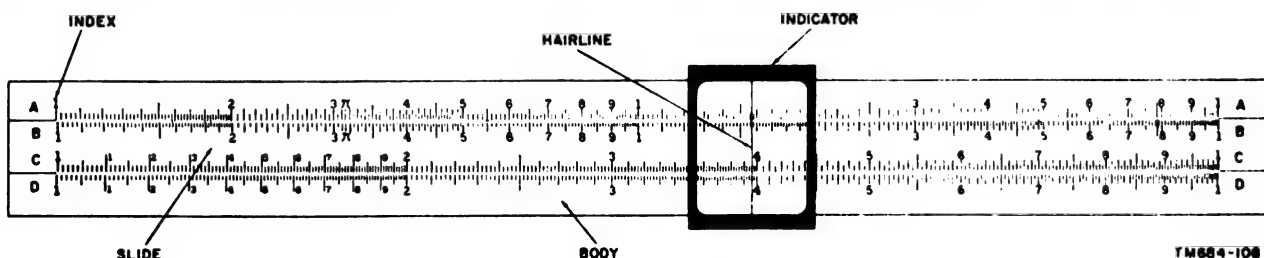


Figure 95. Typical slide rule, simplified drawing.

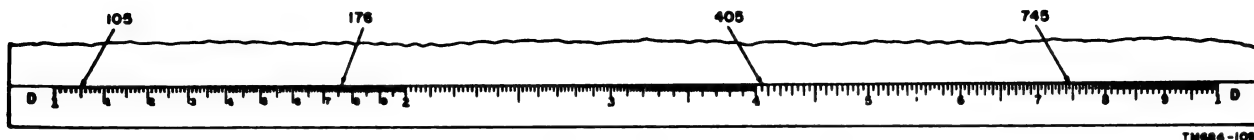


Figure 96. Locating numbers on the D scale.

be careful when approximating numbers at different points on the scales. For example, the space between the larger numbers 1 and 2 on the D scale (fig. 96) is divided into 10 sub-groups (identified by the small numbers 1 through 10) of 10 increments each; thus there are 100 increments between 1 and 2 on the D scale, and each increment is equal to one one-hundredth of the difference. Between 4 and 5 on the D scale, however, there are only 20 increments, and each increment therefore, is equal to five one-hundredths of the difference. Consequently, the number 105 would be located 5 increments above 1, whereas 405 would be 1 increment above 4 on the scale. Figure 96 shows the location of these and other numbers on the D scale.

b. To locate a number on a scale, first determine its general location between two of the numbers on that scale; then determine the value of each increment between the numbers. Finally, determine its exact location based on the value of the increments.

c. In reading the scale, as in logarithms, the decimal point is neglected until after the absolute value of the result is obtained; therefore, in figure 96, the number 1245 could actually represent 1.245, 12.45, 124.5, .001245, etc. The use of scientific notation (par. 106) will greatly simplify the handling of very large or very small numbers.

6. Multiplication

a. Normally, the process of multiplication is performed by using the C and D scales. The A and B scales may also be used, but they are not as accurate because the increments are

smaller. To multiply two numbers, proceed as follows:

- (1) Locate one number on the D scale. Slide the indicator until the hairline is over the number to mark its location.
- (2) Place one of the indexes of the C scale above the number on the D scale. Use the hairline of the indicator to align the index and the number.
- (3) Locate the second number on the C scale. If the number is located on the portion of the C scale beyond the end of the D scale, reposition the slide so that the other index on the C scale is above the number on the D scale.
- (4) Slide the indicator so that the hairline is over the number on the C scale. The product of the two numbers is read under the hairline on the D scale.

b. The two examples below illustrate the method of multiplication described above. They also point out the use of the two indexes on the C scale.

Example 1: Multiply 2 by 3.

- Step 1. Locate the number 2 on the D scale and slide the indicator until the hairline is over it.
- Step 2. Place the left-hand index of the C scale above the number 2 on the D scale. Use the hairline on the indicator for alignment.
- Step 3. Locate the number 3 on the C scale and slide the indicator so that the hairline is over it. The

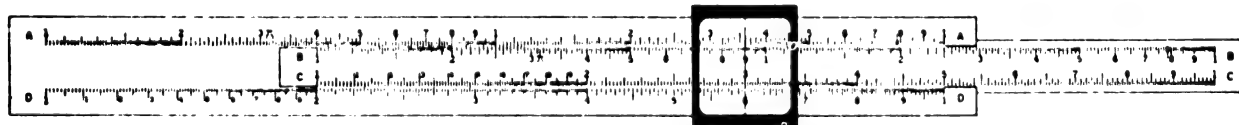
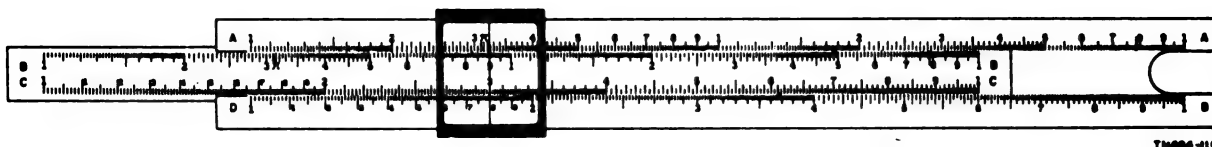


Figure 97. Slide rule arranged for multiplying 2 by 3.



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Figure 98. Slide rule arranged for multiplying 6 by 3.

product of 2 times 3 or 6 is read under the hairline on the D scale. Figure 97 shows a slide rule arranged for this product.

Example 2: Multiply 6 by 3.

- Step 1.** Locate the number 6 on the D scale and slide the indicator so that the hairline is over it.
- Step 2.** Place the right-hand index of the C scale above the number 6 on the D scale. Use the hairline on the indicator for alignment. (The right-hand index is used because the number 3 on the C scale would be beyond the end of the D scale if the left-hand index were used.)
- Step 3.** Locate the number 3 on the C scale and slide the indicator so that the hairline is over it. The product of 6 times 3 or 18 is read under the hairline on the D scale. Figure 98 shows a slide rule arranged for this product.

Move the slide until this number is above the dividend on the D scale. Use the hairline on the indicator for alignment.

- (3) Slide the indicator until the hairline is over the index on the C scale that is above a portion of the D scale. The quotient of the two numbers is read under the hairline on the D scale.

b. The following example illustrates the use of the method of division described above.

Example: Divide 8 by 2.

- Step 1.** Locate the dividend (8) on the D scale and slide the indicator until the hairline is over it.
- Step 2.** Locate the divisor (2) on the C scale and move the slide until this number is above 8 on the D scale. Use the hairline on the indicator for alignment.
- Step 3.** Slide the indicator until the hairline is over the left-hand index. The quotient of 8 divided by 2 is located under the hairline on the D scale. Figure 99 shows a slide rule arranged for this quotient.

7. Division

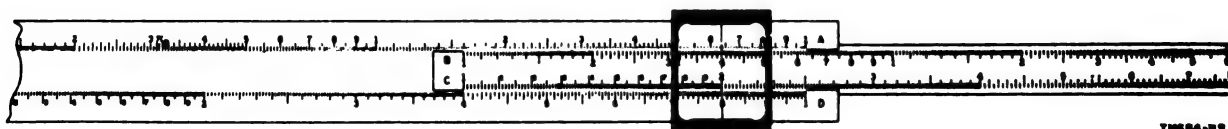
a. The process of division, like multiplication, generally is performed by using the C and D scales. To divide one number by another number, proceed as follows:

- (1) Locate the dividend (number to be divided) on the D scale. Slide the indicator until the hairline is over the number to mark its location.
- (2) Locate the divisor on the C scale.

8. Squaring a Number

a. The process of squaring a number is performed by using the A and D scales. To square a number, proceed as follows:

- (1) Locate the number on the D scale. Slide the indicator until the hairline is over the number.



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Figure 99. Slide rule arranged for dividing 8 by 2.

- (2) Read the square under the hairline on the A scale. Remember that the increments on the A scale are smaller than the increments on the D scale; be sure to evaluate the increment carefully.

b. The following example illustrates the procedure given above.

Example: Find the square of 12.5.

Step 1. Locate 12.5 on the D scale. Slide the indicator until the hairline is over it (fig. 100).

Step 2. Read the square of 12.5 under the hairline on the A scale. The three significant figures that can be obtained from the A scale are 156. To locate the decimal point, estimate the value of the square of 12.5. Since the square of 12 is 144, the square of 12.5 is 156.

9. Square Root of a Number

a. The process of finding the square root of a number is simply the reverse of the proc-

ess of squaring a number (par. 8). To find the square root of a number, proceed as follows:

- (1) Locate the number of the A scale. Slide the indicator until the hairline is over the number.

- (2) Read the square root under the hairline on the D scale.

b. The following example illustrates the procedure given above.

Example: Find the square root of 9.5.

Step 1. Locate 9.5 on the A scale. Note that there are two such numbers on the A scale, one on A-left, and one on A-right. Since the square root of 9 is 3, the number on A-right would not yield the correct result; therefore, slide the indicator until the hairline is over the 9.5 on A-left.

Step 2. Read the square root of 9.5, that is, 3.08, under the hairline on the D scale. Figure 101 shows a slide rule arranged for this square root.

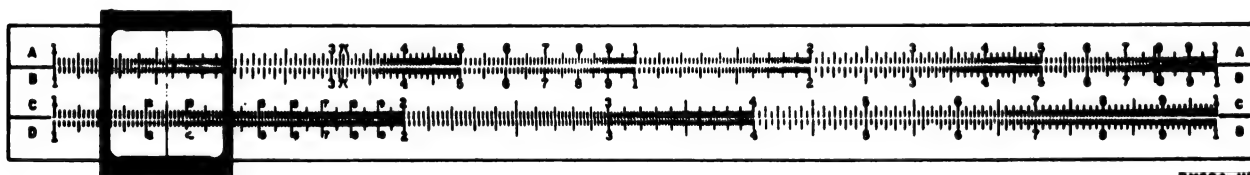


Figure 100. Slide rule arranged for finding the square of 12.5.

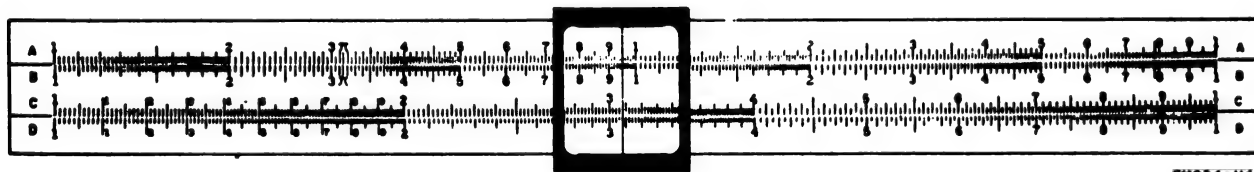


Figure 101. Slide rule arranged for finding the square root of 9.5.

APPENDIX II

SYSTEMS OF MEASUREMENT

1. General

Two systems of measurement are in use in the United States today: the English system, based on the foot and the pound, and the metric system, based on the centimeter (or meter) and the gram (or kilogram). Both systems are used in electronics. For example, the wavelength of an antenna is calculated in the metric system; the physical length of each conductor is stated in feet and inches.

2. Metric Prefixes

In the field of communications, there are often wide ranges in electrical quantities. For example, the input of a radio receiver may be in millionths of a volt, and the output circuit of a transmitter may be in thousands of volts. Thus, metric prefixes are used in electronics in combination with basic units of measurement—volts, ohms, watts, amperes, farads, henrys, and cycles—to facilitate operations. The following chart gives the meaning of these prefixes with respect to various units of measurement.

Metric prefix	Meaning	Associated with
Mega	Million (1,000,000)	Volt, ohms, cycles, amperes
Kilo	Thousand (1,000)	Volts, watts, cycles, meters, amperes
Hecto	Hundred (100)	Meters
Deka	Ten (10)	Meters
Deci	One-tenth (0.1)	Meters
Centi	One-hundredth (0.01)	Meters
Milli	One-thousandth (0.001)	Volts, amperes, meters, henrys, watts, ohms
Micro	One-millionth (0.0000001)	Volts, amperes, farads, henrys, mhos, ohms
Micromicro	One-millionth of one-millionth (0.000,000,000,001)	Volts, amperes, farads, coulombs

3. Conversion Factors

The table below lists the common units of measurement with one set of prefixes and the factor by which these units must be multiplied to convert them to units with another set of prefixes. The examples below illustrate the method in which the table is used.

Example 1: Convert 7.54 megacycles to cycles.

From the table, to convert from megacycles to cycles, multiply by 10^6 .

$$7.54 \times 10^6 = 7.54 \times 1,000,000 = 7,540,000 \text{ cycles}$$

Example 2: Convert 5,500 watts to kilowatts.

From the table, to convert watts to kilowatts, multiply by .001.

$$5,500 \times 0.001 = 5.5 \text{ watts}$$

To convert from	To	Multiply by
Abamperes	Amperes	10.0000
Abamperes	Statampere	2.998×10^{10}
Abcoulombs	Ampere-hours	2.778×10^{-3}
Abcoulombs	Coulombs	10.0000
Abcoulombs	Faradays	1.036×10^{-4}
Abcoulombs	Statcoulombs	2.998×10^{10}
Abfarads	Farads	10^9
Abfarads	Microfarads	10^{15}
Abfarads	Statfarads	8.988×10^{20}
Abhenrys	Henrys	10^{-9}
Abhenrys	Microhenrys	.001
Abhenrys	Millihenrys	10^{-6}
Abhenrys	Stathenrys	1.118×10^{-21}
Abohms	Megohms	10^{-18}
Abohms	Microhms	0.001
Abohms	Ohms	10^{-9}
Abohms	Statohms	1.118×10^{-21}
Abvolts	Microvolts	.01
Abvolts	Millivolts	10^{-6}
Abvolts	Statvolts	3.336×10^{-1}
Abvolts	Volts	10^{-8}
Acres	Ares (square dekameters)	40.46873
Acres	Hectares (square hectometers)	.4046873
Acres	Square feet	4.356×10^4
Acres	Square inches	6,272,640
Acres	Square kilometers	4.047×10^{-8}
Acres	Square meters	4047
Acres	Square miles	1.563×10^{-6}
Acres	Square rods	160
Acres	Square yards	4840
Amperes	Abamperes	.1
Amperes	Milliamperes	1000
Amperes	Statamperes	2.998×10^9
Ampere-hours	Abcoulombs	360
Ampere-hours	Coulombs	3600
Ampere-hours	Faradays	3.731×10^{-2}
Ampere-hours	Statcoulombs	1.080×10^{13}
Ares	Acres (US)	.02471044
Ares	Hectares	.01
Ares	Square feet	1076.4
Ares	Square meters	100
Ares	Square miles	3.861×10^{-6}
Ares	Square yards	119.60
Bushels (dry)	Cubic centimeters	3524×10^4
Bushels (dry)	Cubic feet	1.2444
Bushels (dry)	Cubic inches	2150.4
Bushels (dry)	Cubic meters	3.524×10^{-3}
Bushels (dry)	Liters	35.24
Centimeters	Feet	3.281×10^{-2}
Centimeters	Inches	.3937
Centimeters	Kilometers	10^{-5}
Centimeters	Meters	.01
Centimeters	Mils	393.7
Centimeters	Miles	6.214×10^{-6}
Centimeters	Millimeters	10
Centimeters	Yards	1.094×10^{-2}
Centimeters/second	Feet/minute	1.969
Centimeters/second	Feet/second	3.282×10^{-2}
Centimeters/second	Kilometers/hour	.036

To convert from	To	Multiply by
Centimeters/second	Kilometers/minute	.0006
Centimeters/second	Knots/hour	1.943×10^{-2}
Centimeters/second	Meters/minute	.6
Centimeters/second	Meters/second	.01
Centimeters/second	Miles/hour	2.237×10^{-2}
Centimeters/second	Miles/minute	3.728×10^{-4}
Circular mils	Square centimeters	5.067×10^{-6}
Circular mils	Square inches	7.854×10^{-7}
Circular mils	Square millimeters	5.067×10^{-4}
Circular mils	Square mils	.7854
Coulombs	Abcoulombs	.1
Coulombs	Ampere-hours	2.778×10^{-4}
Coulombs	Faradays	1.036×10^{-5}
Coulombs	Statcoulombs	2.998×10^9
Cubic centimeters	Cubic feet	3.531×10^{-3}
Cubic centimeters	Cubic inches	6.102×10^{-2}
Cubic centimeters	Cubic meters	10^{-6}
Cubic centimeters	Cubic yards	1.308×10^{-4}
Cubic centimeters	Gallons (liquid)	2.642×10^{-4}
Cubic centimeters	Liters	.001
Cubic centimeters	Pints (liquid)	2.113×10^{-4}
Cubic centimeters	Quarts (liquid)	1.057×10^{-4}
Cubic feet	Bushels (dry)	.8036
Cubic feet	Cubic centimeters	2.832×10^4
Cubic feet	Cubic inches	1728
Cubic feet	Cubic meters	2.832×10^{-2}
Cubic feet (US)	Cubic yards	3.704×10^{-2}
Cubic feet	Gallons (liquid)	7.481
Cubic feet	Liters	28.316
Cubic feet	Pints (liquid)	59.84
Cubic feet	Quarts (liquid)	29.922
Cubic hectometers	Cubic meters	10^6
Cubic inches	Bushels (dry)	4.6503×10^{-4}
Cubic inches	Cubic centimeters	16.39
Cubic inches	Cubic feet	5.787×10^{-4}
Cubic inches	Cubic meters	1.639×10^{-3}
Cubic inches (US)	Cubic yards	2.143×10^{-4}
Cubic inches	Gallons	4.329×10^{-4}
Cubic inches	Liters	1.639×10^{-2}
Cubic inches	Pints (liquid)	3.463×10^{-3}
Cubic inches	Quarts (liquid)	1.732×10^{-3}
Cubic meters	Bushels (dry)	28.38
Cubic meters	Cubic centimeters	10^6
Cubic meters	Cubic feet	35.31
Cubic meters	Cubic inches	6.102×10^4
Cubic meters	Cubic yards	1.308
Cubic meters	Gallons (liquid)	264.2
Cubic meters	Liters	1000
Cubic meters	Pints (liquid)	2113
Cubic meters	Quarts (liquid)	1057
Cubic meters	Sterea	1
Cubic yards	Cubic centimeters	7.646×10^5
Cubic yards	Cubic feet	27
Cubic yards	Cubic inches	46656
Cubic yards	Cubic meters	.7646
Cubic yards	Gallons	202.0
Cubic yards	Liters	764.6
Cubic yards	Pints (liquid)	1616
Cubic yards	Quarts (liquid)	807.9

To convert from	To	Multiply by
Decimeters	Meters	.1
Decigrams	Grams	.1
Decisteres	Cubic meters	.1
Degrees	Circumferences* (revolutions)	2.778×10^{-3}
Degrees	Minutes	60
Degrees	Quadrants	1.111×10^{-2}
Degrees	Radians*	1.745×10^{-2}
Degrees	Seconds	3600
Degrees/second	Radians/second	1.745×10^{-2}
Degrees/second	Revolutions/minute	.1667
Degrees/second	Revolutions/second	2.778×10^{-3}
Dekagrams	Grams	10
Dekameters	Meters	10
Faradays	Abcoulombs	9649
Faradays	Ampere-hours	26.81
Faradays	Coulombs	9.649×10^4
Faradays	Statcoulombs	2.893×10^{14}
Farads	Abfarads	10^{-9}
Farads	Microfarads	10^6
Farads	Statfarads	8.988×10^{11}
Feet	Centimeters	30.48
Feet	Inches	12
Feet	Kilometers	3.048×10^{-4}
Feet	Meters	.3048
Feet	Miles (nautical)	1.645×10^{-4}
Feet	Miles (statute)	1.894×10^{-4}
Feet	Mils	1.2×10^4
Feet	Millimeters	304.8
Feet	Yards	.3333
Feet/minute	Centimeter/second	.5080
Feet/minute	Feet/second	1.667×10^{-2}
Feet/minute	Kilometers/hour	1.829×10^{-2}
Feet/minute	Kilometers/second	3.048×10^{-4}
Feet/minute	Knots	9.868×10^{-3}
Feet/minute	Meters/minute	.3048
Feet/minute	Meters/second	5.080×10^{-3}
Feet/minute	Miles/hour	1.136×10^{-2}
Feet/minute	Miles/minute	1.894×10^{-4}
Feet/second	Centimeters/second	30.48
Feet/second	Feet/minute	60
Feet/second	Kilometers/hour	1.097
Feet/second	Kilometers/minute	1.829×10^{-2}
Feet/second	Knots/hour	.5921
Feet/second	Meters/minute	18.29
Feet/second	Meters/second	.3048
Feet/second	Miles/hour	.6818
Feet/second	Miles/minute	1.136×10^{-2}
Gallons (liquid)	Cubic centimeters	3785.
Gallons (liquid)	Cubic feet	.1337
Gallons (liquid)	Cubic inches	231
Gallons (liquid)	Cubic meters	3.785×10^{-3}
Gallons (liquid)	Cubic yards	4.951×10^{-3}
Gallons (liquid)	Liters	3.785
Gallons (liquid)	Pints (liquid)	8
Gallons (liquid)	Quarts (liquid)	4
Grains	Grams	6.480×10^{-2}
Grains	Kilograms	6.481×10^{-5}
Grains	Milligrams	64.81

See notes at end of table.

To convert from	To	Multiply by
Grains	Ounces (avoirdupois)	2.286×10^{-3}
Grains	Pounds (avoirdupois)	1.429×10^{-4}
Grams	Grains	15.43
Grams	Kilograms	6.480×10^{-6}
Grams	Milligrams	64.80
Grams	Ounces (avoirdupois)	3.527×10^{-3}
Grams	Pounds (avoirdupois)	2.205×10^{-6}
Grams	Tons (long)	9.842×10^{-7}
Grams	Tons (metric)	10^{-6}
Grams	Tons (short)	1.102×10^{-6}
Hectares	Acres	2.471
Hectares	Acres	100
Hectares	Square feet	1.076×10^5
Hectares	Square meters	10000
Hectares	Square rods	3.954×10^3
Hectares	Square yards	11959.85
Hectograms	Grams	100
Hectograms	Ounces (avoirdupois)	3.527
Hectoliters	Liters	100
Hectometers	Meters	100
Hectometers	Rods	19.88
Hectometers	Yards	109.4
Hectowatts	Watts	100
Hemispheres	Spheres	.5
Hemispheres	Spherical right angles	4
Hemispheres	Steradians ^b	6.283
Henrys	Abhenrys	10^9
Henrys	Microhenrys	10^6
Henrys	Millihenrys	1000
Henrys	Stathenrys	1.113×10^{-12}
Inches	Centimeters	2.540
Inches	Feet	8.333×10^{-2}
Inches	Kilometers	2.540×10^{-5}
Inches	Meters	2.540×10^{-2}
Inches	Miles	1.578×10^{-5}
Inches	Millimeters	25.40
Inches	Mils	1000
Inches	Yards	2.778×10^{-2}
Kilograms	Grains	1.543×10^4
Kilograms	Grams	1000
Kilograms	Milligrams	10^6
Kilograms	Ounces (avoirdupois)	35.27
Kilograms	Pounds (avoirdupois)	2.205
Kilograms	Tons (long)	9.842×10^{-4}
Kilograms	Tons (metric)	.001
Kilograms	Tons (short)	1.102×10^{-6}
Kiloliters	Gallons (liquid)	264.18
Kiloliters	Liters	1000
Kilometers	Centimeters	10^5
Kilometers	Feet	3281
Kilometers	Inches	3.937×10^4
Kilometers	Meters	1000
Kilometers	Miles (nautical)	.5396
Kilometers	Miles (statute)	.6214
Kilometers	Millimeters	10^6
Kilometers	Mils	3.937×10^7
Kilometers	Yards	1094
Kilometers/hour	Centimeters/second	27.78

^a See notes at end of table.

To convert from	To	Multiply by
Kilometers/hour	Feet/minute	54.68
Kilometers/hour	Feet/second	.9113
Kilometers/hour	Kilometers/minute	1.667×10^{-3}
Kilometers/hour	Knots/hour	.5396
Kilometers/hour	Meters/minute	16.67
Kilometers/hour	Meters/second	.2778
Kilometers/hour	Miles/hour	.6214
Kilometers/hour	Miles/minute	1.036×10^{-3}
Kilometers/minute	Centimeters/second	1667
Kilometers/minute	Feet/minute	3281
Kilometers/minute	Feet/second	54.68
Kilometers/minute	Kilometers/hour	60
Kilometers/minute	Knots/hour	32.38
Kilometers/minute	Meters/minute	1000
Kilometers/minute	Meters/second	16.67
Kilometers/minute	Miles/hour	37.28
Kilometers/minute	Miles/minute	.6214
Kilowatt hours	Watt-hours	1000
Kilowatts	Watts	1000
Knots/hour	Centimeters/second	51.48
Knots/hour	Feet/hour	6080.20
Knots/hour	Feet/minute	101.3
Knots/hour	Feet/second	1.689
Knots/hour	Kilometers/hour	1.853
Knots/hour	Kilometers/minute	3.088×10^{-3}
Knots/hour	Meters/minute	30.88
Knots/hour	Meters/second	.5148
Knots/hour	Miles/hour	1.152
Knots/hour	Miles/minute	1.919×10^{-3}
Liters	Bushels (dry)	2.838×10^{-2}
Liters	Cubic centimeters	1000
Liters	Cubic feet	3.531×10^{-3}
Liters	Cubic inches	61.02
Liters	Cubic meters	.001
Liters	Cubic yards	1.308×10^{-3}
Liters	Gallons (liquid)	.2642
Liters	Pints (liquid)	2.113
Liters	Quarts (liquid)	1.057
Megacycles	Cycles	10^6
Megameters	Meters	10^6
Megohms	Abohms	.001
Megohms	Abohms	10^{15}
Megohms	Microhms	10^{12}
Megohms	Ohms	10^6
Megohms	Statohms	1.112×10^{-3}
Meters	Centimeters	100
Meters	Feet	3.281
Meters	Inches	39.37
Meters	Kilometers	.001
Meters	Megameters	10^{-6}
Meters	Miles (statute)	6.214×10^{-4}
Meters	Millimeters	1000
Meters	Millimicrons	10^9
Meters	Mils	3.937×10^4
Meters	Yards	1.094
Meters/minute	Centimeters/second	1.667
Meters/minute	Feet/minute	3.281
Meters/minute	Feet/second	5.468×10^{-3}
Meters/minute	Kilometers/hour	.06

To convert from	To	Multiply by
Meters/minute	Kilometers/minute	.001
Meters/minute	Knots/hour	3.238×10^{-2}
Meters/minute	Meters/second	1.667×10^{-2}
Meters/minute	Miles/hour	3.728×10^{-3}
Meters/minute	Miles/minute	6.214×10^{-4}
Meters/second	Centimeters/second	100
Meters/second	Feet/minute	196.8
Meters/second	Feet/second	3.281
Meters/second	Kilometers/hour	3.6
Meters/second	Kilometers/minute	.06
Meters/second	Knots/hour	1.943
Meters/second	Meters/minute	60
Meters/second	Miles/hour	2.237
Meters/second	Miles/minute	3.728×10^{-2}
Microfarads	Abfarads	10^{-15}
Microfarads	Farads	10^{-6}
Microfarads	Statfarads	8.988×10^4
Micrograms	Grams	10^{-6}
Milliograms	Milligrams	.001
Microhenrys	Abhenrys	1,000
Microhenrys	Henrys	10^{-6}
Microhenrys	Millihenrys	.001
Microhenrys	Stathenry	1.118×10^{-18}
Microhms	Abohms	1000
Microhms	Megohms	10^{-12}
Microhms	Ohms	10^{-6}
Microhms	Statohms	1.118×10^{-18}
Microliters	Liters	10^{-6}
Micromicrofarads	Farads	10^{-12}
Microvolts	Abvolts	100
Microvolts	Millivolts	.001
Microvolts	Statvolts	3.336×10^{-9}
Microvolts	Volts	10^{-6}
Miles	Centimeters	1.609×10^4
Miles	Feet	5280
Miles	Inches	6.336×10^4
Miles	Kilometers	1.609
Miles	Meters	1609
Miles	Miles (nautical)	.8684
Miles	Rods	320
Miles	Yards	1760
Miles/hour	Centimeters/second	44.70
Miles/hour	Feet/minute	88
Miles/hour	Feet/second	1.467
Miles/hour	Kilometers/hour	1.609
Miles/hour	Kilometers/minute	2.682×10^{-2}
Miles/hour	Knots (per hour)	.8684
Miles/hour	Meters/minute	26.82
Miles/hour	Meters/second	.4470
Miles/hour	Miles/minute	1.667×10^{-2}
Miles/minute	Centimeters/second	2682
Miles/minute	Feet/minute	5280
Miles/minute	Feet/second	88
Miles/minute	Kilometers/hour	96.54
Miles/minute	Kilometers/minute	1.609
Miles/minute	Knots/hour	52.10
Miles/minute	Meters/minute	1609
Miles/minute	Meters/second	26.82
Miles/minute	Miles/hour	60

To convert from	To	Multiply by
Milligrams	Grains	1.543×10^{-2}
Milligrams	Grams	.001
Milligrams	Kilograms	10^{-6}
Milligrams	Ounces (avoirdupois)	3.527×10^{-5}
Milligrams	Pounds (avoirdupois)	2.205×10^{-6}
Milligrams	Tons (long)	9.842×10^{-10}
Milligrams	Tons (metric)	10^{-9}
Milligrams	Tons (short)	1.102×10^{-9}
Millihenrys	Abhenrys	10^6
Millihenrys	Henrys	.001
Millihenrys	Microhenrys	1000
Millihenrys	Stathenrys	1.112×10^{-15}
Milliliters	Liters	.001
Millimeters	Centimeters	.1
Millimeters	Feet	3.281×10^{-3}
Millimeters	Inches	3.937×10^{-2}
Millimeters	Kilometers	10^{-6}
Millimeters	Meters	.001
Millimeters	Miles	6.214×10^{-7}
Millimeters	Mils	39.37
Millimeters	Yards	1.094×10^{-3}
Millimicrons	Microns	.001
Millivolts	Abvolts	10^6
Millivolts	Microvolts	1000
Millivolts	Statvolts	3.336×10^{-6}
Millivolts	Volts	.001
Mils	Centimeters	2.540×10^{-3}
Mils	Feet	8.333×10^{-5}
Mils	Inches	.001
Mils	Kilometers	2.540×10^{-5}
Mils	Millimeters	2.540×10^{-2}
Mils	Yards	2.778×10^{-5}
Minutes (angle)	Degrees	1.667×10^{-2}
Minutes (angle)	Quadrants	1.852×10^{-4}
Minutes (angle)	Radians*	2.909×10^{-4}
Minutes (angle)	Revolutions* (circumferences)	4.630×10^{-5}
Minutes (angle)	Seconds	60
Myriagrams	Grams	10,000
Myriagrams	Kilograms	10
Myriameters	Kilometers	10
Myriameters	Meters	10,000
Myriameters	Miles	6.21370
Ohms	Abohms	10^9
Ohms	Megohms	10^{-6}
Ohms	Microhms	10^6
Ohms	Stathms	1.112×10^{-12}
Ounces (avoirdupois)	Grains	437.5
Ounces (avoirdupois)	Grams	28.35
Ounces (avoirdupois)	Kilograms	2.835×10^{-2}
Ounces (avoirdupois)	Milligrams	2.835×10^4
Ounces (avoirdupois)	Pounds (avoirdupois)	6.250×10^{-2}
Ounces (avoirdupois)	Tons (long)	2.790×10^{-5}
Ounces (avoirdupois)	Tons (metric)	2.835×10^{-5}
Ounces (avoirdupois)	Tons (short)	3.125×10^{-5}
Pints (liquid)	Cubic centimeters	473.2
Pints (liquid)	Cubic feet	1.671×10^{-2}
Pints (liquid)	Cubic inches	28.87
Pints (liquid)	Cubic meters	4.732×10^{-4}

See notes at end of table.

To convert from	To	Multiply by
Pints (liquid)	Cubic yards	6.189×10^{-4}
Pints (liquid)	Gallons (liquid)	.125
Pounds (avoirdupois)	Grains	7000
Pounds (avoirdupois)	Grams	453.6
Pounds (avoirdupois)	Kilograms	.4536
Pounds (avoirdupois)	Milligrams	4.536×10^5
Pounds (avoirdupois)	Ounces (avoirdupois)	16
Pounds (avoirdupois)	Tons (long)	4.464×10^{-4}
Pounds (avoirdupois)	Tons (short)	.0005
Quadrants	Degrees	90
Quadrants	Minutes	5400
Quadrants	Radians*	1.571
Quadrants	Revolutions* (circumferences)	.25
Quadrants	Seconds	3.24×10^5
Quarts (liquid)	Cubic centimeters	946.4
Quarts (liquid)	Cubic feet	3.342×10^{-2}
Quarts (liquid)	Cubic inches	57.75
Quarts (liquid)	Cubic meters	9.464×10^{-4}
Quarts (liquid)	Cubic yards	1.238×10^{-6}
Quarts (liquid)	Gallons (liquid)	.25
Radians*	Circumferences*	.1591
Radians*	Degrees	57.30
Radians*	Degrees, minutes, seconds	57°, 17', 44.8"
Radians*	Minutes	3438
Radians*	Quadrants	.6366
Radians*	Revolutions*	.1591
Radians*	Seconds	2.063×10^5
Radians/second	Degrees/second	57.30
Radians/second	Revolutions/minute	9.549
Radians/second	Revolutions/second	.1592
Revolutions* (circumferences)	Degrees	360
Revolutions* (circumferences)	Minutes	2.16×10^4
Revolutions* (circumferences)	Quadrants	4
Revolutions* (circumferences)	Radians*	6.283
Revolutions* (circumferences)	Seconds	1.296×10^6
Revolutions/minute	Degrees/second	6
Revolutions/minute	Radians/second	.1047
Revolutions/minute ²	Revolutions/second ²	1.667×10^{-2}
Revolutions/second	Degrees/second	360
Revolutions/second	Radians/second	6.283
Revolutions/second	Revolutions/minute	60
Seconds (angle)	Degrees	2.778×10^{-4}
Seconds (angle)	Minutes	1.667×10^{-2}
Seconds (angle)	Quadrants	3.087×10^{-6}
Seconds (angle)	Radians*	4.848×10^{-5}
Seconds (angle)	Revolutions* (circumferences)	7.716×10^{-7}
Spheres	Hemispheres	2
Spheres	Spherical right angles	8
Spheres	Steradians	12.57
Spherical right angles	Hemispheres	.25
Spherical right angles	Spheres	.125
Spherical right angles	Steradians	1.571
Square centimeters	Circular mils	1.973×10^8
Square centimeters	Square decimeters	.01
Square centimeters	Square feet	1.076×10^{-3}
Square centimeters	Square inches	.1550
Square centimeters	Square kilometers	10^{-10}
Square centimeters	Square meters	.0001

See notes at end of table.

To convert from	To	Multiply by
Square centimeters	Square miles	3.861×10^{-11}
Square centimeters	Square millimeters	100
Square centimeters	Square yards	1.196×10^{-4}
Square feet	Acres	2.296×10^{-5}
Square feet	Acres	9.290×10^{-4}
Square feet	Circular mils	1.833×10^8
Square feet	Square centimeters	929.0
Square feet	Square inches	144
Square feet	Square kilometers	9.290×10^{-8}
Square feet	Square meters	9.290×10^{-2}
Square feet	Square miles	3.587×10^{-8}
Square feet	Square millimeters	9.290×10^4
Square inches	Circular mils	1.273×10^6
Square inches	Square centimeters	6.452
Square inches	Square feet	6.944×10^{-3}
Square inches	Square kilometers	6.452×10^{-10}
Square inches	Square meters	6.452×10^{-4}
Square inches	Square millimeters	645.2
Square inches	Square yards	7.716×10^{-4}
Square kilometers	Acres	247.1
Square kilometers	Square centimeters	10^{10}
Square kilometers	Square feet	1.076×10^7
Square kilometers	Square inches	1.550×10^9
Square kilometers	Square meters	10^6
Square kilometers	Square miles	.3861
Square kilometers	Square millimeters	10^{12}
Square kilometers	Square yards	1.196×10^6
Square meters	Acres	2.471×10^{-4}
Square meters	Acres	.01
Square meters	Circular mils	1.973×10^9
Square meters	Square centimeters	10^4
Square meters	Square feet	10.76
Square meters	Square inches	1550
Square meters	Square kilometers	10^{-6}
Square meters	Square miles	3.861×10^{-7}
Square meters	Square millimeters	10^6
Square meters	Square yards	1.196
Square miles	Acres	640
Square miles	Square centimeters	2.590×10^{10}
Square miles	Square feet	2.788×10^7
Square miles	Square inches	4.015×10^9
Square miles	Square kilometers	2.590
Square miles	Square meters	2.590×10^6
Square miles	Square yards	3.098×10^6
Square millimeters	Circular mils	1973
Square millimeters	Square centimeters	.01
Square millimeters	Square feet	1.076×10^{-5}
Square millimeters	Square inches	1.550×10^{-3}
Square millimeters	Square kilometers	10^{-12}
Square millimeters	Square meters	10^{-6}
Square millimeters	Square miles	3.861×10^{-13}
Square millimeters	Square yards	1.196×10^{-6}
Square rods	Acres	.00625
Square rods	Square feet	272.25
Square rods	Square inches	39204
Square rods	Square meters	25.293
Square rods	Square miles	9.766×10^{-6}
Square rods	Square yards	30.25
Square yards	Acres	2.066×10^{-4}

To convert from	To	Multiply by
Square yards	Square centimeters	8361
Square yards	Square feet	9
Square yards	Square inches	1296
Square yards	Square kilometers	8.361×10^{-7}
Square yards	Square meters	8361
Square yards	Square miles	3.228×10^{-7}
Square yards	Square millimeters	8.361×10^{-6}
Statamperes	Abamperes	3.335×10^{-11}
Statamperes	Amperes	3.335×10^{-10}
Statcoulombs	Abcoulombs	3.335×10^{-11}
Statcoulombs	Ampere-hours	9.259×10^{-14}
Statcoulombs	Coulombs	3.335×10^{-10}
Statcoulombs	Faradays	3.457×10^{-15}
Statfarads (or centimeters)	Abfarads	1.112×10^{-21}
Statfarads	Farads	1.112×10^{-12}
Statfarads	Microfarads	1.112×10^{-6}
Stathenrys	Abhenrys	8.988×10^{30}
Stathenrys	Henrys	8.988×10^{11}
Stathenrys	Microhenrys	8.988×10^{17}
Stathenrys	Millihenrys	8.988×10^{14}
Statohms	Abohms	8.988×10^{30}
Statohms	Megohms	8.988×10^8
Statohms	Microhms	8.988×10^{17}
Statohms	Ohms	8.988×10^{11}
Statvolts	Abvolts	2.998×10^{10}
Statvolts	Microvolts	2.998×10^8
Statvolts	Millivolts	2.998×10^6
Statvolts	Volts	299.8
Steradians	Hemispheres	.1592
Steradians	Spheres	7.958×10^{-3}
Steradians	Spherical right angles	.6366
Stercs	Cubic meters	1
Stercs	Liters	999.973
Tons (long)	Grams	1.016×10^6
Tons (long)	Kilograms	1016
Tons (long)	Milligrams	1.016×10^9
Tons (long)	Ounces (avoirdupois)	3.584×10^4
Tons (long)	Pounds (avoirdupois)	2240
Tons (long)	Tons (metric)	1.016
Tons (long)	Tons (short)	1.120
Tons (metric)	Grams	10^6
Tons (metric)	Kilograms	1000
Tons (metric)	Milligrams	10^9
Tons (metric)	Ounces (avoirdupois)	3.527×10^4
Tons (metric)	Pounds (avoirdupois)	2205
Tons (metric)	Tons (long)	.9842
Tons (metric)	Tons (short)	1.102
Tons (short)	Grams	9.072×10^5
Tons (short)	Kilograms	907.2
Tons (short)	Milligrams	9.072×10^8
Tons (short)	Ounces (avoirdupois)	3.2×10^4
Tons (short)	Pounds (avoirdupois)	2000
Tons (short)	Tons (long)	.8929
Tons (short)	Tons (metric)	.9072
Volts	Abvolts	10^8
Volts	Microvolts	10^6
Volts	Millivolts	1000
Volts	Statvolts	3.335×10^{-3}
Watts	Horsepower	.0013410

To convert from	To	Multiply by
Watts	Kilowatts	.001
Yards	Centimeters	91.44
Yards	Feet	3
Yards	Inches	36
Yards	Kilometers	9.144×10^{-4}
Yards	Meters	.9144
Yards	Miles	5.682×10^{-4}
Yards	Miles (nautical)	4.934×10^{-4}
Yards	Millimeters	914.4
Yards	Mils	3.6×10^4

* 2π radians = 1 circumference = 360°

* 4π steradians = 1 sphere.

APPENDIX III

TABLES

1. Squares, Cubes, Square Roots, and Cube Roots

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
1	1	1	1.0000	1.0000	3.142	0.7854	1
2	4	8	1.4142	1.2599	6.283	3.1416	2
3	9	27	1.7321	1.4423	9.425	7.0686	3
4	16	64	2.0000	1.5874	12.566	12.5664	4
5	25	125	2.2361	1.7100	15.708	19.6350	5
6	36	216	2.4495	1.8171	18.850	28.2743	6
7	49	343	2.6458	1.9129	21.991	38.4845	7
8	64	512	2.8284	2.0000	25.133	50.2655	8
9	81	729	3.0000	2.0801	28.274	63.6173	9
10	100	1000	3.1623	2.1544	31.416	78.5398	10
11	121	1331	3.3166	2.2240	34.558	95.0332	11
12	144	1728	3.4641	2.2894	37.699	113.097	12
13	169	2197	3.6056	2.3513	40.841	132.732	13
14	196	2744	3.7417	2.4101	43.982	153.938	14
15	225	3375	3.8730	2.4662	47.124	176.715	15
16	256	4096	4.0000	2.5198	50.265	201.062	16
17	289	4913	4.1231	2.5713	53.407	226.980	17
18	324	5832	4.2426	2.6207	56.549	254.469	18
19	361	6859	4.3589	2.6684	59.690	283.529	19
20	400	8000	4.4721	2.7144	62.832	314.159	20
21	441	9261	4.5826	2.7589	65.973	346.361	21
22	484	10648	4.6904	2.8020	69.115	380.133	22
23	529	12167	4.7958	2.8439	72.257	415.476	23
24	576	13824	4.8990	2.8845	75.398	452.389	24
25	625	15625	5.0000	2.9240	78.540	490.874	25
26	676	17576	5.0990	2.9625	81.681	530.929	26
27	729	19683	5.1962	3.0000	84.823	572.555	27
28	784	21952	5.2915	3.0366	87.965	615.752	28
29	841	24389	5.3852	3.0723	91.106	660.520	29
30	900	27000	5.4772	3.1072	94.248	706.858	30
31	961	29791	5.5678	3.1414	97.389	754.768	31
32	1024	32768	5.6569	3.1748	100.531	804.248	32
33	1089	35937	5.7446	3.2075	103.673	855.299	33
34	1156	39304	5.8310	3.2396	106.814	907.920	34
35	1225	42875	5.9161	3.2711	109.956	962.113	35
36	1296	46656	6.0000	3.3019	113.097	1017.88	36
37	1369	50653	6.0828	3.3322	116.239	1075.21	37
38	1444	54872	6.1644	3.3620	119.381	1134.11	38
39	1521	59319	6.2450	3.3912	122.522	1194.59	39

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No.	Square	Cubo	Square Root	Cubo Root	No.	Square	Cubo	Square Root	Cubo Root	No. = Dim.		No.
										Circum.	Area	
40	1600	64000	6.3246	3.4200	125.66	1256.64	40	80	512000	251.33	5026.55	80
41	1681	68921	6.4031	3.4482	128.81	1320.25	41	81	531441	254.47	5153.00	81
42	1764	74088	6.4807	3.4760	131.95	1385.44	42	82	551368	257.61	5281.02	82
43	1849	79507	6.5574	3.5034	135.09	1452.20	43	83	571787	260.75	5410.61	83
44	1936	85184	6.6332	3.5303	138.23	1520.53	44	84	592704	263.89	5541.77	84
45	2025	91125	6.7082	3.5569	141.37	1590.43	45	85	614125	267.04	5674.50	85
46	2116	97336	6.7823	3.5830	144.51	1661.90	46	86	636056	270.18	5808.80	86
47	2209	103823	6.8557	3.6088	147.65	1734.94	47	87	658503	273.32	5944.68	87
48	2304	110592	6.9282	3.6342	150.80	1809.56	48	88	681472	276.46	6082.12	88
49	2401	117649	7.0000	3.6593	153.94	1885.74	49	89	704969	279.60	6221.14	89
50	2500	125000	7.0711	3.6840	157.08	1963.50	50	90	729000	282.74	6361.73	90
51	2601	132651	7.1414	3.7084	160.22	2042.82	51	91	753571	285.88	6503.88	91
52	2704	140608	7.2111	3.8325	163.36	2123.72	52	92	778688	289.03	6647.61	92
53	2809	148877	7.2801	3.7563	166.50	2206.18	53	93	804357	292.17	6792.91	93
54	2916	157464	7.3485	3.7798	169.65	2290.22	54	94	830584	295.31	6939.78	94
55	3025	166375	7.4162	3.8030	172.79	2375.83	55	95	857375	298.45	7088.22	95
56	3136	175616	7.4833	3.8259	175.93	2463.01	56	96	884736	301.59	7238.23	96
57	3249	185193	7.5498	3.8485	179.07	2551.76	57	97	912673	304.73	7389.81	97
58	3364	195112	7.6158	3.8709	182.21	2642.08	58	98	941192	307.88	7542.96	98
59	3481	205379	7.6811	3.8930	185.35	2733.97	59	99	970299	311.02	7697.69	99
60	3600	216000	7.7460	3.9149	188.50	2827.43	60	100	1000000	314.16	7853.98	100
61	3721	226981	7.8102	3.9365	191.64	2922.47	61	101	1030301	317.30	8011.85	101
62	3844	238328	7.8740	3.9579	194.78	3019.07	62	102	1061208	320.44	8171.28	102
63	3969	250047	7.9373	3.9791	197.92	3117.25	63	103	1092727	323.58	8332.29	103
64	4096	262114	8.0000	4.0000	201.06	3216.99	64	104	1124864	326.73	8494.87	104
65	4225	274625	8.0623	4.0207	204.20	3318.31	65	105	1157625	329.87	8659.01	105
66	4356	287496	8.1240	4.0412	207.35	3421.19	66	106	1191016	333.01	8824.73	106
67	4489	300763	8.1854	4.0615	210.49	3525.65	67	107	1225043	336.15	8992.02	107
68	4624	314432	8.2462	4.0817	213.63	3631.68	68	108	1259712	339.29	9160.88	108
69	4761	328509	8.3066	4.1016	216.77	3739.28	69	109	1295029	342.43	9331.32	109
70	4900	343000	8.3666	4.1213	219.91	3848.45	70	110	1331000	345.58	9503.32	110
71	5041	357911	8.4261	4.1408	223.05	3959.19	71	111	1367631	348.72	9676.89	111
72	5184	373248	8.4853	4.1602	226.19	4071.50	72	112	1404928	351.86	9852.03	112
73	5329	389017	8.5440	4.1793	229.34	4185.39	73	113	1442897	355.00	10028.7	113
74	5476	405224	8.6023	4.1983	232.48	4300.84	74	114	1481544	358.14	10207.0	114
75	5625	421875	8.6603	4.2172	235.62	4417.86	75	115	1520875	361.28	10386.9	115
76	5776	438976	8.7178	4.2358	238.76	4536.46	76	116	1560896	364.42	10568.3	116
77	5929	456533	8.7750	4.2543	241.90	4656.63	77	117	1601613	367.57	10751.3	117
78	6084	474552	8.8318	4.2727	245.04	4778.36	78	118	1643032	370.71	10935.9	118
79	6241	493039	8.8882	4.2908	248.19	4901.67	79	119	1685159	373.85	11122.0	119

TM 684-122

TM 684-123

No.	Square	Cube	Square Feet	Cube Feet	No.	Square	Cube	Square Feet	Cube Feet	No.	Square	Cube	Square Feet	Cube Feet	No. -- Dim.		No.
															Circum.	Area	
120	14400	1728000	10.9545	4.9324	376.99	11309.7	120	12600	4096000	502.65	20106.2	160	126491	5.4288	502.65	20106.2	160
121	14641	1771561	11.0000	4.9461	380.13	11499.0	121	125921	4173281	505.80	20358.3	161	126886	5.4401	505.80	20358.3	161
122	14884	1815848	11.0454	4.9597	383.27	11689.9	122	126244	4251528	508.94	20612.0	162	127279	5.4514	508.94	20612.0	162
123	15129	1860867	11.0905	4.9732	386.42	11882.3	123	126569	4330747	512.08	20867.2	163	127671	5.4626	512.08	20867.2	163
124	15376	1906624	11.1355	4.9866	389.56	12076.3	124	126896	4410944	515.22	21124.1	164	128062	5.4737	515.22	21124.1	164
125	15625	1953125	11.1803	5.0000	392.70	12271.8	125	127225	4492125	518.36	21382.5	165	128452	5.4848	518.36	21382.5	165
126	15876	2000376	11.2250	5.0133	395.84	12469.0	126	127556	4574296	521.50	21642.4	166	128841	5.4959	521.50	21642.4	166
127	16129	2048383	11.2694	5.0265	398.98	12667.7	127	127889	4657463	524.65	21904.0	167	129228	5.5069	524.65	21904.0	167
128	16384	2097152	11.3137	5.0397	402.12	12868.0	128	128224	4741532	527.79	22167.1	168	129615	5.5178	527.79	22167.1	168
129	16641	2146689	11.3578	5.0528	405.27	13069.8	129	128561	4826809	530.93	22431.8	169	130000	5.5288	530.93	22431.8	169
130	16900	2197000	11.4018	5.0658	408.41	13273.2	130	128900	4913000	534.07	22698.0	170	130384	5.5397	534.07	22698.0	170
131	17161	2248091	11.4455	5.0788	411.55	13478.2	131	129241	5000211	537.21	22965.8	171	130767	5.5505	537.21	22965.8	171
132	17424	2299968	11.4891	5.0916	414.69	13684.8	132	129584	5088448	540.35	23235.2	172	131149	5.5613	540.35	23235.2	172
133	17689	2352637	11.5326	5.1045	417.83	13892.9	133	129929	5177717	543.50	23506.2	173	131529	5.5721	543.50	23506.2	173
134	17956	2406104	11.5758	5.1172	420.97	14102.6	134	130276	5268024	546.64	23778.7	174	131909	5.5828	546.64	23778.7	174
135	18225	2460375	11.6190	5.1299	424.12	14313.9	135	130625	5359375	549.78	24052.8	175	132288	5.5934	549.78	24052.8	175
136	18496	2515456	11.6619	5.1426	427.26	14526.7	136	130976	5451776	552.92	24328.5	176	132665	5.6041	552.92	24328.5	176
137	18769	2571353	11.7047	5.1551	430.40	14741.1	137	131329	5545233	556.06	24605.7	177	133041	5.6147	556.06	24605.7	177
138	19044	2628072	11.7473	5.1676	433.54	14957.1	138	131684	5639752	559.20	24884.6	178	133417	5.6252	559.20	24884.6	178
139	19321	2685619	11.7898	5.1801	436.68	15174.7	139	132041	5735339	562.35	25164.9	179	133791	5.6357	562.35	25164.9	179
140	19600	2744000	11.8322	5.1925	439.82	15393.8	140	132400	5832000	565.49	25446.9	180	134164	5.6462	565.49	25446.9	180
141	19881	2803221	11.8743	5.2048	442.96	15614.5	141	132761	5929741	568.63	25730.4	181	134536	5.6567	568.63	25730.4	181
142	20164	2863288	11.9164	5.2171	446.11	15836.8	142	133124	6028568	571.77	26015.5	182	134907	5.6671	571.77	26015.5	182
143	20449	2924207	11.9583	5.2293	449.25	16060.6	143	133489	6128487	574.91	26302.2	183	135277	5.6774	574.91	26302.2	183
144	20736	2985984	12.0000	5.2415	452.39	16286.0	144	133856	6229504	578.05	26590.4	184	135647	5.6877	578.05	26590.4	184
145	21025	3048625	12.0416	5.2536	455.53	16513.0	145	134225	6331625	581.19	26880.3	185	136015	5.6980	581.19	26880.3	185
146	21316	3112136	12.0830	5.2656	458.67	16741.5	146	134596	6434856	584.34	27171.6	186	136382	5.7083	584.34	27171.6	186
147	21609	3176523	12.1244	5.2776	461.81	16971.7	147	134969	6539203	587.48	27464.6	187	136748	5.7185	587.48	27464.6	187
148	21904	3241792	12.1655	5.2896	464.96	17203.4	148	135344	6644672	590.62	27759.1	188	137113	5.7287	590.62	27759.1	188
149	22201	3307949	12.2066	5.3015	468.10	17436.6	149	135721	6751269	593.76	28055.2	189	137477	5.7388	593.76	28055.2	189
150	22500	3375000	12.2474	5.3133	471.24	17671.5	150	136100	6859000	596.90	28352.9	190	137840	5.7489	596.90	28352.9	190
151	22801	3442951	12.2882	5.3251	474.38	17907.9	151	136481	6967871	600.04	28652.1	191	138203	5.7590	600.04	28652.1	191
152	23104	3511808	12.3288	5.3368	477.52	18145.8	152	136864	7077888	603.19	28952.9	192	138564	5.7690	603.19	28952.9	192
153	23409	3581577	12.3693	5.3485	480.66	18385.4	153	137249	7189057	606.33	29255.3	193	138924	5.7790	606.33	29255.3	193
154	23716	3652264	12.4097	5.3601	483.81	18626.5	154	137636	7301384	609.47	29559.2	194	139284	5.7890	609.47	29559.2	194
155	24025	3723875	12.4499	5.3717	486.95	18869.2	155	138025	7414875	612.61	29864.8	195	139642	5.7989	612.61	29864.8	195
156	24336	3796416	12.4900	5.3832	490.09	19113.4	156	138416	7529536	615.75	30171.9	196	140000	5.8088	615.75	30171.9	196
157	24649	3869893	12.5300	5.3947	493.23	19359.3	157	138809	7645373	618.89	30480.5	197	140357	5.8186	618.89	30480.5	197
158	24964	3944312	12.5698	5.4061	496.37	19606.7	158	139204	7762392	622.04	30790.7	198	140712	5.8285	622.04	30790.7	198
159	25281	4019679	12.6095	5.4175	499.51	19855.7	159	139601	7880599	625.18	31102.6	199	141067	5.8383	625.18	31102.6	199

T 10004-123

T 10004-124

No.	Square	Cube	Square Root	Cube Root	No.	Diam.		No.	Square	Cube	Square Root	Cube Root	Diam.		No.
						Circum.	Area						Circum.	Area	
200	40000	8000000	14.1421	5.8480	628.32	31415.9	200	240	57600	13824000	15.4919	6.2145	753.98	45238.9	240
201	40401	8120601	14.1774	5.8578	631.46	31730.9	201	241	58081	13997521	15.5242	6.2231	757.12	45616.7	241
202	40804	8242408	14.2127	5.8675	634.60	32045.4	202	242	58564	14172488	15.5563	6.2317	760.27	45996.1	242
203	41209	8365427	14.2478	5.8771	637.74	32365.5	203	243	59049	14348907	15.5885	6.2403	763.41	46377.0	243
204	41616	8489664	14.2829	5.8868	640.89	32685.1	204	244	59536	14526784	15.6205	6.2488	766.55	46759.5	244
205	42025	8615125	14.3178	5.8964	644.03	33006.4	205	245	60025	14706125	15.6525	6.2573	769.69	47143.5	245
206	42436	8741816	14.3527	5.9059	647.17	33329.2	206	246	60516	14886936	15.6844	6.2658	772.83	47529.2	246
207	42849	8869743	14.3875	5.9155	650.31	33653.5	207	247	61009	15069223	15.7162	6.2743	775.97	47916.4	247
208	43264	8998912	14.4222	5.9250	653.45	33979.5	208	248	61504	15252992	15.7480	6.2828	779.12	48305.1	248
209	43681	9129329	14.4568	5.9345	656.59	34307.0	209	249	62001	15438249	15.7797	6.2912	782.26	48695.5	249
210	44100	9261000	14.4914	5.9439	659.73	34636.1	210	250	62500	15625000	15.8114	6.2996	785.40	49087.4	250
211	44521	9393931	14.5258	5.9533	662.88	34966.7	211	251	63001	15813251	15.8430	6.3080	788.54	49480.9	251
212	44944	9528128	14.5602	5.9627	666.02	35298.9	212	252	63504	16003008	15.8745	6.3164	791.68	49875.9	252
213	45369	9663597	14.5945	5.9721	669.16	35632.7	213	253	64009	16194277	15.9060	6.3247	794.82	50272.6	253
214	45796	9800344	14.6287	5.9814	672.30	35968.1	214	254	64516	16387064	15.9374	6.3330	797.96	50670.7	254
215	46225	9938375	14.6629	5.9907	675.44	36305.0	215	255	65025	16581375	15.9687	6.3413	801.11	51070.5	255
216	46656	10077696	14.6969	6.0000	678.58	36643.5	216	256	65536	16777216	16.0000	6.3496	804.25	51471.9	256
217	47089	10218313	14.7309	6.0092	681.73	36983.6	217	257	66049	16974593	16.0312	6.3579	807.39	51874.8	257
218	47524	10360232	14.7648	6.0185	684.87	37325.3	218	258	66564	17173512	16.0624	6.3661	810.53	52279.2	258
219	47961	10503459	14.7986	6.0277	688.01	37668.5	219	259	67081	17373979	16.0935	6.3743	813.67	52685.3	259
220	48400	10648000	14.8324	6.0368	691.15	38013.3	220	260	67600	17576000	16.1245	6.3825	816.81	53092.9	260
221	48841	10793861	14.8661	6.0459	694.29	38359.6	221	261	68121	17779581	16.1555	6.3907	819.96	53502.1	261
222	49284	10941048	14.8997	6.0550	697.43	38707.6	222	262	68644	17984728	16.1854	6.3988	823.10	53912.9	262
223	49729	11089567	14.9332	6.0641	700.58	39057.1	223	263	69169	18191447	16.2173	6.4070	826.24	54325.2	263
224	50176	11239424	14.9666	6.0732	703.72	39408.1	224	264	69696	18399744	16.2481	6.4151	829.38	54739.1	264
225	50625	11390625	15.0000	6.0822	706.86	39760.8	225	265	70225	18609625	16.2788	6.4232	832.52	55154.6	265
226	51076	11543176	15.0333	6.0912	710.00	40115.0	226	266	70756	18821096	16.3095	6.4312	835.66	55571.6	266
227	51529	11697083	15.0665	6.1002	713.14	40470.8	227	267	71289	19034163	16.3401	6.4393	838.81	55990.3	267
228	51984	11852352	15.0997	6.1091	716.28	40828.1	228	268	71824	19248832	16.3707	6.4473	841.95	56410.4	268
229	52441	12008989	15.1327	6.1180	719.42	41187.1	229	269	72361	19465109	16.4012	6.4553	845.09	56832.2	269
230	52900	12167000	15.1658	6.1269	722.57	41547.6	230	270	72900	19683000	16.4317	6.4633	848.23	57255.5	270
231	53361	12326391	15.1987	6.1358	725.71	41909.6	231	271	73441	19902511	16.4621	6.4713	851.37	57680.4	271
232	53824	12487168	15.2315	6.1446	728.85	42273.3	232	272	73984	20123648	16.4924	6.4792	854.51	58106.9	272
233	54289	12649337	15.2643	6.1534	731.99	42638.5	233	273	74529	20346417	16.5227	6.4872	857.66	58534.9	273
234	54756	12812904	15.2971	6.1622	735.13	43005.3	234	274	75076	20570824	16.5529	6.4951	860.80	58964.6	274
235	55225	12977875	15.3297	6.1710	738.27	43373.6	235	275	75625	20796875	16.5831	6.5030	863.94	59395.7	275
236	55696	13144256	15.3623	6.1797	741.42	43743.5	236	276	76176	21024576	16.6132	6.5108	867.08	59828.5	276
237	56169	13312053	15.3948	6.1885	744.56	44115.0	237	277	76729	21253933	16.6433	6.5187	870.22	60262.8	277
238	56644	13481272	15.4272	6.1972	747.70	44488.1	238	278	77284	21484952	16.6733	6.5265	873.36	60698.7	278
239	57121	13651919	15.4596	6.2058	750.84	44862.7	239	279	77841	21717639	16.7033	6.5343	876.50	61136.2	279

TM604-127

TM604-126

No.	Square	Cube	Square Root	Cube Root	No.		No.	Square	Cube	Square Root	Cube Root	No.		No.
					Circum.	Area						Circum.	Area	
280	78400	21952000	16.7332	6.5421	879.65	61575.2	280	102400	32768000	17.8885	6.8399	1005.3	80424.8	320
281	78961	22188041	16.7631	6.5499	882.79	62015.8	281	103041	33076161	17.9165	6.8470	1008.5	80928.2	321
282	79524	22425768	16.7929	6.5577	885.93	62458.0	282	103684	33386248	17.9444	6.8541	1011.6	81433.2	322
283	80089	22665187	16.8226	6.5654	889.07	62901.8	283	104329	33698267	17.9722	6.8612	1014.7	81939.8	323
284	80656	22906304	16.8523	6.5731	892.21	63347.1	284	104976	34012224	18.0000	6.8683	1017.9	82448.0	324
285	81225	23149125	16.8819	6.5808	895.35	63794.0	285	105625	34328125	18.0278	6.8753	1021.0	82957.7	325
286	81796	23393056	16.9115	6.5885	898.50	64242.4	286	106276	34645976	18.0555	6.8824	1024.2	83469.0	326
287	82369	23639903	16.9411	6.5962	901.64	64692.5	287	106929	34965783	18.0831	6.8894	1027.3	83981.8	327
288	82944	23887872	16.9706	6.6039	904.78	65144.1	288	107584	35285752	18.1108	6.8964	1030.4	84496.3	328
289	83521	24137569	17.0000	6.6115	907.92	65597.2	289	108241	35611289	18.1384	6.9034	1033.6	85012.3	329
290	84100	24389000	17.0294	6.6191	911.06	66052.0	290	108900	35937000	18.1659	6.9104	1036.7	85529.9	330
291	84681	24642171	17.0587	6.6267	914.20	66508.3	291	109561	36264691	18.1934	6.9174	1039.9	86049.0	331
292	85264	24897088	17.0880	6.6343	917.35	66966.2	292	110224	36594368	18.2209	6.9244	1043.0	86569.7	332
293	85849	25153757	17.1172	6.6419	920.49	67425.6	293	110889	36926037	18.2483	6.9313	1046.2	87092.0	333
294	86436	25412184	17.1464	6.6494	923.63	67886.7	294	111556	37259704	18.2757	6.9382	1049.3	87615.9	334
295	87025	25672375	17.1756	6.6569	926.77	68349.3	295	112225	37595375	18.3030	6.9451	1052.4	88141.3	335
296	87616	25934336	17.2047	6.6644	929.91	68813.5	296	112896	37933056	18.3303	6.9521	1055.6	88668.3	336
297	88209	26198073	17.2337	6.6719	933.05	69279.2	297	113569	38272753	18.3576	6.9589	1058.7	89196.9	337
298	88804	26463592	17.2627	6.6794	936.19	69746.5	298	114244	38614472	18.3848	6.9658	1061.9	89727.0	338
299	89401	26730899	17.2916	6.6869	939.34	70215.4	299	114921	38958219	18.4120	6.9727	1065.0	90258.7	339
300	90000	27000000	17.3205	6.6943	942.48	70685.8	300	115600	39304000	18.4391	6.9795	1068.1	90792.0	340
301	90601	27270901	17.3494	6.7018	945.62	71157.9	301	116281	39651821	18.4662	6.9864	1071.3	91326.9	341
302	91204	27543608	17.3781	6.7092	948.76	71631.5	302	116964	40001688	18.4932	6.9932	1074.4	91863.3	342
303	91809	27818127	17.4069	6.7166	951.90	72106.6	303	117649	40353607	18.5203	7.0000	1077.6	92401.3	343
304	92416	28094464	17.4356	6.7240	955.04	72583.4	304	118336	40707584	18.5472	7.0068	1080.7	92940.9	344
305	93025	28372625	17.4642	6.7313	958.19	73061.7	305	119025	41063625	18.5742	7.0136	1083.8	93482.0	345
306	93636	28652616	17.4929	6.7387	961.33	73541.5	306	119716	41421736	18.6011	7.0203	1087.0	94024.7	346
307	94249	28934443	17.5214	6.7460	964.47	74023.0	307	120409	41781923	18.6279	7.0271	1090.1	94569.0	347
308	94864	29218112	17.5499	6.7533	967.61	74506.0	308	121104	42144192	18.6548	7.0338	1093.3	95114.9	348
309	95481	29503629	17.5784	6.7606	970.75	74990.6	309	121801	42508549	18.6815	7.0406	1096.4	95662.3	349
310	96100	29791000	17.6068	6.7679	973.89	75476.8	310	122500	42875000	18.7083	7.0473	1099.6	96211.3	350
311	96721	30080231	17.6352	6.7752	977.04	75964.5	311	123201	43243551	18.7350	7.0540	1102.7	96761.8	351
312	97344	30371328	17.6635	6.7824	980.18	76453.8	312	123904	43614208	18.7617	7.0607	1105.8	97314.0	352
313	97969	30664297	17.6918	6.7897	983.32	76944.7	313	124609	43986977	18.7883	7.0674	1109.0	97867.7	353
314	98596	30959144	17.7200	6.7969	986.46	77437.1	314	125316	44361864	18.8149	7.0740	1112.1	98423.0	354
315	99225	31255875	17.7482	6.8041	989.60	77931.1	315	126025	44738875	18.8414	7.0807	1115.3	98979.8	355
316	99856	31554496	17.7764	6.8113	992.74	78426.7	316	126736	45118016	18.8680	7.0873	1118.4	99538.2	356
317	100489	31855013	17.8045	6.8185	995.88	78923.9	317	127449	45499293	18.8944	7.0940	1121.5	100098	357
318	101124	32157432	17.8326	6.8256	999.03	79422.6	318	128164	45882712	18.9209	7.1006	1124.7	100660	358
319	101761	32461759	17.8606	6.8328	1002.2	79922.9	319	128881	46268279	18.9473	7.1072	1127.8	101223	359

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No.	Square	Cubo	Square Root	Cubo Root	No.		No.	Square	Cubo	Square Root	Cubo Root	No.		No.
					Circum.	Area						Circum.	Area	
360	129600	46656000	18.9737	7.1138	1131.0	101788	360	160000	64000000	20.0000	7.3681	1256.6	125664	400
361	130321	47045881	19.0000	7.1204	1134.1	102354	361	160801	64481201	20.0250	7.3742	1259.8	126293	401
362	131044	47437928	19.0263	7.1269	1137.3	102922	362	161604	64964808	20.0499	7.3803	1262.9	126923	402
363	131769	47832147	19.0526	7.1335	1140.4	103491	363	162409	65450827	20.0749	7.3864	1266.1	127556	403
364	132496	48228544	19.0788	7.1400	1143.5	104062	364	163216	65939264	20.0998	7.3925	1269.2	128190	404
365	133225	48627125	19.1050	7.1466	1146.7	104635	365	164025	66430125	20.1246	7.3986	1272.3	128825	405
366	133956	49027896	19.1311	7.1531	1149.8	105209	366	164836	66923416	20.1494	7.4047	1275.5	129462	406
367	134689	49430863	19.1572	7.1596	1153.0	105785	367	165649	67419143	20.1742	7.4108	1278.6	130100	407
368	135424	49836032	19.1833	7.1661	1156.1	106362	368	166464	67917312	20.1990	7.4169	1281.8	130741	408
369	136161	50243409	19.2094	7.1726	1159.2	106941	369	167281	68417929	20.2237	7.4229	1284.9	131382	409
370	136900	50653000	19.2354	7.1791	1162.4	107521	370	168100	68921000	20.2485	7.4290	1288.1	132025	410
371	137641	51064811	19.2614	7.1855	1165.5	108103	371	168921	69426531	20.2731	7.4350	1291.2	132670	411
372	138384	51478848	19.2873	7.1920	1168.7	108687	372	169744	69934528	20.2978	7.4410	1294.3	133317	412
373	139129	51895117	19.3132	7.1984	1171.8	109272	373	170569	70444997	20.3224	7.4470	1297.5	133965	413
374	139876	52313624	19.3391	7.2048	1175.0	109858	374	171396	70957944	20.3470	7.4530	1300.6	134614	414
375	140625	52734375	19.3649	7.2112	1178.1	110447	375	172225	71473375	20.3715	7.4590	1303.8	135265	415
376	141376	53157376	19.3907	7.2177	1181.2	111036	376	173056	71991296	20.3961	7.4650	1306.9	135918	416
377	142129	53582633	19.4165	7.2240	1184.4	111628	377	173889	72511713	20.4206	7.4710	1310.0	136572	417
378	142884	54010152	19.4422	7.2304	1187.5	112221	378	174724	73034632	20.4450	7.4770	1313.2	137228	418
379	143641	54439939	19.4679	7.2368	1190.7	112815	379	175561	73560059	20.4695	7.4829	1316.3	137885	419
380	144400	54872000	19.4936	7.2432	1193.8	113411	380	176400	74088000	20.4939	7.4889	1319.5	138544	420
381	145161	55306341	19.5192	7.2495	1196.9	114009	381	177241	74618461	20.5183	7.4948	1322.6	139205	421
382	145924	55742968	19.5448	7.2558	1200.1	114608	382	178084	75151448	20.5426	7.5007	1325.8	139867	422
383	146689	56181887	19.5704	7.2622	1203.2	115209	383	178929	75686967	20.5670	7.5067	1328.9	140531	423
384	147456	56623104	19.5959	7.2685	1206.4	115812	384	179776	76225024	20.5913	7.5126	1332.0	141196	424
385	148225	57066625	19.6214	7.2748	1209.5	116416	385	180625	76765625	20.6155	7.5185	1335.2	141863	425
386	148996	57512456	19.6469	7.2811	1212.7	117021	386	181476	77308776	20.6398	7.5244	1338.3	142531	426
387	149769	57960603	19.6723	7.2874	1215.8	117628	387	182329	77854483	20.6640	7.5302	1341.5	143201	427
388	150544	58411072	19.6977	7.2936	1218.9	118237	388	183184	78402752	20.6882	7.5361	1344.6	143872	428
389	151321	58863869	19.7231	7.2999	1222.1	118847	389	184041	78953589	20.7123	7.5420	1347.7	144545	429
390	152100	59319000	19.7484	7.3061	1225.2	119459	390	184900	79507000	20.7364	7.5478	1350.9	145220	430
391	152881	59776471	19.7737	7.3124	1228.4	120072	391	185761	80062991	20.7605	7.5537	1354.0	145896	431
392	153664	60236288	19.7990	7.3186	1231.5	120687	392	186624	80621568	20.7846	7.5595	1357.2	146574	432
393	154449	60698457	19.8242	7.3248	1234.6	121304	393	187489	81182737	20.8087	7.5654	1360.3	147254	433
394	155236	61162984	19.8494	7.3310	1237.8	121922	394	188356	81746504	20.8327	7.5712	1363.5	147934	434
395	156025	61629875	19.8746	7.3372	1240.9	122542	395	189225	82312875	20.8567	7.5770	1366.6	148617	435
396	156816	62099136	19.8997	7.3434	1244.1	123163	396	190096	82881856	20.8806	7.5828	1369.7	149301	436
397	157609	62570773	19.9249	7.3496	1247.2	123786	397	190969	83453453	20.9045	7.5886	1372.9	149987	437
398	158404	63047992	19.9499	7.3558	1250.4	124410	398	191844	84027672	20.9284	7.5944	1376.0	150674	438
399	159201	63522199	19.9750	7.3619	1253.5	125036	399	192721	84604519	20.9523	7.6001	1379.2	151363	439

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No.	Square	Cubo	Square Root	Cubo Root	No. = Biom.		No.
					Chrom.	Area	
440	193600	85184000	20.9762	7.6059	1382.3	152053	440
441	194481	85766121	21.0000	7.6117	1385.4	152745	441
442	195364	86350888	21.0238	7.6174	1388.6	153439	442
443	196249	86938307	21.0476	7.6232	1391.7	154134	443
444	197136	87528384	21.0713	7.6289	1394.9	154830	444
445	198025	88121125	21.0950	7.6346	1398.0	155528	445
446	198916	88716536	21.1187	7.6403	1401.2	156228	446
447	199809	89314623	21.1424	7.6460	1404.3	156933	447
448	200704	89915392	21.1660	7.6517	1407.4	157633	448
449	201601	90518449	21.1896	7.6574	1410.6	158337	449
450	202500	91125000	21.2132	7.6631	1413.7	159043	450
451	203401	91733851	21.2368	7.6688	1416.9	159751	451
452	204304	92345408	21.2603	7.6744	1420.0	160460	452
453	205209	92959577	21.2838	7.6801	1423.1	161171	453
454	206116	93576564	21.3073	7.6857	1426.3	161883	454
455	207025	94196375	21.3307	7.6914	1429.4	162597	455
456	207936	94818316	21.3542	7.6970	1432.6	163313	456
457	208849	95443393	21.3776	7.7026	1435.7	164033	457
458	209764	96071912	21.4009	7.7082	1438.9	164748	458
459	210681	96702579	21.4243	7.7138	1442.0	165468	459
460	211600	97336000	21.4476	7.7194	1445.1	166190	460
461	212521	97972181	21.4709	7.7250	1448.3	166914	461
462	213444	98611128	21.4942	7.7306	1451.4	167639	462
463	214369	99252847	21.5174	7.7362	1454.6	168365	463
464	215296	99897344	21.5407	7.7418	1457.7	169093	464
465	216225	100544625	21.5639	7.7473	1460.8	169823	465
466	217156	101194696	21.5870	7.7529	1464.0	170554	466
467	218089	101847563	21.6102	7.7584	1467.1	171287	467
468	219024	102503232	21.6333	7.7639	1470.3	172021	468
469	219961	103161709	21.6564	7.7695	1473.4	172757	469
470	220900	103823000	21.6795	7.7750	1476.5	173494	470
471	221841	104487111	21.7025	7.7805	1479.7	174234	471
472	222784	105154048	21.7256	7.7860	1482.8	174974	472
473	223729	105823817	21.7486	7.7915	1486.0	175716	473
474	224676	106496424	21.7715	7.7970	1489.1	176460	474
475	225625	107171875	21.7945	7.8025	1492.3	177205	475
476	226576	107850176	21.8174	7.8079	1495.4	177952	476
477	227529	108533333	21.8403	7.8134	1498.5	178701	477
478	228484	109215352	21.8632	7.8188	1501.7	179451	478
479	229441	109902739	21.8861	7.8243	1504.8	180203	479
480	230400	110592000	21.9089	7.8297	1508.0	180956	480
481	231361	111284641	21.9317	7.8352	1511.1	181711	481
482	232324	111980168	21.9545	7.8406	1514.3	182467	482
483	233289	112678587	21.9773	7.8460	1517.4	183225	483
484	234256	113379904	22.0000	7.8514	1520.5	183984	484
485	235225	114084125	22.0227	7.8568	1523.7	184745	485
486	236196	114791256	22.0454	7.8622	1526.8	185508	486
487	237169	115501303	22.0681	7.8676	1530.0	186272	487
488	238144	116214272	22.0907	7.8730	1533.1	187038	488
489	239121	116930169	22.1133	7.8784	1536.2	187805	489
490	240100	117649000	22.1359	7.8837	1539.4	188574	490
491	241081	118370771	22.1585	7.8891	1542.5	189345	491
492	242064	119095488	22.1811	7.8944	1545.7	190117	492
493	243049	119823157	22.2036	7.8998	1548.8	190890	493
494	244036	120553784	22.2261	7.9051	1551.9	191665	494
495	245025	121287375	22.2486	7.9105	1555.1	192442	495
496	246016	122023936	22.2711	7.9158	1558.2	193221	496
497	247009	122763473	22.2935	7.9211	1561.4	194000	497
498	248004	123505992	22.3159	7.9264	1564.5	194782	498
499	249001	124251499	22.3383	7.9317	1567.7	195565	499
500	250000	125000000	22.3607	7.9370	1570.8	196350	500
501	251001	125751501	22.3830	7.9423	1573.9	197136	501
502	252004	126506008	22.4054	7.9476	1577.1	197923	502
503	253009	127263527	22.4277	7.9528	1580.2	198713	503
504	254016	128024064	22.4499	7.9581	1583.4	199504	504
505	255025	128787625	22.4722	7.9634	1586.5	200296	505
506	256036	129554216	22.4944	7.9686	1589.7	201090	506
507	257049	130323843	22.5167	7.9739	1592.8	201886	507
508	258064	131096512	22.5389	7.9791	1595.9	202683	508
509	259081	131872229	22.5610	7.9843	1599.1	203482	509
510	260100	132651000	22.5832	7.9896	1602.2	204282	510
511	261121	133432831	22.6053	7.9948	1605.4	205084	511
512	262144	134217728	22.6274	8.0000	1608.5	205887	512
513	263169	135005697	22.6495	8.0052	1611.6	206692	513
514	264196	135796744	22.6716	8.0104	1614.8	207499	514
515	265225	136590875	22.6936	8.0156	1617.9	208307	515
516	266256	137388096	22.7156	8.0208	1621.1	209117	516
517	267289	138188413	22.7376	8.0260	1624.2	209928	517
518	268324	138991832	22.7596	8.0311	1627.3	210741	518
519	269361	139798359	22.7816	8.0363	1630.5	211556	519

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No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.	Square	Cube	Square Root	Cube Root	No.
					Circum.	Area						
520	270400	140608000	22.8035	8.0415	1633.6	212372	520	313600	175616000	23.6643	8.2426	560
521	271441	141420761	22.8254	8.0466	1636.8	213189	521	314721	176558481	23.6854	8.2475	561
522	272484	142236648	22.8473	8.0517	1639.9	214008	522	315844	177504328	23.7065	8.2524	562
523	273529	143055667	22.8692	8.0569	1643.1	214829	523	316969	178453547	23.7276	8.2573	563
524	274576	143877824	22.8910	8.0620	1646.2	215651	524	318096	179406144	23.7487	8.2621	564
525	275625	144703125	22.9129	8.0671	1649.3	216475	525	319225	180362125	23.7697	8.2670	565
526	276676	145531576	22.9347	8.0723	1652.5	217301	526	320356	181321496	23.7908	8.2719	566
527	277729	146363183	22.9565	8.0774	1655.6	218128	527	321489	182284263	23.8118	8.2768	567
528	278784	147197952	22.9783	8.0825	1658.8	218956	528	322624	183250432	23.8328	8.2816	568
529	279841	148035889	23.0000	8.0876	1661.9	219787	529	323761	184220009	23.8537	8.2865	569
530	280900	148877000	23.0217	8.0927	1665.0	220618	530	324900	185193000	23.8747	8.2913	570
531	281961	149721291	23.0434	8.0978	1668.2	221452	531	326041	186169411	23.8956	8.2962	571
532	283024	150568768	23.0651	8.1028	1671.3	222287	532	327184	187149248	23.9165	8.3010	572
533	284089	151419437	23.0868	8.1079	1674.5	223123	533	328329	188132517	23.9374	8.3059	573
534	285156	152277304	23.1084	8.1130	1677.6	223961	534	329476	189119224	23.9583	8.3107	574
535	286225	153130375	23.1301	8.1180	1680.8	224801	535	330625	190109375	23.9792	8.3155	575
536	287296	153990656	23.1517	8.1231	1683.9	225642	536	331776	191102976	24.0000	8.3203	576
537	288369	154854153	23.1733	8.1281	1687.0	226484	537	332929	192100033	24.0208	8.3251	577
538	289444	155720872	23.1948	8.1332	1690.2	227329	538	334084	193100552	24.0416	8.3300	578
539	290521	156590819	23.2164	8.1382	1693.3	228175	539	335241	194104539	24.0624	8.3348	579
540	291600	157464000	23.2379	8.1433	1696.5	229022	540	336400	195112000	24.0832	8.3396	580
541	292681	158340421	23.2594	8.1483	1699.6	229871	541	337561	196122941	24.1039	8.3443	581
542	293764	159220088	23.2809	8.1533	1702.7	230722	542	338724	197137368	24.1247	8.3491	582
543	294849	160103007	23.3024	8.1583	1705.9	231574	543	339889	198155287	24.1454	8.3539	583
544	295936	160989184	23.3238	8.1633	1709.0	232428	544	341056	199176704	24.1661	8.3587	584
545	297025	161878625	23.3452	8.1683	1712.2	233283	545	342225	200201625	24.1868	8.3634	585
546	298116	162771336	23.3666	8.1733	1715.3	234140	546	343396	201230056	24.2074	8.3682	586
547	299209	163667323	23.3880	8.1783	1718.5	234998	547	344569	202262003	24.2281	8.3730	587
548	300304	164566592	23.4094	8.1833	1721.6	235858	548	345744	203297472	24.2487	8.3777	588
549	301401	165469149	23.4307	8.1882	1724.7	236720	549	346921	204336469	24.2693	8.3825	589
550	302500	166375000	23.4521	8.1932	1727.9	237583	550	348100	205379000	24.2899	8.3872	590
551	303601	167284151	23.4734	8.1982	1731.0	238448	551	349281	206425071	24.3105	8.3919	591
552	304704	168196608	23.4947	8.2031	1734.2	239314	552	350464	207474688	24.3311	8.3967	592
553	305809	169112377	23.5160	8.2081	1737.3	240182	553	351649	208527857	24.3516	8.4014	593
554	306916	170031464	23.5372	8.2130	1740.4	241051	554	352836	209584584	24.3721	8.4061	594
555	308025	170953875	23.5584	8.2180	1743.6	241922	555	354025	210644875	24.3926	8.4108	595
556	309136	171879616	23.5797	8.2229	1746.7	242795	556	355216	211708736	24.4131	8.4155	596
557	310249	172808693	23.6008	8.2278	1749.9	243669	557	356409	212776173	24.4336	8.4202	597
558	311364	173741112	23.6220	8.2327	1753.0	244545	558	357604	213847192	24.4540	8.4249	598
559	312481	174676879	23.6432	8.2377	1756.2	245422	559	358801	214921799	24.4745	8.4296	599

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No.	Square	Cube	Square Root	Cube Root	No. = 1000		No.
					Circum.	Area	
600	360000	216000000	24.4949	8.4343	1885.0	282743	600
601	361201	217081801	24.5153	8.4390	1888.1	283687	601
602	362404	218167208	24.5357	8.4437	1891.2	284631	602
603	363609	219256227	24.5561	8.4484	1894.4	285578	603
604	364816	220348864	24.5764	8.4530	1897.5	286526	604
605	366025	221445125	24.5967	8.4577	1900.7	287475	605
606	367236	222545016	24.6171	8.4623	1903.8	288426	606
607	368449	223648543	24.6374	8.4670	1907.0	289379	607
608	369664	224755712	24.6577	8.4716	1910.1	290333	608
609	370881	225866529	24.6779	8.4763	1913.2	291289	609
610	372100	226981000	24.6982	8.4809	1916.4	292247	610
611	373321	228099131	24.7184	8.4856	1919.5	293206	611
612	374544	229220928	24.7386	8.4902	1922.7	294166	612
613	375769	230346397	24.7588	8.4948	1925.8	295128	613
614	376996	231475544	24.7790	8.4994	1928.9	296092	614
615	378225	232608375	24.7992	8.5040	1932.1	297057	615
616	379456	233744896	24.8193	8.5086	1935.2	298024	616
617	380689	234885113	24.8395	8.5132	1938.4	298992	617
618	381924	236029032	24.8596	8.5178	1941.5	299962	618
619	383161	237176659	24.8797	8.5224	1944.7	300934	619
620	384400	238328000	24.8998	8.5270	1947.8	301907	620
621	385641	239483061	24.9199	8.5316	1950.9	302882	621
622	386884	240641848	24.9399	8.5362	1954.1	303858	622
623	388129	241804367	24.9600	8.5408	1957.2	304836	623
624	389376	242970624	24.9800	8.5453	1960.4	305815	624
625	390625	244140625	25.0000	8.5499	1963.5	306796	625
626	391876	245314376	25.0200	8.5544	1966.6	307779	626
627	393129	246491883	25.0400	8.5590	1969.8	308763	627
628	394384	247673152	25.0599	8.5635	1972.9	309748	628
629	395641	248858189	25.0799	8.5681	1976.1	310736	629
630	396900	250047000	25.0998	8.5726	1979.2	311725	630
631	398161	251239591	25.1197	8.5772	1982.4	312715	631
632	399424	252435968	25.1396	8.5817	1985.5	313707	632
633	400689	253636137	25.1595	8.5862	1988.6	314700	633
634	401956	254840104	25.1794	8.5907	1991.8	315696	634
635	403225	256047875	25.1992	8.5952	1994.9	316692	635
636	404496	257259456	25.2190	8.5997	1998.1	317690	636
637	405769	258474853	25.2389	8.6043	2001.2	318690	637
638	407044	259694072	25.2587	8.6088	2004.3	319692	638
639	408321	260917119	25.2784	8.6132	2007.5	320695	639
640	409600	262144000	25.2982	8.6177	2010.6	321699	640
641	410881	263374721	25.3180	8.6222	2013.8	322705	641
642	412164	264609288	25.3377	8.6267	2016.9	323713	642
643	413449	265847707	25.3574	8.6312	2020.0	324722	643
644	414736	267089984	25.3772	8.6357	2023.2	325733	644
645	416025	268336125	25.3969	8.6401	2026.3	326745	645
646	417316	269586136	25.4165	8.6446	2029.5	327759	646
647	418609	270840023	25.4362	8.6490	2032.6	328775	647
648	419904	272097792	25.4558	8.6535	2035.8	329792	648
649	421201	273359449	25.4755	8.6579	2038.9	330810	649
650	422500	274625000	25.4951	8.6624	2042.0	331831	650
651	423801	275894451	25.5147	8.6668	2045.2	332853	651
652	425104	277167808	25.5343	8.6713	2048.3	333876	652
653	426409	278445077	25.5539	8.6757	2051.5	334901	653
654	427716	279726264	25.5734	8.6801	2054.6	335927	654
655	429025	281011375	25.5930	8.6845	2057.7	336955	655
656	430336	282300416	25.6125	8.6890	2060.9	337985	656
657	431649	283593393	25.6320	8.6934	2064.0	339016	657
658	432964	284890312	25.6515	8.6978	2067.2	340049	658
659	434281	286191179	25.6710	8.7022	2070.3	341084	659
660	435600	287496000	25.6905	8.7066	2073.5	342119	660
661	436921	288804781	25.7099	8.7110	2076.6	343157	661
662	438244	290117528	25.7294	8.7154	2079.7	344196	662
663	439569	291434247	25.7488	8.7198	2082.9	345237	663
664	440896	292754944	25.7682	8.7241	2086.0	346279	664
665	442225	294079625	25.7876	8.7285	2089.2	347323	665
666	443556	295408296	25.8070	8.7329	2092.3	348368	666
667	444889	296740963	25.8263	8.7373	2095.4	349415	667
668	446224	298077632	25.8457	8.7416	2098.6	350464	668
669	447561	299418309	25.8650	8.7460	2101.7	351514	669
670	448900	300763000	25.8844	8.7503	2104.9	352565	670
671	450241	302111711	25.9037	8.7547	2108.0	353618	671
672	451584	303464448	25.9230	8.7590	2111.2	354673	672
673	452929	304821217	25.9422	8.7634	2114.3	355730	673
674	454276	306182024	25.9615	8.7677	2117.4	356788	674
675	455625	307546875	25.9808	8.7721	2120.6	357847	675
676	456976	308915776	26.0000	8.7764	2123.7	358908	676
677	458329	310288733	26.0192	8.7807	2126.9	359971	677
678	459684	311665752	26.0384	8.7850	2130.0	361035	678
679	461041	313046839	26.0576	8.7893	2133.1	362101	679

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No.	Square	Cubo	Square Root	Cubo Root	No. = Diam.		No.
					Circum.	Area	
680	462400	314332000	26.0768	8.7937	2136.3	363168	680
681	463761	315821241	26.0960	8.7980	2139.4	364237	681
682	465124	317214568	26.1151	8.8023	2142.6	365308	682
683	466489	318611987	26.1343	8.8066	2145.7	366380	683
684	467856	320013504	26.1534	8.8109	2148.9	367453	684
685	469225	321419125	26.1725	8.8152	2152.0	368528	685
686	470596	322828856	26.1916	8.8194	2155.1	369605	686
687	471969	324242703	26.2107	8.8237	2158.3	370684	687
688	473344	325660672	26.2298	8.8280	2161.4	371764	688
689	474721	327082769	26.2488	8.8323	2164.6	372845	689
690	476100	328509000	26.2679	8.8366	2167.7	373928	690
691	477481	329939371	26.2869	8.8408	2170.8	375013	691
692	478864	331373888	26.3059	8.8451	2174.0	376099	692
693	480249	332812557	26.3249	8.8493	2177.1	377187	693
694	481636	334255384	26.3439	8.8536	2180.3	378276	694
695	483025	335702375	26.3629	8.8578	2183.4	379367	695
696	484416	337153536	26.3818	8.8621	2186.6	380459	696
697	485809	338608873	26.4008	8.8663	2189.7	381554	697
698	487204	340068392	26.4197	8.8706	2192.8	382649	698
699	488601	341532099	26.4386	8.8748	2196.0	383746	699
700	490000	343000000	26.4575	8.8790	2199.1	384845	700
701	491401	344472101	26.4764	8.8833	2202.3	385945	701
702	492804	345948408	26.4953	8.8875	2205.4	387047	702
703	494209	347428927	26.5141	8.8917	2208.5	388151	703
704	495616	348913664	26.5330	8.8959	2211.7	389256	704
705	497025	350402625	26.5518	8.9001	2214.8	390363	705
706	498436	351895816	26.5707	8.9043	2218.0	391471	706
707	499849	353393243	26.5895	8.9085	2221.1	392580	707
708	501264	354894912	26.6083	8.9127	2224.3	393692	708
709	502681	356400829	26.6271	8.9169	2227.4	394805	709
710	504100	357911000	26.6458	8.9211	2230.5	395919	710
711	505521	359425431	26.6646	8.9253	2233.7	397035	711
712	506944	360944128	26.6833	8.9295	2236.8	398153	712
713	508369	362467097	26.7021	8.9337	2240.0	399272	713
714	509796	363994344	26.7208	8.9378	2243.1	400393	714
715	511225	365525875	26.7395	8.9420	2246.2	401515	715
716	512656	367061696	26.7582	8.9462	2249.4	402639	716
717	514089	368601813	26.7769	8.9503	2252.5	403765	717
718	515524	370146232	26.7955	8.9545	2255.7	404892	718
719	516961	371694959	26.8142	8.9587	2258.8	406020	719
720	518400	373248000	26.8328	8.9628	2261.9	407150	720
721	519841	374805361	26.8514	8.9670	2265.1	408282	721
722	521284	376367048	26.8701	8.9711	2268.2	409416	722
723	522729	377933067	26.8887	8.9752	2271.4	410550	723
724	524176	379503424	26.9072	8.9794	2274.5	411687	724
725	525625	381078125	26.9258	8.9835	2277.7	412825	725
726	527076	382657176	26.9444	8.9876	2280.8	413965	726
727	528529	384240583	26.9629	8.9918	2283.9	415106	727
728	529984	385828352	26.9815	8.9959	2287.1	416248	728
729	531441	387420489	27.0000	9.0000	2290.2	417393	729
730	532900	389017000	27.0185	9.0041	2293.4	418539	730
731	534361	390617891	27.0370	9.0082	2296.5	419686	731
732	535824	392223168	27.0555	9.0123	2299.7	420835	732
733	537289	393832837	27.0740	9.0164	2302.8	421986	733
734	538756	395446904	27.0924	9.0205	2305.9	423138	734
735	540225	397065375	27.1109	9.0246	2309.1	424293	735
736	541696	398688256	27.1293	9.0287	2312.2	425448	736
737	543169	400315553	27.1477	9.0328	2315.4	426604	737
738	544644	401947272	27.1662	9.0369	2318.5	427762	738
739	546121	403583419	27.1846	9.0410	2321.6	428922	739
740	547600	405224000	27.2029	9.0450	2324.8	430084	740
741	549081	406869021	27.2213	9.0491	2327.9	431247	741
742	550564	408518488	27.2397	9.0532	2331.1	432412	742
743	552049	410172407	27.2580	9.0572	2334.2	433578	743
744	553536	411830784	27.2764	9.0613	2337.3	434746	744
745	555025	413493625	27.2947	9.0654	2340.5	435916	745
746	556516	415160936	27.3130	9.0694	2343.6	437087	746
747	558009	416832723	27.3313	9.0735	2346.8	438259	747
748	559504	418508992	27.3496	9.0775	2349.9	439433	748
749	561001	420189749	27.3679	9.0816	2353.1	440609	749
750	562500	421875000	27.3861	9.0856	2356.2	441786	750
751	564001	423564751	27.4044	9.0896	2359.3	442965	751
752	565504	425259008	27.4226	9.0937	2362.5	444146	752
753	567009	426957777	27.4408	9.0977	2365.6	445328	753
754	568516	428661064	27.4591	9.1017	2368.8	446511	754
755	570025	430368875	27.4775	9.1057	2371.9	447697	755
756	571536	432081216	27.4955	9.1098	2375.0	448883	756
757	573049	433798093	27.5136	9.1138	2378.2	450072	757
758	574564	435519512	27.5318	9.1178	2381.3	451262	758
759	576081	437245479	27.5500	9.1218	2384.5	452453	759

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No.	Square Feet	Cubic	Square Feet	Cubic Feet	No. = Diam.		No.	Square Feet	Cubic	Square Feet	Cubic Feet	No. = Diam.		No.
					Circum.	Area						Circum.	Area	
760	577600	438976000	27.5681	9.1258	2387.6	453646	760	640000	512000000	28.2843	9.2832	2513.3	502655	800
761	579121	440711081	27.5862	9.1298	2390.8	454841	761	641601	513922401	28.3019	9.2870	2516.4	503912	801
762	580644	442450728	27.6043	9.1338	2393.9	456037	762	643204	515849608	28.3196	9.2909	2519.6	505171	802
763	582169	444194947	27.6225	9.1378	2397.0	457234	763	644809	517781627	28.3373	9.2948	2522.7	506432	803
764	583696	445943744	27.6405	9.1418	2400.2	458434	764	646416	519718464	28.3549	9.2986	2525.8	507694	804
765	585225	447697125	27.6586	9.1458	2402.3	459635	765	648025	521660125	28.3725	9.3025	2529.0	508958	805
766	586756	449455096	27.6767	9.1498	2406.5	460837	766	649636	523606616	28.3901	9.3063	2532.1	510223	806
767	588289	451217663	27.6948	9.1537	2409.6	462042	767	651249	525557943	28.4077	9.3102	2535.3	511490	807
768	589824	452984832	27.7128	9.1577	2412.7	463247	768	652864	527514112	28.4253	9.3140	2538.4	512758	808
769	591361	454756609	27.7308	9.1617	2415.9	464454	769	654481	529475129	28.4429	9.3179	2541.5	514028	809
770	592900	456533000	27.7489	9.1657	2419.0	465663	770	656100	531441000	28.4605	9.3217	2544.7	515300	810
771	594441	458314011	27.7669	9.1696	2422.2	466873	771	657721	533411731	28.4781	9.3255	2547.8	516573	811
772	595984	460099648	27.7849	9.1736	2425.3	468085	772	659344	535387328	28.4956	9.3294	2551.0	517848	812
773	597529	461889917	27.8029	9.1775	2428.5	469298	773	660969	537367797	28.5132	9.3332	2554.1	519124	813
774	599076	463684824	27.8209	9.1815	2431.6	470513	774	662596	539353144	28.5307	9.3370	2557.3	520402	814
775	600625	465484375	27.8388	9.1855	2434.7	471730	775	664225	541343375	28.5482	9.3408	2560.4	521681	815
776	602176	467288576	27.8568	9.1894	2437.9	472948	776	665856	543338496	28.5657	9.3447	2563.5	522962	816
777	603729	469097433	27.8747	9.1933	2441.0	474168	777	667489	545338513	28.5832	9.3485	2566.7	524245	817
778	605284	470910952	27.8927	9.1973	2444.2	475389	778	669124	547343432	28.6007	9.3523	2569.8	525529	818
779	606841	472729139	27.9106	9.2012	2447.3	476612	779	670761	549353259	28.6182	9.3561	2573.0	526814	819
780	608400	474552000	27.9285	9.2052	2450.4	477836	780	672400	551368000	28.6356	9.3599	2576.1	528102	820
781	609961	476379541	27.9464	9.2091	2453.6	479062	781	674041	553387661	28.6531	9.3637	2579.2	529391	821
782	611524	478211768	27.9643	9.2130	2456.7	480290	782	675684	555412248	28.6705	9.3675	2582.4	530681	822
783	613089	480048687	27.9821	9.2170	2459.9	481519	783	677329	557441767	28.6880	9.3713	2585.5	531973	823
784	614656	481890304	28.0000	9.2209	2463.0	482750	784	678976	559476224	28.7054	9.3751	2588.7	533267	824
785	616225	483736625	28.0179	9.2248	2466.2	483982	785	680625	561515625	28.7228	9.3789	2591.8	534562	825
786	617796	485587656	28.0357	9.2287	2469.3	485216	786	682276	563559976	28.7402	9.3827	2595.0	535858	826
787	619369	487443403	28.0535	9.2326	2472.4	486451	787	683929	565609283	28.7576	9.3865	2598.1	537157	827
788	620944	489303672	28.0713	9.2365	2475.6	487688	788	685584	567663552	28.7750	9.3902	2601.2	538456	828
789	622521	491169069	28.0891	9.2404	2478.7	488927	789	687241	569722789	28.7924	9.3940	2604.4	539758	829
790	624100	493039000	28.1069	9.2443	2481.9	490167	790	688900	571787000	28.8097	9.3978	2607.5	541061	830
791	625681	494913671	28.1247	9.2482	2485.0	491409	791	690561	573856191	28.8271	9.4016	2610.7	542365	831
792	627264	496793088	28.1425	9.2521	2488.1	492652	792	692224	575930368	28.8444	9.4053	2613.8	543671	832
793	628849	498677257	28.1603	9.2560	2491.3	493897	793	693889	578009537	28.8617	9.4091	2616.9	544979	833
794	630436	500566184	28.1780	9.2599	2494.4	495143	794	695556	580093704	28.8791	9.4129	2620.1	546288	834
795	632025	502459875	28.1957	9.2638	2497.6	496391	795	697225	582182875	28.8964	9.4166	2623.2	547599	835
796	633616	504358336	28.2135	9.2677	2500.7	497641	796	698896	584277056	28.9137	9.4204	2626.4	548912	836
797	635209	506261573	28.2312	9.2716	2503.8	498892	797	700569	586376253	28.9310	9.4241	2629.5	550226	837
798	636804	508169592	28.2489	9.2754	2507.0	500145	798	702244	588480472	28.9482	9.4279	2632.7	551541	838
799	638401	510082399	28.2666	9.2793	2510.1	501399	799	703921	590589719	28.9655	9.4316	2635.8	552858	839

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No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
840	705600	592704000	28.9828	9.4354	2638.9	554177	840
841	707281	594823321	29.0000	9.4391	2642.1	555497	841
842	708964	596947688	29.0172	9.4429	2648.2	556819	842
843	710649	599077107	29.0345	9.4466	2654.4	558142	843
844	712336	6012111584	29.0517	9.4503	2660.6	559465	844
845	714025	603351125	29.0689	9.4541	2666.8	560788	845
846	715716	605495736	29.0861	9.4578	2673.0	562111	846
847	717409	607645423	29.1033	9.4615	2679.2	563434	847
848	719104	609800192	29.1204	9.4652	2685.4	564757	848
849	720801	611960049	29.1376	9.4690	2691.6	566080	849
850	722500	614125000	29.1548	9.4727	2697.8	567403	850
851	724201	616295051	29.1719	9.4764	2704.0	568726	851
852	725904	618470208	29.1890	9.4801	2710.2	570049	852
853	727609	620650477	29.2062	9.4838	2716.4	571372	853
854	729316	622835864	29.2233	9.4875	2722.6	572695	854
855	731025	625026375	29.2404	9.4912	2728.8	574018	855
856	732736	627222016	29.2575	9.4949	2735.0	575341	856
857	734449	629422793	29.2746	9.4986	2741.2	576664	857
858	736164	631628712	29.2916	9.5023	2747.4	577987	858
859	737881	633839779	29.3087	9.5060	2753.6	579310	859
860	739600	636056000	29.3258	9.5097	2759.8	580633	860
861	741321	638277381	29.3428	9.5134	2766.0	581956	861
862	743044	640503928	29.3598	9.5171	2772.2	583279	862
863	744769	642735647	29.3769	9.5207	2778.4	584602	863
864	746496	644972544	29.3939	9.5244	2784.6	585925	864
865	748225	647214625	29.4109	9.5281	2790.8	587248	865
866	749956	649461896	29.4279	9.5317	2797.0	588571	866
867	751689	651714363	29.4449	9.5354	2803.2	589894	867
868	753424	653972032	29.4618	9.5391	2809.4	591217	868
869	755161	656234909	29.4788	9.5427	2815.6	592540	869
870	756900	658503000	29.4958	9.5464	2821.8	593863	870
871	758641	660776311	29.5127	9.5501	2828.0	595186	871
872	760384	663054848	29.5296	9.5537	2834.2	596509	872
873	762129	665338617	29.5466	9.5574	2840.4	597832	873
874	763876	667627624	29.5635	9.5610	2846.6	599155	874
875	765625	669921875	29.5804	9.5647	2852.8	600478	875
876	767376	672221376	29.5973	9.5683	2859.0	601801	876
877	769129	674526133	29.6142	9.5719	2865.2	603124	877
878	770884	676836152	29.6311	9.5756	2871.4	604447	878
879	772641	679151439	29.6479	9.5792	2877.6	605770	879
880	774400	681472000	29.6648	9.5828	2883.8	607093	880
881	776161	683797841	29.6816	9.5865	2890.0	608416	881
882	777924	686128968	29.6985	9.5901	2896.2	609739	882
883	779689	688465387	29.7153	9.5937	2902.4	611062	883
884	781456	690807104	29.7321	9.5973	2908.6	612385	884
885	783225	693154125	29.7489	9.6010	2914.8	613708	885
886	784996	695506456	29.7658	9.6046	2921.0	615031	886
887	786769	697864103	29.7825	9.6082	2927.2	616354	887
888	788544	700227072	29.7993	9.6118	2933.4	617677	888
889	790321	702595369	29.8161	9.6154	2939.6	619000	889
890	792100	704969000	29.8329	9.6190	2945.8	620323	890
891	793881	707347971	29.8496	9.6226	2952.0	621646	891
892	795664	709732288	29.8664	9.6262	2958.2	622969	892
893	797449	712121957	29.8831	9.6298	2964.4	624292	893
894	799236	714516984	29.8998	9.6334	2970.6	625615	894
895	801025	716917375	29.9166	9.6370	2976.8	626938	895
896	802816	719323136	29.9333	9.6406	2983.0	628261	896
897	804609	721734273	29.9500	9.6442	2989.2	629584	897
898	806404	724150792	29.9666	9.6477	2995.4	630907	898
899	808201	726572699	29.9833	9.6513	3001.6	632230	899
900	810000	729000000	30.0000	9.6549	3007.8	633553	900
901	811801	731432701	30.0167	9.6585	3014.0	634876	901
902	813604	733870808	30.0333	9.6620	3020.2	636199	902
903	815409	736314327	30.0500	9.6656	3026.4	637522	903
904	817216	738763264	30.0666	9.6692	3032.6	638845	904
905	819025	741217625	30.0832	9.6727	3038.8	640168	905
906	820836	743677416	30.0998	9.6763	3045.0	641491	906
907	822649	746142643	30.1164	9.6799	3051.2	642814	907
908	824464	748613312	30.1330	9.6834	3057.4	644137	908
909	826281	751089429	30.1496	9.6870	3063.6	645460	909
910	828100	753571000	30.1662	9.6905	3069.8	646783	910
911	829921	756058031	30.1828	9.6941	3076.0	648106	911
912	831744	758550528	30.1993	9.6976	3082.2	649429	912
913	833569	761048497	30.2159	9.7012	3088.4	650752	913
914	835396	763551944	30.2324	9.7047	3094.6	652075	914
915	837225	766060875	30.2490	9.7082	3100.8	653398	915
916	839056	768575296	30.2655	9.7118	3107.0	654721	916
917	840889	771095213	30.2820	9.7153	3113.2	656044	917
918	842724	773620632	30.2985	9.7188	3119.4	657367	918
919	844561	776151559	30.3150	9.7224	3125.6	658690	919

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No.	Square	Cube	Square Root	Cube Root	No. = Dim.		No.
					Circum.	Area	
920	846400	778688000	30.3315	9.7259	2890.3	664761	920
921	848241	781229961	30.3480	9.7294	2893.4	666207	921
922	850084	783777448	30.3645	9.7329	2896.5	667654	922
923	851929	786330467	30.3809	9.7364	2899.7	669103	923
924	853776	788889024	30.3974	9.7400	2902.8	670554	924
925	855625	791453125	30.4138	9.7435	2906.0	672006	925
926	857476	794022776	30.4302	9.7470	2909.1	673460	926
927	859329	796597983	30.4467	9.7505	2912.3	674915	927
928	861184	799178752	30.4631	9.7540	2915.4	676372	928
929	863041	801765089	30.4795	9.7575	2918.5	677831	929
930	864900	804357000	30.4959	9.7610	2921.7	679291	930
931	866761	806954491	30.5123	9.7645	2924.8	680752	931
932	868624	809557568	30.5287	9.7680	2928.0	682216	932
933	870489	812166237	30.5450	9.7715	2931.1	683680	933
934	872356	814780504	30.5614	9.7750	2934.2	685147	934
935	874225	817400375	30.5778	9.7785	2937.4	686615	935
936	876096	820025856	30.5941	9.7819	2940.5	688084	936
937	877969	822656953	30.6105	9.7854	2943.7	689555	937
938	879844	825293672	30.6268	9.7889	2946.8	691028	938
939	881721	827936019	30.6431	9.7924	2950.0	692502	939
940	883600	830584000	30.6594	9.7959	2953.1	693978	940
941	885481	833237621	30.6757	9.7993	2956.2	695455	941
942	887364	835896888	30.6920	9.8028	2959.4	696934	942
943	889249	838561807	30.7083	9.8063	2962.5	698415	943
944	891136	841232384	30.7246	9.8097	2965.7	699897	944
945	893025	843908625	30.7409	9.8132	2968.8	701380	945
946	894915	846590536	30.7571	9.8167	2971.9	702865	946
947	896809	849278123	30.7733	9.8201	2975.1	704352	947
948	898704	851971392	30.7896	9.8236	2978.2	705840	948
949	900600	854670349	30.8058	9.8270	2981.4	707330	949
950	902500	857375000	30.8221	9.8305	2984.5	708822	950
951	904401	860085351	30.8383	9.8339	2987.7	710315	951
952	906304	862801408	30.8545	9.8374	2990.8	711809	952
953	908209	865523177	30.8707	9.8408	2993.9	713306	953
954	910116	868250664	30.8869	9.8443	2997.1	714803	954
955	912025	870983875	30.9031	9.8477	3000.2	716303	955
956	913936	873722816	30.9192	9.8511	3003.4	717804	956
957	915849	876467493	30.9354	9.8546	3006.5	719306	957
958	917764	879217912	30.9516	9.8580	3009.6	720810	958
959	919681	881974079	30.9677	9.8614	3012.8	722316	959

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2. Common Logarithms

N	0	1	2	3	4	5	6	7	8	9	d
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4.3
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	3.8
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3.5
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3.2
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3.0
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	2.8
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	2.6
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2.5
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2.4
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2.2
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2.1
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2.0
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	1.9
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	1.8
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	1.8
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	1.7
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	1.6
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	1.6
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	1.5
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1.5
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1.4
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1.4
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1.3
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1.3
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1.3
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1.2
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1.2
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1.2
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1.1
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1.1
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1.1
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1.0
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1.0
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1.0
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1.0
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1.0
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	.9
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	.9
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	.9
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	.9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	.9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	.8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	.8
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	.8
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	.8

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°	0°		1°		2°		3°		4°		°
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
°	89°		88°		87°		86°		85°		°
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	

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AGO 686A

°	5°			6°			7°			8°			9°			10°			11°			12°			13°			14°		
	Sine	Cosine	Shine	Sine	Cosine	Shine	Sine	Cosine	Shine	Sine	Cosine	Shine	Sine	Cosine	Shine	Sine	Cosine	Shine	Sine	Cosine	Shine	Sine	Cosine	Shine	Sine	Cosine	Shine			
0	.08716	.99619	.10452	.99452	.13917	.99027	.15643	.98769	.60	1	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	2	.24192	.97030	3	.25887	.96653	4	.27579	.96278	5	.29268	.95904
1	.08745	.99617	.10482	.99449	.12187	.99023	.15672	.98768	50	2	.17393	.98477	.19108	.98157	.20820	.97803	.22523	.97430	51	.24220	.97027	52	.25913	.96648	53	.27603	.96273	54	.29292	.95899
2	.08774	.99614	.10511	.99446	.12245	.99019	.15701	.98765	40	3	.17422	.98471	.19136	.98152	.20848	.97803	.22552	.97424	41	.24249	.97023	42	.25941	.96643	43	.27630	.96268	44	.29320	.95894
3	.08803	.99611	.10540	.99443	.12302	.99015	.15730	.98762	30	4	.17451	.98466	.19167	.98147	.20905	.97797	.22608	.97411	31	.24277	.97018	32	.25969	.96638	33	.27657	.96263	34	.29348	.95889
4	.08831	.99609	.10569	.99440	.12360	.99011	.15758	.98759	20	5	.17480	.98461	.19198	.98142	.20939	.97788	.22665	.97398	21	.24305	.97015	22	.25997	.96633	23	.27685	.96258	24	.29369	.95884
5	.08860	.99607	.10598	.99437	.12418	.99006	.15787	.98756	10	6	.17509	.98456	.19224	.98137	.20962	.97778	.22693	.97384	11	.24333	.97012	12	.26029	.96628	13	.27716	.96253	14	.29391	.95879
6	.08889	.99604	.10627	.99434	.12476	.99002	.15816	.98753	0	7	.17538	.98451	.19252	.98128	.21019	.97766	.22722	.97384	0	.24361	.97009	0	.26057	.96623	0	.27742	.96248	0	.29402	.95874
7	.08918	.99601	.10656	.99431	.12534	.99000	.15845	.98750	9	8	.17567	.98446	.19281	.98119	.21059	.97755	.22778	.97375	9	.24389	.97006	9	.26083	.96618	9	.27768	.96243	9	.29413	.95869
8	.08947	.99599	.10685	.99428	.12592	.98998	.15874	.98747	9	9	.17596	.98441	.19312	.98110	.21099	.97744	.22833	.97364	9	.24417	.97003	9	.26109	.96613	9	.27793	.96238	9	.29424	.95864
9	.08976	.99596	.10714	.99425	.12650	.98990	.15903	.98744	9	10	.17625	.98436	.19343	.98101	.21139	.97733	.22888	.97353	9	.24445	.97000	9	.26135	.96608	9	.27819	.96233	9	.29435	.95859
10	.09005	.99593	.10743	.99422	.12708	.98986	.15932	.98741	9	11	.17654	.98431	.19374	.98092	.21179	.97722	.22943	.97342	9	.24473	.97000	9	.26161	.96603	9	.27845	.96228	9	.29446	.95854
11	.09034	.99590	.10772	.99419	.12766	.98982	.15961	.98738	9	12	.17683	.98426	.19405	.98083	.21219	.97711	.23000	.97331	9	.24501	.97000	9	.26187	.96598	9	.27871	.96223	9	.29457	.95849
12	.09063	.99587	.10801	.99416	.12824	.98978	.15990	.98735	9	13	.17712	.98421	.19436	.98074	.21259	.97700	.23055	.97320	9	.24529	.97000	9	.26213	.96593	9	.27897	.96218	9	.29468	.95844
13	.09092	.99584	.10830	.99413	.12882	.98973	.16019	.98732	9	14	.17741	.98416	.19467	.98065	.21299	.97689	.23110	.97287	9	.24557	.97000	9	.26239	.96588	9	.27923	.96213	9	.29479	.95839
14	.09121	.99581	.10859	.99410	.12940	.98969	.16048	.98729	9	15	.17770	.98411	.19498	.98056	.21339	.97678	.23165	.97276	9	.24585	.97000	9	.26265	.96583	9	.27949	.96208	9	.29490	.95834
15	.09150	.99578	.10888	.99407	.13000	.98965	.16077	.98726	9	16	.17799	.98406	.19529	.98047	.21379	.97667	.23220	.97265	9	.24613	.97000	9	.26291	.96578	9	.27975	.96203	9	.29501	.95829
16	.09179	.99575	.10917	.99404	.13060	.98961	.16106	.98723	9	17	.17828	.98401	.19560	.98038	.21419	.97656	.23275	.97254	9	.24641	.97000	9	.26317	.96573	9	.28001	.96198	9	.29512	.95824
17	.09208	.99572	.10946	.99401	.13120	.98957	.16135	.98720	9	18	.17857	.98396	.19591	.98029	.21459	.97645	.23330	.97243	9	.24669	.97000	9	.26343	.96568	9	.28027	.96193	9	.29523	.95819
18	.09237	.99569	.10975	.99398	.13180	.98953	.16164	.98717	9	19	.17886	.98391	.19622	.98020	.21499	.97634	.23385	.97232	9	.24697	.97000	9	.26369	.96563	9	.28053	.96188	9	.29534	.95814
19	.09266	.99566	.11004	.99395	.13240	.98948	.16193	.98714	9	20	.17915	.98386	.19653	.98011	.21539	.97623	.23440	.97221	9	.24725	.97000	9	.26395	.96558	9	.28079	.96183	9	.29545	.95809
20	.09295	.99563	.11033	.99392	.13300	.98944	.16222	.98711	9	21	.17944	.98381	.19684	.98002	.21579	.97612	.23495	.97210	9	.24753	.97000	9	.26421	.96553	9	.28105	.96178	9	.29556	.95804
21	.09324	.99560	.11062	.99389	.13360	.98940	.16251	.98708	9	22	.17973	.98376	.19715	.97993	.21619	.97601	.23550	.97199	9	.24781	.97000	9	.26447	.96548	9	.28131	.96173	9	.29567	.95799
22	.09353	.99557	.11091	.99386	.13420	.98936	.16280	.98705	9	23	.18002	.98371	.19746	.97984	.21659	.97590	.23605	.97188	9	.24809	.97000	9	.26473	.96543	9	.28157	.96168	9	.29578	.95794
23	.09382	.99554	.11120	.99383	.13480	.98931	.16309	.98702	9	24	.18031	.98366	.19777	.97975	.21699	.97579	.23660	.97177	9	.24837	.97000	9	.26499	.96538	9	.28183	.96163	9	.29589	.95789
24	.09411	.99551	.11149	.99380	.13540	.98927	.16338	.98699	9	25	.18060	.98361	.19808	.97966	.21739	.97568	.23715	.97166	9	.24865	.97000	9	.26525	.96533	9	.28209	.96158	9	.29600	.95784
25	.09440	.99548	.11178	.99377	.13600	.98923	.16367	.98696	9	26	.18089	.98356	.19839	.97957	.21779	.97557	.23770	.97155	9	.24893	.97000	9	.26551	.96528	9	.28235	.96153	9	.29611	.95779
26	.09469	.99545	.11207	.99374	.13660	.98919	.16396	.98693	9	27	.18118	.98351	.19870	.97948	.21819	.97546	.23825	.97144	9	.24921	.97000	9	.26577	.96523	9	.28261	.96148	9	.29622	.95774
27	.09498	.99542	.11236	.99371	.13720	.98916	.16425	.98690	9	28	.18147	.98346	.19901	.97939	.21859	.97535	.23880	.97133	9	.24949	.97000	9	.26603	.96518	9	.28287	.96143	9	.29633	.95769
28	.09527	.99539	.11265	.99368	.13780	.98913	.16454	.98687	9	29	.18176	.98341	.19932	.97930	.21899	.97524	.23935	.97122	9	.24977	.97000	9	.26629	.96513	9	.28313	.96138	9	.29644	.95764
29	.09556	.99536	.11294	.99365	.13840	.98909	.16483	.98684	9	30	.18205	.98336	.19963	.97921	.21939	.97513	.23990	.97111	9	.25005	.97000	9	.26655	.96508	9	.28339	.96133	9	.29655	.95759
30	.09585	.99533	.11323	.99362	.13900	.98905	.16512	.98681	9	31	.18234	.98331	.20025	.97912	.21979	.97502	.24045	.97100	9	.25033	.97000	9	.26681	.96503	9	.28365	.96128	9	.29666	.95754
31	.09614	.99530	.11352	.99359	.13960	.98901	.16541	.98678	9	32	.18263	.98326	.20056	.97903	.22019	.97491	.24100	.97089	9	.25061	.97000	9	.26707	.96498	9	.28391	.96123	9	.29677	.95749
32	.09643	.99527	.11381	.99356	.14020	.98897	.16570	.98675	9	33	.18292	.98321	.20087	.97894	.22059	.97480	.24155	.97078	9	.25089	.97000	9	.26733	.96493	9	.28417	.96118	9	.29688	.95744
33	.09672	.99524	.11410	.99353	.14080	.98893	.16599	.98672	9	34	.18321	.98316	.20118	.97885	.22099	.97469	.24210	.97067	9	.25117	.97000	9	.26759	.96488	9	.28443	.96113	9	.29699	.95739
34	.09701	.99521	.11439	.99350	.14140	.98889	.16628	.98669	9	35	.18350	.98311	.20149	.97876	.22139	.97458	.24265	.97056	9	.25145	.97000	9	.26785	.96483	9	.28469	.96108	9	.29710	.95734
35	.09730	.99518	.11468	.99347	.14200	.98885	.16657	.98666	9	36	.18379	.98306	.20180	.97867	.22179	.97447	.24320	.97045	9	.25173	.97000	9	.26811	.9						

°	18°		19°		20°		21°		22°		23°		24°	
	Shine	Coast	Shine	Coast	Shine	Coast	Shine	Coast	Shine	Coast	Shine	Coast	Shine	Coast
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	34202	93969	35837	93358
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	34229	93959	35864	93348
2	25958	96578	27620	96110	29293	95613	30957	95088	32612	94533	34257	93949	35891	93337
3	26006	96570	27648	96102	29321	95605	30985	95079	32639	94523	34284	93939	35918	93327
4	26054	96562	27676	96094	29348	95596	31012	95070	32667	94514	34311	93929	35945	93317
5	26102	96555	27704	96086	29376	95588	31040	95061	32694	94504	34338	93919	35972	93307
6	26150	96547	27731	96078	29404	95579	31068	95052	32722	94495	34366	93909	36000	93297
7	26198	96540	27759	96070	29432	95571	31095	95043	32749	94485	34393	93899	36027	93287
8	26246	96532	27787	96062	29460	95562	31123	95033	32777	94475	34421	93889	36054	93277
9	26294	96524	27815	96054	29487	95554	31151	95024	32804	94465	34448	93879	36081	93267
10	26342	96516	27843	96046	29515	95545	31178	95015	32832	94455	34475	93869	36108	93257
11	26390	96509	27871	96037	29543	95536	31206	95006	32859	94447	34503	93859	36135	93247
12	26438	96502	27899	96029	29571	95528	31233	94997	32887	94438	34530	93849	36162	93237
13	26486	96494	27927	96021	29599	95519	31261	94988	32914	94428	34557	93839	36189	93227
14	26534	96486	27955	96013	29626	95511	31289	94979	32942	94418	34584	93829	36217	93217
15	26582	96479	27983	96005	29654	95502	31316	94970	32969	94408	34611	93819	36244	93207
16	26630	96471	28011	95997	29682	95493	31344	94961	32997	94399	34638	93809	36271	93197
17	26678	96463	28039	95989	29710	95485	31372	94952	33024	94390	34665	93799	36298	93187
18	26726	96455	28067	95981	29737	95476	31399	94943	33051	94380	34692	93789	36325	93177
19	26774	96448	28095	95972	29765	95467	31427	94933	33079	94370	34719	93779	36352	93167
20	26822	96440	28123	95964	29793	95459	31454	94924	33106	94360	34746	93769	36379	93157
21	26870	96433	28151	95956	29821	95450	31482	94915	33134	94351	34773	93759	36406	93147
22	26918	96425	28178	95948	29849	95441	31510	94906	33161	94342	34800	93749	36433	93137
23	26966	96417	28206	95940	29876	95433	31537	94897	33189	94332	34827	93739	36460	93127
24	27014	96410	28234	95931	29904	95424	31565	94888	33216	94322	34854	93729	36487	93117
25	27062	96402	28262	95923	29932	95415	31593	94879	33244	94312	34881	93719	36514	93107
26	27110	96394	28290	95915	29960	95407	31620	94869	33271	94302	34908	93709	36541	93097
27	27158	96386	28318	95907	29987	95398	31648	94860	33299	94292	34935	93699	36568	93087
28	27206	96378	28346	95898	30015	95389	31675	94851	33326	94282	34962	93689	36595	93077
29	27254	96370	28374	95890	30043	95380	31703	94842	33353	94272	34989	93679	36622	93067
30	27302	96362	28402	95882	30071	95372	31730	94832	33381	94262	35016	93669	36649	93057
31	27350	96354	28429	95874	30098	95363	31758	94823	33408	94252	35043	93659	36676	93047
32	27398	96346	28457	95865	30126	95354	31786	94814	33436	94242	35070	93649	36703	93037
33	27446	96338	28485	95857	30154	95345	31813	94805	33463	94232	35097	93639	36730	93027
34	27494	96330	28513	95849	30182	95337	31841	94796	33490	94222	35124	93629	36757	93017
35	27542	96322	28541	95841	30209	95328	31868	94786	33518	94212	35151	93619	36784	93007
36	27590	96314	28569	95832	30237	95319	31896	94777	33545	94202	35178	93609	36811	92997
37	27638	96306	28597	95824	30265	95310	31923	94768	33573	94192	35205	93599	36838	92987
38	27686	96298	28625	95816	30292	95301	31951	94758	33600	94182	35232	93589	36865	92977
39	27734	96290	28653	95807	30320	95292	31979	94749	33627	94172	35259	93579	36892	92967
40	27782	96282	28680	95799	30348	95284	32006	94740	33655	94162	35286	93569	36919	92957
41	27830	96274	28708	95791	30376	95275	32034	94730	33682	94152	35313	93559	36946	92947
42	27878	96266	28736	95782	30403	95266	32061	94721	33710	94142	35340	93549	36973	92937
43	27926	96258	28764	95774	30431	95257	32089	94712	33737	94132	35367	93539	36999	92927
44	27974	96250	28792	95766	30459	95248	32116	94702	33764	94122	35394	93529	37026	92917
45	28022	96242	28820	95757	30486	95240	32144	94693	33791	94112	35421	93519	37053	92907
46	28070	96234	28847	95749	30514	95231	32171	94684	33818	94102	35448	93509	37080	92897
47	28118	96226	28875	95740	30542	95222	32199	94674	33845	94092	35475	93499	37107	92887
48	28166	96218	28903	95732	30570	95213	32227	94665	33872	94082	35502	93489	37134	92877
49	28214	96210	28931	95724	30597	95204	32254	94656	33899	94072	35529	93479	37161	92867
50	28262	96202	28959	95715	30625	95195	32282	94646	33926	94062	35556	93469	37188	92857
51	28310	96194	28987	95707	30653	95186	32309	94637	33953	94052	35583	93459	37215	92847
52	28358	96186	29015	95698	30680	95177	32337	94627	33980	94042	35610	93449	37242	92837
53	28406	96178	29043	95690	30708	95168	32364	94618	34008	94032	35637	93439	37269	92827
54	28454	96170	29070	95681	30736	95159	32392	94609	34035	94022	35664	93429	37296	92817
55	28502	96162	29098	95673	30764	95150	32419	94600	34062	94012	35691	93419	37323	92807
56	28550	96154	29126	95664	30791	95142	32447	94591	34089	94002	35718	93409	37350	92797
57	28598	96146	29154	95656	30819	95133	32474	94582	34116	93992	35745	93399	37377	92787
58	28646	96138	29182	95647	30846	95124	32502	94573	34143	93982	35772	93389	37404	92777
59	28694	96130	29210	95639	30874	95115	32529	94564	34170	93972	35799	93379	37431	92767
60	28742	96122	29237	95630	30902	95106	32557	94555	34202	93962	35826	93369	37458	92757

TMO-4-181

TMO-4-182

°	25°		26°		27°		28°		29°		30°		31°		32°		33°		34°		°
	Line	Coast	Line	Coast	Line	Coast	Line	Coast	Line	Coast	Line	Coast	Line	Coast	Line	Coast	Line	Coast	Line	Coast	
0	41262	90631	43837	89879	45390	89101	46947	88291	48481	87462	49900	86603	51504	85717	53092	84805	54464	83867	55919	82904	40
1	41268	90618	43863	89867	45425	89074	46973	88285	48506	87448	49925	86588	51530	85702	53117	84789	54488	83851	55943	82887	41
2	41273	90606	43889	89854	45451	89059	47000	88267	48532	87430	49949	86573	51554	85687	53141	84774	54513	83835	55968	82871	42
3	41279	90594	43916	89841	45477	89061	47024	88254	48557	87420	50006	86559	51579	85672	53166	84759	54537	83819	55992	82855	43
4	41284	90582	43942	89828	45503	89048	47050	88246	48583	87406	50031	86544	51604	85657	53193	84743	54561	83804	56016	82839	44
5	41290	90569	43969	89816	45529	89035	47076	88230	48608	87391	50058	86530	51635	85642	53221	84730	54586	83788	56040	82822	45
6	41296	90557	43994	89803	45554	89021	47101	88213	48634	87377	50080	86515	51663	85627	53248	84717	54610	83772	56064	82806	46
7	41302	90545	44020	89790	45580	89008	47127	88199	48659	87363	50102	86496	51691	85618	53276	84704	54635	83756	56088	82790	47
8	41308	90532	44046	89777	45606	88995	47153	88185	48684	87349	50128	86481	51718	85597	53303	84691	54659	83740	56112	82773	48
9	41314	90520	44072	89764	45632	88981	47178	88178	48710	87335	50153	86467	51748	85582	53329	84680	54683	83724	56136	82757	49
10	41320	90507	44098	89752	45658	88968	47204	88158	48735	87321	50179	86457	51778	85567	53356	84666	54708	83708	56160	82741	50
11	41326	90495	44124	89739	45684	88955	47229	88144	48761	87306	50204	86442	51778	85551	53382	84655	54732	83692	56184	82724	51
12	41332	90483	44151	89726	45710	88942	47255	88130	48786	87292	50229	86427	51803	85536	53408	84644	54756	83676	56208	82708	52
13	41338	90471	44177	89713	45736	88928	47281	88117	48811	87278	50254	86413	51828	85521	53434	84632	54781	83658	56232	82692	53
14	41344	90459	44203	89700	45762	88915	47306	88103	48837	87264	50279	86400	51853	85506	53460	84620	54805	83645	56256	82678	54
15	41350	90447	44229	89687	45787	88902	47332	88090	48862	87250	50304	86387	51879	85491	53486	84608	54829	83632	56280	82664	55
16	41356	90435	44255	89674	45813	88888	47358	88078	48888	87238	50329	86374	51902	85476	53512	84596	54854	83619	56304	82650	56
17	41362	90423	44281	89662	45839	88875	47383	88062	48913	87221	50354	86361	51927	85461	53538	84584	54878	83606	56328	82636	57
18	41368	90411	44307	89649	45865	88862	47409	88048	48938	87207	50379	86350	51952	85446	53564	84572	54902	83592	56352	82622	58
19	41374	90399	44333	89636	45891	88848	47434	88034	48964	87193	50404	86339	51977	85431	53590	84560	54927	83579	56376	82608	59
20	41380	90387	44359	89623	45917	88835	47460	88020	48989	87178	50429	86328	52002	85416	53616	84548	54951	83565	56400	82594	60
21	41386	90375	44385	89610	45942	88822	47486	88006	49014	87164	50454	86317	52026	85401	53642	84536	54975	83553	56424	82580	61
22	41392	90363	44411	89597	45968	88808	47511	87993	49040	87150	50479	86306	52051	85385	53668	84524	54999	83541	56448	82566	62
23	41398	90351	44437	89584	45994	88795	47537	87979	49065	87136	50504	86295	52076	85370	53694	84512	55024	83529	56472	82552	63
24	41404	90339	44463	89571	46020	88782	47562	87965	49090	87125	50529	86284	52101	85355	53720	84500	55048	83517	56496	82538	64
25	41410	90327	44489	89558	46046	88768	47588	87951	49116	87110	50554	86273	52126	85339	53746	84488	55072	83504	56520	82524	65
26	41416	90315	44515	89545	46072	88755	47614	87937	49141	87093	50579	86262	52151	85323	53772	84476	55097	83492	56544	82510	66
27	41422	90303	44541	89532	46097	88742	47639	87923	49166	87079	50604	86251	52176	85310	53798	84464	55121	83480	56568	82496	67
28	41428	90291	44567	89519	46123	88728	47665	87909	49191	87064	50629	86240	52201	85294	53824	84452	55145	83468	56592	82482	68
29	41434	90279	44593	89506	46149	88715	47690	87896	49217	87050	50654	86229	52226	85282	53850	84440	55169	83456	56616	82468	69
30	41440	90267	44619	89493	46175	88701	47716	87882	49242	87036	50679	86218	52251	85269	53876	84432	55193	83444	56640	82454	70
31	41446	90255	44645	89480	46201	88688	47741	87868	49268	87021	50704	86207	52276	85257	53902	84420	55217	83432	56664	82440	71
32	41452	90243	44671	89467	46226	88674	47767	87854	49293	87007	50729	86196	52301	85248	53928	84408	55241	83420	56688	82426	72
33	41458	90231	44697	89454	46252	88661	47793	87840	49318	86993	50754	86185	52326	85234	53954	84396	55265	83408	56712	82412	73
34	41464	90219	44723	89441	46278	88647	47818	87826	49343	86978	50779	86174	52351	85221	53980	84384	55289	83396	56736	82398	74
35	41470	90207	44749	89428	46304	88634	47844	87812	49368	86964	50804	86163	52376	85208	54006	84372	55313	83384	56760	82384	75
36	41476	90195	44775	89415	46330	88620	47869	87798	49394	86950	50829	86152	52401	85195	54032	84360	55337	83372	56784	82370	76
37	41482	90183	44801	89402	46356	88607	47895	87784	49419	86935	50854	86141	52426	85183	54058	84348	55361	83360	56808	82356	77
38	41488	90171	44827	89389	46381	88593	47920	87770	49444	86921	50879	86130	52451	85174	54084	84336	55385	83348	56832	82342	78
39	41494	90159	44853	89376	46407	88580	47946	87756	49470	86906	50904	86119	52476	85165	54110	84324	55409	83336	56856	82328	79
40	41500	90147	44879	89363	46433	88566	47971	87743	49495	86892	50929	86108	52501	85156	54136	84312	55433	83324	56880	82314	80
41	41506	90135	44905	89350	46458	88553	47997	87729	49521	86878	50954	86097	52526	85147	54162	84300	55457	83312	56904	82300	81
42	41512	90123	44931	89337	46484	88539	48022	87715	49546	86863	50979	86086	52551	85138	54188	84288	55481	83300	56928	82286	82
43	41518	90111	44957	89324	46510	88526	48048	87701	49571	86849	51004	86075	52576	85129	54214	84276	55505	83288	56952	82272	83
44	41524	90099	44983	89311	46536	88512	48073	87687	49596	86834	51029	86064	52601	85120	54240	84264	55529	83276	56976	82258	84
45	41530	90087	45009	89298	46561	88499	48099	87673	49622	86820	51054	86053	52626	85111	54266	84252	55553	83264	57000	82244	85
46	41536	90075	45035	89285	46587	88485	48124	87659	49647	86805	51079	86042	52651	85102	54292	84240	55577	83252	57024	82230	86
47	41542	90063	45061	89272	46613	88472	48149	87645	49672	86791	51104	86031	52676	85093	54318	84228	55601	83240	57048	82216	87
48	41548	90051	45087	89259	46639	88458	48175	87631	49697	86777	51129	86020	52701	85084	54344	84216	55625	83228	57072	82202	88
49	41554	90039	45113	89246	46665	88445	48201	87617	49723	86762	51154	86009	52726	85075	54370	84204	55649	83216	57096	82188	89
50	41560	90027	45139	89233	46690	88431	48226	87603	49748	86748	51179	86000	52751	85066	54396	84192	55673	83204	57120	82174	90
51	41566	90015	45165	89219	46716	88417	48252	87589	49773	86733	51204	85985	52776	85057	54422	84180	55697	83192	57144	82160	91
52	41572	90003	45191	89206	46742	88404	48277	87575	49798	86719	51229	85974	52801	85048	54448	84168	55721	83180	57168	82146	92
53	41578	89991	45217	89193	46767	88390	48303	87561	49824	86704	51254	85963	52826	85039	54474	84156	55745	83168	57192	82132	93
54	41584</																				

°	34°		35°		36°		37°		38°		39°		40°		41°		42°		43°		44°	
	Sho	Cosin	Sho	Cosin	Sho	Cosin	Sho	Cosin	Sho	Cosin	Sho	Cosin	Sho	Cosin	Sho	Cosin	Sho	Cosin	Sho	Cosin	Sho	Cosin
0	57358	81915	58779	80902	60182	78664	61566	78801	62932	77715	60	77715	62932	77715	60182	78664	61566	78801	62932	77715	60182	78664
1	57361	81899	58782	80885	60205	78646	61549	78783	62915	77696	59	77696	62915	77696	60205	78646	61549	78783	62915	77696	60205	78646
2	57345	81882	58765	80868	60228	78629	61612	78765	62907	77680	58	77680	62907	77680	60228	78629	61612	78765	62907	77680	60228	78629
3	57329	81865	58749	80850	60251	78611	61635	78747	62897	77663	57	77663	62897	77663	60251	78611	61635	78747	62897	77663	60251	78611
4	57313	81848	58733	80833	60274	78593	61658	78729	62887	77645	56	77645	62887	77645	60274	78593	61658	78729	62887	77645	60274	78593
5	57297	81832	58717	80816	60298	78576	61681	78715	62878	77627	55	77627	62878	77627	60298	78576	61681	78715	62878	77627	60298	78576
6	57281	81815	58701	80799	60321	78559	61704	78694	62869	77609	54	77609	62869	77609	60321	78559	61704	78694	62869	77609	60321	78559
7	57265	81798	58685	80782	60344	78542	61728	78676	62860	77591	53	77591	62860	77591	60344	78542	61728	78676	62860	77591	60344	78542
8	57249	81782	58669	80765	60367	78525	61751	78658	62851	77575	52	77575	62851	77575	60367	78525	61751	78658	62851	77575	60367	78525
9	57233	81765	58653	80748	60390	78508	61774	78640	62842	77559	51	77559	62842	77559	60390	78508	61774	78640	62842	77559	60390	78508
10	57217	81748	58637	80731	60414	78491	61793	78622	62833	77542	50	77542	62833	77542	60414	78491	61793	78622	62833	77542	60414	78491
11	57201	81731	58621	80714	60437	78474	61816	78604	62824	77526	49	77526	62824	77526	60437	78474	61816	78604	62824	77526	60437	78474
12	57185	81714	58605	80697	60460	78457	61841	78586	62815	77510	48	77510	62815	77510	60460	78457	61841	78586	62815	77510	60460	78457
13	57169	81697	58589	80680	60483	78440	61866	78568	62806	77494	47	77494	62806	77494	60483	78440	61866	78568	62806	77494	60483	78440
14	57153	81680	58573	80663	60506	78423	61891	78550	62797	77478	46	77478	62797	77478	60506	78423	61891	78550	62797	77478	60506	78423
15	57137	81663	58557	80646	60529	78406	61916	78532	62788	77462	45	77462	62788	77462	60529	78406	61916	78532	62788	77462	60529	78406
16	57121	81646	58541	80629	60552	78389	61941	78514	62779	77446	44	77446	62779	77446	60552	78389	61941	78514	62779	77446	60552	78389
17	57105	81629	58525	80612	60575	78372	61966	78496	62770	77430	43	77430	62770	77430	60575	78372	61966	78496	62770	77430	60575	78372
18	57089	81612	58509	80595	60598	78355	61991	78478	62761	77414	42	77414	62761	77414	60598	78355	61991	78478	62761	77414	60598	78355
19	57073	81595	58493	80578	60622	78338	62016	78460	62752	77398	41	77398	62752	77398	60622	78338	62016	78460	62752	77398	60622	78338
20	57057	81578	58477	80561	60645	78321	62041	78442	62743	77382	40	77382	62743	77382	60645	78321	62041	78442	62743	77382	60645	78321
21	57041	81561	58461	80544	60668	78304	62066	78424	62734	77366	39	77366	62734	77366	60668	78304	62066	78424	62734	77366	60668	78304
22	57025	81544	58445	80527	60691	78287	62091	78406	62725	77350	38	77350	62725	77350	60691	78287	62091	78406	62725	77350	60691	78287
23	57009	81527	58429	80510	60714	78270	62116	78388	62716	77334	37	77334	62716	77334	60714	78270	62116	78388	62716	77334	60714	78270
24	57000	81510	58412	80493	60737	78253	62141	78370	62707	77318	36	77318	62707	77318	60737	78253	62141	78370	62707	77318	60737	78253
25	56984	81493	58396	80476	60760	78236	62166	78352	62698	77302	35	77302	62698	77302	60760	78236	62166	78352	62698	77302	60760	78236
26	56968	81476	58380	80459	60783	78219	62191	78334	62689	77286	34	77286	62689	77286	60783	78219	62191	78334	62689	77286	60783	78219
27	56952	81459	58364	80442	60806	78202	62216	78316	62680	77270	33	77270	62680	77270	60806	78202	62216	78316	62680	77270	60806	78202
28	56936	81442	58348	80425	60829	78185	62241	78298	62671	77254	32	77254	62671	77254	60829	78185	62241	78298	62671	77254	60829	78185
29	56920	81425	58332	80408	60852	78168	62266	78280	62662	77238	31	77238	62662	77238	60852	78168	62266	78280	62662	77238	60852	78168
30	56904	81408	58316	80391	60875	78151	62291	78262	62653	77222	30	77222	62653	77222	60875	78151	62291	78262	62653	77222	60875	78151
31	56888	81391	58300	80374	60898	78134	62316	78244	62644	77206	29	77206	62644	77206	60898	78134	62316	78244	62644	77206	60898	78134
32	56872	81374	58284	80357	60921	78117	62341	78226	62635	77190	28	77190	62635	77190	60921	78117	62341	78226	62635	77190	60921	78117
33	56856	81357	58268	80340	60944	78100	62366	78208	62626	77174	27	77174	62626	77174	60944	78100	62366	78208	62626	77174	60944	78100
34	56840	81340	58252	80323	60967	78083	62391	78190	62617	77158	26	77158	62617	77158	60967	78083	62391	78190	62617	77158	60967	78083
35	56824	81323	58236	80306	60990	78066	62416	78172	62608	77142	25	77142	62608	77142	60990	78066	62416	78172	62608	77142	60990	78066
36	56808	81306	58220	80289	61013	78049	62441	78154	62599	77126	24	77126	62599	77126	61013	78049	62441	78154	62599	77126	61013	78049
37	56792	81289	58204	80272	61036	78032	62466	78136	62590	77110	23	77110	62590	77110	61036	78032	62466	78136	62590	77110	61036	78032
38	56776	81272	58188	80255	61059	78015	62491	78118	62581	77094	22	77094	62581	77094	61059	78015	62491	78118	62581	77094	61059	78015
39	56760	81255	58172	80238	61082	78000	62516	78100	62572	77078	21	77078	62572	77078	61082	78000	62516	78100	62572	77078	61082	78000
40	56744	81238	58156	80221	61105	77983	62541	78082	62563	77062	20	77062	62563	77062	61105	77983	62541	78082	62563	77062	61105	77983
41	56728	81221	58140	80204	61128	77966	62566	78064	62554	77046	19	77046	62554	77046	61128	77966	62566	78064	62554	77046	61128	77966
42	56712	81204	58124	80187	61151	77949	62591	78046	62545	77030	18	77030	62545	77030	61151	77949	62591	78046	62545	77030	61151	77949
43	56696	81187	58108	80170	61174	77932	62616	78028	62536	77014	17	77014	62536	77014	61174	77932	62616	78028	62536	77014	61174	77932
44	56680	81170	58092	80153	61197	77915	62641	78010	62527	77000	16	77000	62527	77000	61197	77915	62641	78010	62527	77000	61197	77915
45	56664	81153	58076	80136	61220	77898	62666	77992	62518	76984	15	76984	62518	76984	61220	77898	62666	77992	62518	76984	61220	77898
46	56648	81136	58060	80119	61243	77881	62691	77974	62509	76968	14	76968	62509	76968	61243	77881	62691	77974	62509	76968	61243	77881
47	56632	81119	58044	80102	61266	77864	62716	77956	62500	76952	13	76952	62500	76952	61266	77864	62716	77956	62500	76952	61266	77864
48	56616	81102	58028	80085	61289	77847	62741	77938	62491	76936	12	76936	62491	76936	61289	77847	62741	77938	62491	76936	61289	77847
49	56600	81085	58012	80068	61312	77829	62766	77920	62482	76920	11	76920	62482	76920	61312	77829	62766	77920	62482	76920	61312	77829
50	56584	81068	57996	80051	61335	77812	62791	77902	62473	76904	10	76904	62473	76904	61335	77812	62791	77902	62473	76904	61335	77812
51	56568	81051	57980	80034	61358	77795	62816	77884	62464	76888	9	76888	62464									

		0°		1°		2°		3°		4°		5°		6°		7°	
	Temp	Celcius	F	Temp	Celcius	F	Temp	Celcius	F	Temp	Celcius	F	Temp	Celcius	F	Temp	Celcius
0	0.0000	Infinit.	32.0000	0.0000	32.0000	32.0000	0.0000	32.0000	32.0000	0.0000	32.0000	32.0000	0.0000	32.0000	32.0000	0.0000	32.0000
1	0.0001	34.3775	93.8815	0.0002	34.7550	94.5590	0.0003	35.1325	95.2365	0.0004	35.5100	95.9140	0.0005	35.8875	96.5915	0.0006	36.2650
2	0.0007	35.7550	96.3590	0.0008	36.1325	97.0365	0.0009	36.5100	97.7140	0.0010	36.8875	98.3915	0.0011	37.2650	99.0690	0.0012	37.6425
3	0.0013	37.6425	100.0000	0.0014	38.0200	100.4360	0.0015	38.3975	100.9120	0.0016	38.7750	101.3880	0.0017	39.1525	101.8700	0.0018	39.5300
4	0.0019	39.5300	103.1425	0.0020	39.9075	103.8200	0.0021	40.2850	104.5075	0.0022	40.6625	105.1950	0.0023	41.0400	105.8725	0.0024	41.4175
5	0.0025	41.4175	106.5525	0.0026	41.7950	107.2300	0.0027	42.1725	107.9075	0.0028	42.5500	108.5850	0.0029	42.9275	109.2625	0.0030	43.3050
6	0.0031	43.3050	110.0000	0.0032	43.6825	110.6775	0.0033	44.0600	111.3550	0.0034	44.4375	112.0325	0.0035	44.8150	112.7100	0.0036	45.1925
7	0.0037	45.1925	113.5475	0.0038	45.5700	114.2250	0.0039	45.9475	114.9025	0.0040	46.3250	115.5800	0.0041	46.7025	116.2575	0.0042	47.0800
8	0.0043	47.0800	116.9425	0.0044	47.4575	117.6200	0.0045	47.8350	118.2975	0.0046	48.2125	118.9750	0.0047	48.5900	119.6525	0.0048	49.0675
9	0.0049	49.0675	120.5200	0.0050	49.4450	121.1975	0.0051	49.8225	121.8750	0.0052	50.2000	122.5525	0.0053	50.5775	123.2300	0.0054	50.9550
10	0.0055	50.9550	124.1325	0.0056	51.3325	124.8100	0.0057	51.7100	125.4875	0.0058	52.0875	126.1650	0.0059	52.4650	126.8425	0.0060	52.8425
11	0.0061	52.8425	127.5200	0.0062	53.2200	128.1975	0.0063	53.5975	128.8750	0.0064	53.9750	129.5525	0.0065	54.3525	130.2300	0.0066	54.7300
12	0.0067	54.7300	130.9025	0.0068	55.1075	131.5800	0.0069	55.4850	132.2575	0.0070	55.8625	132.9350	0.0071	56.2400	133.6125	0.0072	56.6175
13	0.0073	56.6175	134.2900	0.0074	57.0200	134.9425	0.0075	57.4025	135.6200	0.0076	57.7800	136.2975	0.0077	58.1575	136.9750	0.0078	58.5350
14	0.0079	58.5350	137.6525	0.0080	58.9125	138.3300	0.0081	59.2900	139.0075	0.0082	59.6675	139.6850	0.0083	60.0450	140.3625	0.0084	60.4225
15	0.0085	60.4225	141.0500	0.0086	60.8000	141.7275	0.0087	61.1775	142.4050	0.0088	61.5550	143.0825	0.0089	61.9325	143.7600	0.0090	62.3375
16	0.0091	62.3375	144.4325	0.0092	62.7150	145.1100	0.0093	63.0925	145.7875	0.0094	63.4700	146.4650	0.0095	63.8475	147.1425	0.0096	64.2250
17	0.0097	64.2250	147.8200	0.0098	64.6025	148.4975	0.0099	64.9800	149.1750	0.0100	65.3575	149.8525	0.0101	65.7350	150.5300	0.0102	65.9125
18	0.0103	65.9125	151.2075	0.0104	66.2900	151.8850	0.0105	66.6675	152.5625	0.0106	67.0450	153.2400	0.0107	67.4225	153.9175	0.0108	67.8000
19	0.0109	67.8000	154.5800	0.0110	68.1775	155.2575	0.0111	68.5550	155.9350	0.0112	68.9325	156.6125	0.0113	69.3100	157.2900	0.0114	69.6875
20	0.0115	69.6875	157.9625	0.0116	70.0650	158.6400	0.0117	70.4425	159.3175	0.0118	70.8200	160.0000	0.0119	71.1975	160.6825	0.0120	71.5700
21	0.0121	71.5700	161.3575	0.0122	71.9475	162.0350	0.0123	72.3250	162.7125	0.0124	72.7025	163.3900	0.0125	73.0800	164.0675	0.0126	73.4550
22	0.0127	73.4550	164.7425	0.0128	73.8325	165.4200	0.0129	74.2100	166.0975	0.0130	74.5875	166.7750	0.0131	74.9650	167.4525	0.0132	75.3400
23	0.0133	75.3400	168.1200	0.0134	75.7175	168.8000	0.0135	76.0950	169.4775	0.0136	76.4725	170.1550	0.0137	76.8500	170.8325	0.0138	77.2275
24	0.0139	77.2275	171.5100	0.0140	77.6050	172.1875	0.0141	77.9825	172.8650	0.0142	78.3600	173.5425	0.0143	78.7375	174.2200	0.0144	79.1050
25	0.0145	79.1050	174.9025	0.0146	79.4825	175.5800	0.0147	79.8600	176.2575	0.0148	80.2375	176.9350	0.0149	80.6150	177.6125	0.0150	80.9900
26	0.0151	80.9900	178.2900	0.0152	81.3675	178.9675	0.0153	81.7450	179.6450	0.0154	82.1225	180.3225	0.0155	82.5000	181.0000	0.0156	82.3775
27	0.0157	82.3775	181.6775	0.0158	82.7550	182.3550	0.0159	83.1325	183.0325	0.0160	83.5100	183.7100	0.0161	83.8875	184.3875	0.0162	84.2650
28	0.0163	84.2650	185.0625	0.0164	84.6425	185.7400	0.0165	85.0200	186.4175	0.0166	85.3975	187.0950	0.0167	85.7750	187.7725	0.0168	86.1525
29	0.0169	86.1525	188.4500	0.0170	86.5300	189.1275	0.0171	86.9075	189.8050	0.0172	87.2850	190.4825	0.0173	87.6625	191.1600	0.0174	87.5375
30	0.0175	87.5375	191.8375	0.0176	87.9150	192.5150	0.0177	88.2925	193.1925	0.0178	88.6700	193.8700	0.0179	89.0475	194.5475	0.0180	89.4250
31	0.0181	89.4250	195.2200	0.0182	89.8025	195.9000	0.0183	90.1800	196.5775	0.0184	90.5575	197.2550	0.0185	90.9350	197.9325	0.0186	91.3100
32	0.0187	91.3100	198.6100	0.0188	91.6875	199.2875	0.0189	92.0650	199.9650	0.0190	92.4425	200.6425	0.0191	92.8200	201.3200	0.0192	93.1975
33	0.0193	93.1975	202.0000	0.0194	93.5750	202.6775	0.0195	93.9525	203.3550	0.0196	94.3300	204.0325	0.0197	94.7075	204.7100	0.0198	95.0850
34	0.0199	95.0850	205.3875	0.0200	95.4625	206.0650	0.0201	95.8400	206.7425	0.0202	96.2175	207.4200	0.0203	96.5950	208.0975	0.0204	96.9725
35	0.0205	96.9725	208.7750	0.0206	97.3500	209.4525	0.0207	97.7275	210.1300	0.0208	98.1050	210.8075	0.0209	98.4825	211.4850	0.0210	98.8600
36	0.0211	98.8600	212.1625	0.0212	99.2375	212.8400	0.0213	99.6150	213.5175	0.0214	99.9925	214.1950	0.0215	100.3700	214.8725	0.0216	100.7475
37	0.0217	100.7475	215.5500	0.0218	101.1250	216.2275	0.0219	101.5025	216.9050	0.0220	101.8800	217.5825	0.0221	102.2575	218.2600	0.0222	102.6375
38	0.0223	102.6375	218.9375	0.0224	103.0150	219.6150	0.0225	103.3925	220.2925	0.0226	103.7700	220.9700	0.0227	104.1475	221.6475	0.0228	104.5250
39	0.0229	104.5250	222.3200	0.0230	104.9025	223.0000	0.0231	105.2800	223.6775	0.0232	105.6575	224.3550	0.0233	106.0350	225.0325	0.0234	106.4125
40	0.0235	106.4125	225.7100	0.0236	106.7900	226.3875	0.0237	107.1675	227.0650	0.0238	107.5450	227.7425	0.0239	107.9225	228.4200	0.0240	108.7975
41	0.0241	108.7975	229.1000	0.0242	109.1750	229.7800	0.0243	109.5525	230.4575	0.0244	109.9300	231.1350	0.0245	110.3075	231.8125	0.0246	110.6900
42	0.0247	110.6900	232.4900	0.0248	111.0675	233.1675	0.0249	111.4450	233.8450	0.0250	111.8225	234.5225	0.0251	112.2000	235.2000	0.0252	112.5775
43	0.0253	112.5775	235.8775	0.0254	112.9550	236.5550	0.0255	113.3325	237.2325	0.0256	113.7100	237.9100	0.0257	114.0875	238.5875	0.0258	114.4650
44	0.0259	114.4650	239.2625	0.0260	114.8425	240.0000	0.0261	115.2200	240.6775	0.0262	115.5975	241.3550	0.0263	115.9750	242.0325	0.0264	116.4100
45	0.0265	116.4100	242.7100	0.0266	116.7875	243.3875	0.0267	117.1650	244.0650	0.0268	117.5425	244.7425	0.0269	117.9200	245.4200	0.0270	118.2975
46	0.0271	118.2975	246.1000	0.0272	118.6750	246.7800	0.0273	119.0525	247.4575	0.0274	119.4300	248.1350	0.0275	119.8075	248.8125	0.0276	120.1900
47	0.0277	120.1900	249.4875	0.0278	120.5675	250.1650	0.0279	120.9450	250.8425	0.0280	121.3225	251.5200	0.0281	121.7000	252.1975	0.0282	122.5725
48	0.0283	122.5725	252.8700	0.0284	122.9500	253.5500	0.0285	123.3275	254.2275	0.0286	123.7050	254.9050	0.0287	124.0825	255.5825	0.0288	124.9575
49	0.0289	124.9575	256.2600	0.0290	125.3350	256.9400	0.0291	125.7125	257.6175	0.0292	126.0900	258.2950	0.0293	126.4675	258.9725	0.0294	127.3475
50	0.0295	127.3475	259.6625	0.0296	127.7250	260.3400	0.0297	128.1025	261.0175	0.0298	128.4800	261.6950	0.0299	128.8575	262.3725	0.0300	129.2500
51	0.0301	129.2500	263.0500	0.0302	129.6275	263.7275	0.0303	130.0050	264.4050	0.0304	130.3825	265.0825	0.0305	130.7600	265.7600	0.0306	131.1375
52	0.0307	131.1375	266.4375	0.0308	131.5150	267.1050	0.0309	131.8925	267.7825	0.0310	132.2700	268.4600	0.0311	132.6475	269.1375	0.0312	132.5225
53	0.0313	132.5225	269.8100	0.0314	132.9000	270.4900	0.0315	133.2775	271.1675	0.0316	133.6550	271.8450	0.0317	134.0325	272.5225	0.0318	134.9000
54	0.0319	134.9000	273.2100	0.0320	135.2775	273.8900	0.0321	135.6550	274.5675	0.0322	136.0325	275.2450	0.0323	136.4100	275.9225	0.0324	137.7900
55	0.0325	137.7900	276.6125	0.0326	138.1675	277.2900	0.0327	138.5450									

°	8°		9°		10°		11°		12°		13°		14°		15°	
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang
0	1.4034	7.11537	1.5638	6.31375	1.7633	5.67128	1.9438	5.14455	2.1256	4.70463	2.3087	4.33148	2.4933	4.01078	2.6795	3.73205
1	1.4084	7.10038	1.5688	6.30189	1.7683	5.66165	1.9488	5.13658	2.1286	4.69791	2.3117	4.32701	2.4964	4.00582	2.6826	3.72771
2	1.4133	7.08546	1.5748	6.29077	1.7733	5.64825	1.9538	5.12862	2.1316	4.69121	2.3148	4.32252	2.5014	4.00086	2.6857	3.72338
3	1.4183	7.07059	1.5808	6.28029	1.7783	5.63498	1.9588	5.12069	2.1347	4.68452	2.3179	4.31803	2.5062	3.99592	2.6888	3.71907
4	1.4232	7.05579	1.5868	6.27000	1.7833	5.62176	1.9638	5.11279	2.1377	4.67786	2.3209	4.31354	2.5110	3.99099	2.6920	3.71476
5	1.4282	7.04105	1.5928	6.26000	1.7883	5.60884	1.9688	5.10490	2.1408	4.67121	2.3240	4.30905	2.5158	3.98607	2.6951	3.71046
6	1.4332	7.02637	1.6000	6.25000	1.7933	5.59598	1.9738	5.09704	2.1438	4.66458	2.3271	4.30456	2.5206	3.98117	2.6982	3.70616
7	1.4382	7.01174	1.6072	6.24000	1.7983	5.58312	1.9788	5.08919	2.1469	4.65792	2.3301	4.29999	2.5254	3.97627	2.7013	3.70186
8	1.4431	6.99718	1.6144	6.23000	1.8033	5.57026	1.9838	5.08139	2.1499	4.65126	2.3332	4.29550	2.5302	3.97139	2.7044	3.69756
9	1.4481	6.98261	1.6216	6.22000	1.8083	5.55740	1.9888	5.07360	2.1529	4.64460	2.3363	4.29101	2.5350	3.96651	2.7075	3.69325
10	1.4531	6.96805	1.6288	6.21000	1.8133	5.54454	1.9938	5.06580	2.1560	4.63794	2.3394	4.28652	2.5398	3.96165	2.7106	3.68894
11	1.4581	6.95349	1.6360	6.20000	1.8183	5.53168	1.9988	5.05809	2.1590	4.63128	2.3425	4.28203	2.5446	3.95680	2.7137	3.68463
12	1.4631	6.93893	1.6432	6.19000	1.8233	5.51882	2.0038	5.05037	2.1621	4.62462	2.3456	4.27754	2.5494	3.95196	2.7168	3.68032
13	1.4681	6.92437	1.6504	6.18000	1.8283	5.50596	2.0088	5.04266	2.1651	4.61796	2.3487	4.27305	2.5542	3.94713	2.7199	3.67601
14	1.4731	6.90981	1.6576	6.17000	1.8333	5.49310	2.0138	5.03495	2.1682	4.61130	2.3518	4.26856	2.5590	3.94230	2.7230	3.67170
15	1.4781	6.89525	1.6648	6.16000	1.8383	5.48024	2.0188	5.02724	2.1712	4.60464	2.3549	4.26407	2.5638	3.93747	2.7261	3.66739
16	1.4831	6.88069	1.6720	6.15000	1.8433	5.46738	2.0238	5.01953	2.1743	4.59798	2.3580	4.25958	2.5686	3.93264	2.7292	3.66308
17	1.4881	6.86613	1.6792	6.14000	1.8483	5.45452	2.0288	5.01182	2.1774	4.59132	2.3611	4.25509	2.5734	3.92781	2.7323	3.65877
18	1.4931	6.85157	1.6864	6.13000	1.8533	5.44166	2.0338	5.00411	2.1804	4.58466	2.3642	4.25060	2.5782	3.92298	2.7354	3.65446
19	1.4981	6.83701	1.6936	6.12000	1.8583	5.42880	2.0388	4.99640	2.1835	4.57800	2.3673	4.24611	2.5830	3.91815	2.7385	3.65015
20	1.5031	6.82245	1.7008	6.11000	1.8633	5.41594	2.0438	4.98869	2.1866	4.57134	2.3704	4.24162	2.5878	3.91332	2.7416	3.64584
21	1.5081	6.80789	1.7080	6.10000	1.8683	5.40308	2.0488	4.98098	2.1897	4.56468	2.3735	4.23713	2.5926	3.90849	2.7447	3.64153
22	1.5131	6.79333	1.7152	6.09000	1.8733	5.39022	2.0538	4.97327	2.1928	4.55802	2.3766	4.23264	2.5974	3.90366	2.7478	3.63722
23	1.5181	6.77877	1.7224	6.08000	1.8783	5.37736	2.0588	4.96556	2.1959	4.55136	2.3797	4.22815	2.6022	3.89883	2.7509	3.63291
24	1.5231	6.76421	1.7296	6.07000	1.8833	5.36450	2.0638	4.95800	2.1990	4.54470	2.3828	4.22366	2.6070	3.89400	2.7540	3.62860
25	1.5281	6.74965	1.7368	6.06000	1.8883	5.35164	2.0688	4.95044	2.2021	4.53804	2.3859	4.21917	2.6118	3.88917	2.7571	3.62429
26	1.5331	6.73509	1.7440	6.05000	1.8933	5.33878	2.0738	4.94288	2.2052	4.53138	2.3890	4.21468	2.6166	3.88434	2.7602	3.61998
27	1.5381	6.72053	1.7512	6.04000	1.8983	5.32592	2.0788	4.93532	2.2083	4.52472	2.3921	4.21019	2.6214	3.87951	2.7633	3.61567
28	1.5431	6.70597	1.7584	6.03000	1.9033	5.31306	2.0838	4.92776	2.2114	4.51806	2.3952	4.20570	2.6262	3.87468	2.7664	3.61136
29	1.5481	6.69141	1.7656	6.02000	1.9083	5.30020	2.0888	4.92020	2.2145	4.51140	2.3983	4.20121	2.6310	3.86985	2.7695	3.60705
30	1.5531	6.67685	1.7728	6.01000	1.9133	5.28734	2.0938	4.91264	2.2176	4.50474	2.4014	4.19672	2.6358	3.86502	2.7726	3.60274
31	1.5581	6.66229	1.7800	6.00000	1.9183	5.27448	2.0988	4.90508	2.2207	4.49808	2.4045	4.19223	2.6406	3.86019	2.7757	3.59843
32	1.5631	6.64773	1.7872	5.99000	1.9233	5.26162	2.1038	4.89752	2.2238	4.49142	2.4076	4.18774	2.6454	3.85536	2.7788	3.59412
33	1.5681	6.63317	1.7944	5.98000	1.9283	5.24876	2.1088	4.88996	2.2269	4.48476	2.4107	4.18325	2.6502	3.85053	2.7819	3.58981
34	1.5731	6.61861	1.8016	5.97000	1.9333	5.23590	2.1138	4.88240	2.2300	4.47810	2.4138	4.17876	2.6550	3.84570	2.7850	3.58550
35	1.5781	6.60405	1.8088	5.96000	1.9383	5.22304	2.1188	4.87484	2.2331	4.47144	2.4169	4.17427	2.6598	3.84087	2.7881	3.58119
36	1.5831	6.58949	1.8160	5.95000	1.9433	5.21018	2.1238	4.86728	2.2362	4.46478	2.4200	4.16978	2.6646	3.83604	2.7912	3.57688
37	1.5881	6.57493	1.8232	5.94000	1.9483	5.19732	2.1288	4.85972	2.2393	4.45812	2.4231	4.16529	2.6694	3.83121	2.7943	3.57257
38	1.5931	6.56037	1.8304	5.93000	1.9533	5.18446	2.1338	4.85216	2.2424	4.45146	2.4262	4.16080	2.6742	3.82638	2.7974	3.56826
39	1.5981	6.54581	1.8376	5.92000	1.9583	5.17160	2.1388	4.84460	2.2455	4.44480	2.4293	4.15631	2.6790	3.82155	2.8005	3.56395
40	1.6031	6.53125	1.8448	5.91000	1.9633	5.15874	2.1438	4.83704	2.2486	4.43814	2.4324	4.15182	2.6838	3.81672	2.8036	3.55964
41	1.6081	6.51669	1.8520	5.90000	1.9683	5.14588	2.1488	4.82948	2.2517	4.43148	2.4355	4.14733	2.6886	3.81189	2.8067	3.55533
42	1.6131	6.50213	1.8592	5.89000	1.9733	5.13302	2.1538	4.82192	2.2548	4.42482	2.4386	4.14284	2.6934	3.80706	2.8098	3.55102
43	1.6181	6.48757	1.8664	5.88000	1.9783	5.12016	2.1588	4.81436	2.2579	4.41816	2.4417	4.13835	2.6982	3.80223	2.8129	3.54671
44	1.6231	6.47301	1.8736	5.87000	1.9833	5.10730	2.1638	4.80680	2.2610	4.41150	2.4448	4.13386	2.7030	3.79740	2.8160	3.54240
45	1.6281	6.45845	1.8808	5.86000	1.9883	5.09444	2.1688	4.79924	2.2641	4.40484	2.4479	4.12937	2.7078	3.79257	2.8191	3.53809
46	1.6331	6.44389	1.8880	5.85000	1.9933	5.08158	2.1738	4.79168	2.2672	4.39818	2.4510	4.12488	2.7126	3.78774	2.8222	3.53378
47	1.6381	6.42933	1.8952	5.84000	1.9983	5.06872	2.1788	4.78412	2.2703	4.39152	2.4541	4.12039	2.7174	3.78291	2.8253	3.52947
48	1.6431	6.41477	1.9024	5.83000	2.0033	5.05586	2.1838	4.77656	2.2734	4.38486	2.4572	4.11590	2.7222	3.77808	2.8284	3.52516
49	1.6481	6.40021	1.9096	5.82000	2.0083	5.04300	2.1888	4.76900	2.2765	4.37820	2.4603	4.11141	2.7270	3.77325	2.8315	3.52085
50	1.6531	6.38565	1.9168	5.81000	2.0133	5.03014	2.1938	4.76144	2.2796	4.37154	2.4634	4.10692	2.7318	3.76842	2.8346	3.51654
51	1.6581	6.37109	1.9240	5.80000	2.0183	5.01728	2.1988	4.75388	2.2827	4.36488	2.4665	4.10243	2.7366	3.76359	2.8377	3.51223
52	1.6631	6.35653	1.9312	5.79000	2.0233	5.00442	2.2038	4.74632	2.2858	4.35822	2.4696	4.09794	2.7414	3.75876	2.8408	3.50792
53	1.6681	6.34197	1.9384	5.78000	2.0283	4.99156	2.2088	4.73876	2.2889	4.35156	2.4727	4.09345	2.7462	3.75393	2.8439	3.50361
54	1.6731	6.32741	1.9456	5.77000	2.0333	4.97870	2.2138	4.73120	2.2920	4.34490	2.4758	4.08896	2.7510	3.74910	2.8470	3.49930
55	1.6781	6.31285	1.9528	5.76000	2.0383	4.96584	2.2188	4.72364	2.2951	4.33824	2.4789	4.08447	2.7558	3.74427	2.8501	3.49499
56	1.6831	6.29829	1.9600	5.75000	2.0433	4.95298	2.2238	4.71608	2.2982	4.33158	2.4820	4.08000	2.7606	3.73944	2.8532	3.49068
57	1.6881	6.28373	1.9672	5.74000	2.0483	4.94012	2.2288	4.70852	2.3013	4.32492	2.4851	4.07551	2.7654	3.73461	2.8563	3.48637
58	1.6931	6.26917	1.9744	5.73000	2.0533	4.92726	2.2338	4.70096	2.3044	4.31826	2.4882	4.07102	2.7702	3.72978	2.8594	3.48206
59	1.6981	6.25461	1.9816	5.72000	2.0583	4.91440	2.2388	4.69340	2.3075	4.31160	2.4913	4.06653	2.7750	3.72495	2.8625	3.47775
60	1.7031	6.24005	1.9888	5.71000	2.0633	4.90154	2.2438	4.68584	2.3106	4.30494	2.4944	4.06204	2.7798	3.72012	2.8656	3.47344

°	16°		17°		18°		19°		20°		21°		22°		23°		24°	
	Temp	Conting	Temp	Conting	Temp	Conting	Temp	Conting	Temp	Conting	Temp	Conting	Temp	Conting	Temp	Conting	Temp	Conting
0	28475	348701	30573	327085	32492	34433	34433	280431	16397	274748	31386	260509	40403	247509	42447	235383	49	
1	28706	348359	30605	327454	32524	34465	34465	280478	16430	274999	31420	260557	40436	247557	42482	235431	50	
2	28738	347977	30637	327823	32556	34498	34498	280525	16463	275250	31453	260605	40469	247605	42516	235479	51	
3	28769	347596	30669	328192	32588	34530	34530	280572	16496	275501	31486	260653	40502	247653	42551	235527	52	
4	28800	347216	30701	328561	32620	34563	34563	280619	16529	275752	31519	260701	40535	247701	42586	235575	53	
5	28832	346835	30733	328930	32653	34596	34596	280666	16562	276003	31552	260749	40568	247749	42621	235623	54	
6	28864	346454	30765	329299	32685	34628	34628	280713	16595	276254	31585	260797	40601	247797	42656	235671	55	
7	28895	346073	30797	329668	32717	34661	34661	280760	16628	276505	31618	260845	40634	247845	42691	235719	56	
8	28927	345693	30829	330037	32749	34693	34693	280807	16661	276756	31651	260893	40667	247893	42726	235767	57	
9	28958	345312	30861	330406	32781	34726	34726	280854	16694	277007	31684	260941	40700	247941	42761	235815	58	
10	28990	344931	30893	330775	32814	34758	34758	280901	16727	277258	31717	260989	40733	247989	42796	235863	59	
11	29021	344550	30925	331144	32846	34791	34791	280948	16760	277509	31750	261037	40766	248037	42831	235911	60	
12	29053	344169	30957	331513	32878	34824	34824	280995	16793	277760	31783	261085	40799	248085	42866	235959	61	
13	29084	343788	30989	331882	32911	34856	34856	281042	16826	278011	31816	261133	40832	248133	42901	236007	62	
14	29116	343407	31021	332251	32943	34889	34889	281089	16859	278262	31849	261181	40865	248181	42936	236055	63	
15	29147	343026	31053	332620	32975	34922	34922	281136	16892	278513	31882	261229	40898	248229	42971	236103	64	
16	29179	342645	31085	332989	33007	34955	34955	281183	16925	278764	31915	261277	40931	248277	43006	236151	65	
17	29210	342264	31117	333358	33040	34987	34987	281230	16958	279015	31948	261325	40964	248325	43041	236199	66	
18	29242	341883	31149	333727	33072	35020	35020	281277	16991	279266	31981	261373	40997	248373	43076	236247	67	
19	29274	341502	31181	334096	33104	35053	35053	281324	17024	279517	32014	261421	41030	248421	43111	236295	68	
20	29305	341121	31213	334465	33136	35085	35085	281371	17057	279768	32047	261469	41063	248469	43146	236343	69	
21	29337	340740	31245	334834	33169	35117	35117	281418	17090	280019	32080	261517	41096	248517	43181	236391	70	
22	29368	340359	31277	335203	33201	35150	35150	281465	17123	280270	32113	261565	41129	248565	43216	236439	71	
23	29399	340000	31309	335572	33233	35183	35183	281512	17156	280521	32146	261613	41162	248613	43251	236487	72	
24	29430	339641	31341	335941	33266	35216	35216	281559	17189	280772	32179	261661	41195	248661	43286	236535	73	
25	29461	339282	31373	336310	33298	35248	35248	281606	17222	281023	32212	261709	41228	248709	43321	236583	74	
26	29492	338923	31405	336679	33331	35281	35281	281653	17255	281274	32245	261757	41261	248757	43356	236631	75	
27	29523	338564	31437	337048	33363	35313	35313	281700	17288	281525	32278	261805	41294	248805	43391	236679	76	
28	29554	338205	31469	337417	33396	35346	35346	281747	17321	281776	32311	261853	41327	248853	43426	236727	77	
29	29585	337846	31501	337786	33428	35378	35378	281794	17354	282027	32344	261901	41360	248901	43461	236775	78	
30	29616	337487	31533	338155	33461	35411	35411	281841	17387	282278	32377	261949	41393	248949	43496	236823	79	
31	29647	337128	31565	338524	33493	35443	35443	281888	17420	282529	32410	261997	41426	248997	43531	236871	80	
32	29678	336769	31597	338893	33526	35476	35476	281935	17453	282780	32443	262045	41459	249045	43566	236919	81	
33	29709	336410	31629	339262	33558	35508	35508	281982	17486	283031	32476	262093	41492	249093	43601	236967	82	
34	29740	336051	31661	339631	33591	35541	35541	282029	17519	283282	32509	262141	41525	249141	43636	237015	83	
35	29771	335692	31693	340000	33623	35573	35573	282076	17552	283533	32542	262189	41558	249189	43671	237063	84	
36	29802	335333	31725	340369	33656	35606	35606	282123	17585	283784	32575	262237	41591	249237	43706	237111	85	
37	29833	334974	31757	340738	33688	35638	35638	282170	17618	284035	32608	262285	41624	249285	43741	237159	86	
38	29864	334615	31789	341107	33721	35671	35671	282217	17651	284286	32641	262333	41657	249333	43776	237207	87	
39	29895	334256	31821	341476	33753	35703	35703	282264	17684	284537	32674	262381	41690	249381	43811	237255	88	
40	29926	333897	31853	341845	33786	35736	35736	282311	17717	284788	32707	262429	41723	249429	43846	237303	89	
41	29957	333538	31885	342214	33818	35768	35768	282358	17750	285039	32740	262477	41756	249477	43881	237351	90	
42	29988	333179	31917	342583	33851	35801	35801	282405	17783	285290	32773	262525	41789	249525	43916	237399	91	
43	30019	332820	31949	342952	33883	35833	35833	282452	17816	285541	32806	262573	41822	249573	43951	237447	92	
44	30050	332461	31981	343321	33916	35866	35866	282499	17849	285792	32839	262621	41855	249621	43986	237495	93	
45	30081	332102	32013	343690	33948	35898	35898	282546	17882	286043	32872	262669	41888	249669	44021	237543	94	
46	30112	331743	32045	344059	33981	35931	35931	282593	17915	286294	32905	262717	41921	249717	44056	237591	95	
47	30143	331384	32077	344428	34013	35963	35963	282640	17948	286545	32938	262765	41954	249765	44091	237639	96	
48	30174	331025	32109	344797	34046	36000	36000	282687	17981	286796	32971	262813	41987	249813	44126	237687	97	
49	30205	330666	32141	345166	34078	36032	36032	282734	18014	287047	33004	262861	42020	249861	44161	237735	98	
50	30236	330307	32173	345535	34111	36065	36065	282781	18047	287298	33037	262909	42053	249909	44196	237783	99	
51	30267	330000	32205	345904	34143	36098	36098	282828	18080	287549	33070	262957	42086	249957	44231	237831	100	
52	30298	329693	32237	346273	34176	36131	36131	282875	18113	287800	33103	263005	42119	249999	44266	237879	101	
53	30329	329386	32269	346642	34208	36164	36164	282922	18146	288051	33136	263053	42152	250047	44301	237927	102	
54	30360	329079	32301	347011	34241	36197	36197	282969	18179	288302	33169	263101	42185	250095	44336	237975	103	
55	30391	328772	32333	347380	34273	36230	36230	283016	18212	288553	33202	263149	42218	250143	44371	238023	104	
56	30422	328465	32365	347749	34306	36263	36263	283063	18245	288804	33235	263197	42251	250191	44406	238071	105	
57	30453	328158	32397	348118	34338	36296	36296	283110	18278	289055	33268	263245	42284	250239	44441	238119	106	
58	30484	327851	32429	348487	34371	36329	36329	283157	18311	289306	33301	263293	42317	250287	44476	238167	107	
59	30515	327544	32461	348856	34403	36362	36362	283204	18344	289557	33334	263341	42350	250335	44511	238215	108	
60	30546	327237	32493	349225	34436	36395	36395	283251	18377	289808	33367	263389	42383	250383	44546	238263	109	

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°	24°		25°		26°		27°		28°		29°		30°		31°	
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang
0	445.3	2.14604	466.31	2.14451	487.73	2.05030	509.53	1.96261	531.71	1.88073	554.31	1.80405	577.35	1.73205	600.86	1.66428
1	445.58	2.14248	466.66	2.14248	488.09	2.04728	510.26	1.95979	532.08	1.87941	554.69	1.80281	577.74	1.72973	601.26	1.66318
2	445.85	2.13892	467.02	2.13892	488.45	2.04428	510.53	1.95688	532.35	1.87650	554.96	1.80000	578.13	1.72673	601.63	1.66209
3	446.12	2.13536	467.37	2.13536	488.81	2.04128	510.80	1.95397	532.62	1.87351	555.23	1.79711	578.52	1.72373	602.00	1.66100
4	446.39	2.13180	467.72	2.13180	489.17	2.03828	511.07	1.95106	532.89	1.87052	555.50	1.79422	578.91	1.72073	602.37	1.65991
5	446.66	2.12824	468.08	2.12824	489.53	2.03528	511.34	1.94815	533.16	1.86753	555.77	1.79133	579.30	1.71773	602.74	1.65882
6	446.93	2.12468	468.43	2.12468	489.89	2.03228	511.61	1.94524	533.43	1.86454	556.04	1.78844	579.69	1.71473	603.11	1.65773
7	447.20	2.12112	468.78	2.12112	490.25	2.02928	511.88	1.94233	533.70	1.86154	556.31	1.78554	580.04	1.71173	603.48	1.65664
8	447.47	2.11756	469.14	2.11756	490.61	2.02628	512.15	1.93942	533.97	1.85854	556.58	1.78264	580.40	1.70873	603.85	1.65555
9	447.74	2.11400	469.49	2.11400	490.97	2.02328	512.42	1.93651	534.24	1.85554	556.85	1.77973	580.76	1.70573	604.22	1.65446
10	448.01	2.11044	469.85	2.11044	491.33	2.02028	512.69	1.93360	534.51	1.85254	557.12	1.77683	581.12	1.70273	604.59	1.65337
11	448.28	2.10688	470.21	2.10688	491.69	2.01728	512.96	1.93069	534.78	1.84954	557.39	1.77393	581.48	1.69973	604.96	1.65228
12	448.55	2.10332	470.56	2.10332	492.05	2.01428	513.23	1.92778	535.05	1.84654	557.66	1.77103	581.84	1.69673	605.33	1.65119
13	448.82	2.09976	470.92	2.09976	492.41	2.01128	513.50	1.92487	535.32	1.84354	557.93	1.76813	582.20	1.69373	605.70	1.65010
14	449.09	2.09620	471.28	2.09620	492.77	2.00828	513.77	1.92196	535.59	1.84054	558.20	1.76523	582.56	1.69073	606.07	1.64901
15	449.36	2.09264	471.63	2.09264	493.13	2.00528	514.04	1.91905	535.86	1.83754	558.47	1.76233	582.92	1.68773	606.44	1.64792
16	449.63	2.08908	471.99	2.08908	493.49	2.00228	514.31	1.91614	536.13	1.83454	558.74	1.75943	583.28	1.68473	606.81	1.64683
17	449.90	2.08552	472.34	2.08552	493.85	1.99928	514.58	1.91323	536.40	1.83154	559.01	1.75653	583.64	1.68173	607.18	1.64574
18	450.17	2.08196	472.70	2.08196	494.21	1.99628	514.85	1.91032	536.67	1.82854	559.28	1.75363	584.00	1.67873	607.55	1.64465
19	450.44	2.07840	473.05	2.07840	494.57	1.99328	515.12	1.90741	536.94	1.82554	559.55	1.75073	584.36	1.67573	607.92	1.64356
20	450.71	2.07484	473.41	2.07484	494.93	1.99028	515.39	1.90450	537.21	1.82254	559.82	1.74783	584.72	1.67273	608.29	1.64247
21	450.98	2.07128	473.77	2.07128	495.29	1.98728	515.66	1.90159	537.48	1.81954	560.09	1.74493	585.08	1.66973	608.66	1.64138
22	451.25	2.06772	474.12	2.06772	495.65	1.98428	515.93	1.89868	537.75	1.81654	560.36	1.74203	585.44	1.66673	609.03	1.64029
23	451.52	2.06416	474.48	2.06416	496.01	1.98128	516.20	1.89577	538.02	1.81354	560.63	1.73913	585.80	1.66373	609.40	1.63920
24	451.79	2.06060	474.83	2.06060	496.37	1.97828	516.47	1.89286	538.29	1.81054	560.90	1.73623	586.16	1.66073	609.77	1.63811
25	452.06	2.05704	475.19	2.05704	496.73	1.97528	516.74	1.88995	538.56	1.80754	561.17	1.73333	586.52	1.65773	610.14	1.63702
26	452.33	2.05348	475.54	2.05348	497.09	1.97228	517.01	1.88704	538.83	1.80454	561.44	1.73043	586.88	1.65473	610.51	1.63593
27	452.60	2.04992	475.90	2.04992	497.45	1.96928	517.28	1.88413	539.10	1.80154	561.71	1.72753	587.24	1.65173	610.88	1.63484
28	452.87	2.04636	476.26	2.04636	497.81	1.96628	517.55	1.88122	539.37	1.79854	561.98	1.72463	587.60	1.64873	611.25	1.63375
29	453.14	2.04280	476.62	2.04280	498.17	1.96328	517.82	1.87831	539.64	1.79554	562.25	1.72173	587.96	1.64573	611.62	1.63266
30	453.41	2.03924	476.98	2.03924	498.53	1.96028	518.09	1.87540	539.91	1.79254	562.52	1.71883	588.32	1.64273	611.99	1.63157
31	453.68	2.03568	477.33	2.03568	498.89	1.95728	518.36	1.87249	540.18	1.78954	562.79	1.71593	588.68	1.63973	612.36	1.63048
32	453.95	2.03212	477.69	2.03212	499.25	1.95428	518.63	1.86958	540.45	1.78654	563.06	1.71303	589.04	1.63673	612.73	1.62939
33	454.22	2.02856	478.05	2.02856	499.61	1.95128	518.90	1.86667	540.72	1.78354	563.33	1.71013	589.40	1.63373	613.10	1.62830
34	454.49	2.02500	478.40	2.02500	500.00	1.94828	519.17	1.86376	540.99	1.78054	563.60	1.70723	589.76	1.63073	613.47	1.62721
35	454.76	2.02144	478.76	2.02144	500.36	1.94528	519.44	1.86085	541.26	1.77754	563.87	1.70433	590.12	1.62773	613.84	1.62612
36	455.03	2.01788	479.12	2.01788	500.72	1.94228	519.71	1.85794	541.53	1.77454	564.14	1.70143	590.48	1.62473	614.21	1.62503
37	455.30	2.01432	479.48	2.01432	501.08	1.93928	520.00	1.85503	541.80	1.77154	564.41	1.69853	590.84	1.62173	614.58	1.62394
38	455.57	2.01076	479.84	2.01076	501.44	1.93628	520.27	1.85212	542.07	1.76854	564.68	1.69563	591.20	1.61873	614.95	1.62285
39	455.84	2.00720	480.20	2.00720	501.80	1.93328	520.54	1.84921	542.34	1.76554	564.95	1.69273	591.56	1.61573	615.32	1.62176
40	456.11	2.00364	480.56	2.00364	502.16	1.93028	520.81	1.84630	542.61	1.76254	565.22	1.68983	591.92	1.61273	615.69	1.62067
41	456.38	1.99992	480.92	1.99992	502.52	1.92728	521.08	1.84339	542.88	1.75954	565.49	1.68693	592.28	1.60973	616.06	1.61958
42	456.65	1.99636	481.28	1.99636	502.88	1.92428	521.35	1.84048	543.15	1.75654	565.76	1.68403	592.64	1.60673	616.43	1.61849
43	456.92	1.99280	481.63	1.99280	503.24	1.92128	521.62	1.83757	543.42	1.75354	566.03	1.68113	593.00	1.60373	616.80	1.61740
44	457.19	1.98924	481.99	1.98924	503.60	1.91828	521.89	1.83466	543.69	1.75054	566.30	1.67823	593.36	1.60073	617.17	1.61631
45	457.46	1.98568	482.34	1.98568	503.96	1.91528	522.16	1.83175	543.96	1.74754	566.57	1.67533	593.72	1.59773	617.54	1.61522
46	457.73	1.98212	482.70	1.98212	504.32	1.91228	522.43	1.82884	544.23	1.74454	566.84	1.67243	594.08	1.59473	617.91	1.61413
47	457.99	1.97856	483.06	1.97856	504.68	1.90928	522.70	1.82593	544.50	1.74154	567.11	1.66953	594.44	1.59173	618.28	1.61304
48	458.26	1.97500	483.42	1.97500	505.04	1.90628	522.97	1.82302	544.77	1.73854	567.38	1.66663	594.80	1.58873	618.65	1.61195
49	458.53	1.97144	483.78	1.97144	505.40	1.90328	523.24	1.82011	545.04	1.73554	567.65	1.66373	595.16	1.58573	619.02	1.61086
50	458.80	1.96788	484.14	1.96788	505.76	1.90028	523.51	1.81720	545.31	1.73254	567.92	1.66083	595.52	1.58273	619.39	1.60977
51	459.07	1.96432	484.50	1.96432	506.12	1.89728	523.78	1.81429	545.58	1.72954	568.19	1.65793	595.88	1.57973	619.76	1.60868
52	459.34	1.96076	484.86	1.96076	506.48	1.89428	524.05	1.81138	545.85	1.72654	568.46	1.65503	596.24	1.57673	620.13	1.60759
53	459.61	1.95720	485.21	1.95720	506.84	1.89128	524.32	1.80847	546.12	1.72354	568.73	1.65213	596.60	1.57373	620.50	1.60650
54	459.88	1.95364	485.57	1.95364	507.20	1.88828	524.59	1.80556	546.39	1.72054	569.00	1.64923	596.96	1.57073	620.87	1.60541
55	460.15	1.95008	485.93	1.95008	507.56	1.88528	524.86	1.80265	546.66	1.71754	569.27	1.64633	597.32	1.56773	621.24	1.60432
56	460.42	1.94652	486.29	1.94652	507.92	1.88228	525.13	1.80000	546.93	1.71454	569.54	1.64343	597.68	1.56473	621.61	1.60323
57	460.69	1.94296	486.65	1.94296	508.28	1.87928	525.40	1.79709	547.20	1.71154	569.81	1.64053	598.04	1.56173	621.98	1.60214
58	460.96	1.93940	487.01	1.93940	508.64	1.87628	525.67	1.79418	547.47	1.70854	570.08	1.63763	598.40	1.55873	622.35	1.60105
59	461.23	1.93584	487.37	1.93584	509.00	1.87328	525.94	1.79127	547.74	1.70554	570.35	1.63473	598.76	1.55573	622.72	1.60000
60	461.50	1.93228	487.73	1.93228	509.36	1.87028	526.21	1.78836	548.01	1.70254	570.62	1.63183	599.12	1.55273	623.09	

°	32°		33°		34°		35°		36°		37°		38°		39°		40°	
	Tong	Ceting	Tong	Ceting	Tong	Ceting	Tong	Ceting	Tong	Ceting	Tong	Ceting	Tong	Ceting	Tong	Ceting	Tong	Ceting
0	62487	1.60033	64941	1.53966	67451	1.48256	70021	1.42815	72654	1.37638	75358	1.32764	78129	1.27994	80978	1.23490	83910	1.19175
1	62527	1.59930	64982	1.53888	67493	1.48163	70064	1.42726	72699	1.37554	75401	1.32624	78175	1.27917	81027	1.23416	83959	1.19128
2	62568	1.59826	65023	1.53791	67536	1.48070	70107	1.42638	72743	1.37470	75447	1.32544	78222	1.27841	81075	1.23340	84001	1.19080
3	62608	1.59723	65065	1.53693	67578	1.47977	70151	1.42505	72785	1.37306	75492	1.32466	78267	1.27764	81123	1.23240	84043	1.19032
4	62649	1.59620	65106	1.53595	67620	1.47883	70194	1.42432	72827	1.37132	75538	1.32330	78316	1.27688	81171	1.23140	84085	1.18984
5	62689	1.59517	65148	1.53498	67663	1.47792	70238	1.42364	72874	1.36978	75584	1.32244	78361	1.27611	81220	1.23050	84127	1.18936
6	62730	1.59414	65189	1.53400	67706	1.47700	70281	1.42296	72921	1.36811	75629	1.32158	78410	1.27535	81268	1.22960	84169	1.18888
7	62770	1.59311	65231	1.53302	67749	1.47607	70325	1.42228	72966	1.36718	75675	1.32064	78454	1.27458	81316	1.22877	84211	1.18840
8	62811	1.59208	65272	1.53205	67790	1.47514	70368	1.42160	73005	1.36625	75721	1.31984	78504	1.27382	81364	1.22784	84253	1.18792
9	62852	1.59105	65314	1.53107	67832	1.47422	70412	1.42092	73051	1.36532	75767	1.31904	78551	1.27306	81413	1.22691	84295	1.18744
10	62892	1.59002	65355	1.53010	67875	1.47330	70455	1.42024	73100	1.36440	75812	1.31825	78598	1.27230	81461	1.22598	84337	1.18696
11	62933	1.58900	65397	1.52913	67917	1.47238	70499	1.41957	73144	1.36348	75858	1.31745	78645	1.27153	81510	1.22505	84379	1.18648
12	62973	1.58797	65438	1.52816	67960	1.47146	70542	1.41889	73189	1.36256	75904	1.31666	78692	1.27077	81558	1.22412	84421	1.18600
13	63014	1.58695	65480	1.52719	68002	1.47053	70586	1.41822	73234	1.36164	75950	1.31588	78739	1.27001	81606	1.22320	84463	1.18552
14	63055	1.58593	65521	1.52622	68045	1.46962	70629	1.41755	73278	1.36072	76002	1.31507	78786	1.26925	81655	1.22227	84505	1.18504
15	63095	1.58490	65563	1.52525	68088	1.46870	70673	1.41688	73321	1.35980	76046	1.31427	78834	1.26849	81703	1.22134	84547	1.18456
16	63136	1.58388	65604	1.52429	68130	1.46778	70717	1.41621	73368	1.35888	76088	1.31348	78881	1.26774	81752	1.22041	84589	1.18408
17	63177	1.58286	65646	1.52332	68173	1.46686	70760	1.41554	73413	1.35796	76134	1.31269	78928	1.26698	81800	1.21948	84631	1.18360
18	63218	1.58184	65688	1.52235	68215	1.46594	70804	1.41487	73457	1.35704	76180	1.31190	78975	1.26622	81849	1.21855	84673	1.18312
19	63258	1.58083	65729	1.52139	68258	1.46503	70848	1.41420	73502	1.35612	76226	1.31110	79022	1.26546	81898	1.21762	84715	1.18264
20	63299	1.57981	65771	1.52043	68301	1.46411	70891	1.41353	73547	1.35520	76272	1.31031	79070	1.26471	81946	1.21669	84757	1.18216
21	63340	1.57879	65813	1.51946	68343	1.46320	70935	1.41286	73592	1.35428	76318	1.30952	79117	1.26395	81995	1.21576	84799	1.18168
22	63380	1.57778	65854	1.51850	68386	1.46229	70979	1.41219	73637	1.35336	76364	1.30873	79164	1.26319	82044	1.21483	84841	1.18120
23	63421	1.57676	65896	1.51754	68429	1.46137	71023	1.41152	73681	1.35244	76410	1.30795	79212	1.26244	82092	1.21390	84883	1.18072
24	63462	1.57575	65938	1.51658	68471	1.46046	71066	1.41085	73726	1.35152	76456	1.30716	79259	1.26169	82141	1.21297	84925	1.18024
25	63503	1.57474	65980	1.51562	68514	1.45955	71110	1.41018	73771	1.35060	76502	1.30637	79306	1.26093	82190	1.21204	84967	1.17976
26	63544	1.57372	66021	1.51466	68557	1.45864	71154	1.40951	73816	1.34968	76548	1.30558	79354	1.26018	82238	1.21111	85009	1.17928
27	63584	1.57271	66063	1.51370	68600	1.45773	71198	1.40884	73861	1.34876	76594	1.30480	79402	1.25943	82287	1.21018	85051	1.17880
28	63625	1.57170	66105	1.51275	68642	1.45682	71242	1.40817	73906	1.34784	76640	1.30403	79450	1.25868	82336	1.20925	85093	1.17832
29	63666	1.57069	66147	1.51179	68685	1.45592	71285	1.40750	73951	1.34692	76686	1.30325	79496	1.25792	82385	1.20832	85135	1.17784
30	63707	1.56969	66189	1.51084	68728	1.45501	71329	1.40683	73996	1.34600	76733	1.30248	79544	1.25717	82434	1.20739	85177	1.17736
31	63748	1.56868	66230	1.50988	68771	1.45410	71373	1.40616	74041	1.34508	76779	1.30166	79591	1.25642	82483	1.20646	85219	1.17688
32	63789	1.56767	66272	1.50893	68814	1.45319	71417	1.40549	74086	1.34416	76825	1.30087	79639	1.25567	82532	1.20553	85261	1.17640
33	63830	1.56666	66314	1.50799	68857	1.45228	71461	1.40482	74131	1.34324	76871	1.30009	79686	1.25492	82580	1.20460	85303	1.17592
34	63871	1.56566	66356	1.50706	68900	1.45137	71505	1.40415	74176	1.34232	76918	1.30009	79734	1.25417	82629	1.20367	85345	1.17544
35	63912	1.56466	66398	1.50613	68942	1.45046	71549	1.40348	74221	1.34140	76964	1.29931	79781	1.25342	82678	1.20274	85387	1.17496
36	63953	1.56366	66440	1.50519	68985	1.44955	71593	1.40281	74266	1.34048	77010	1.29853	79829	1.25268	82727	1.20181	85429	1.17448
37	63994	1.56266	66482	1.50426	69028	1.44864	71637	1.40214	74312	1.33956	77057	1.29775	79877	1.25193	82776	1.20088	85471	1.17400
38	64035	1.56165	66524	1.50332	69071	1.44773	71681	1.40147	74357	1.33864	77103	1.29696	79924	1.25118	82825	1.20000	85513	1.17352
39	64076	1.56065	66566	1.50238	69114	1.44682	71725	1.40080	74402	1.33772	77149	1.29618	79972	1.25044	82874	1.19907	85555	1.17304
40	64117	1.55966	66608	1.50143	69157	1.44591	71769	1.40013	74447	1.33680	77196	1.29541	80020	1.24969	82923	1.19814	85597	1.17256
41	64158	1.55866	66650	1.50048	69200	1.44500	71813	1.39946	74492	1.33588	77242	1.29463	80067	1.24895	82972	1.19721	85639	1.17208
42	64199	1.55766	66692	1.49954	69243	1.44410	71857	1.39879	74538	1.33496	77289	1.29385	80115	1.24820	83021	1.19628	85681	1.17160
43	64240	1.55666	66734	1.49860	69286	1.44319	71901	1.39812	74583	1.33404	77335	1.29307	80163	1.24746	83072	1.19535	85723	1.17112
44	64281	1.55567	66776	1.49765	69329	1.44228	71946	1.39745	74628	1.33312	77382	1.29229	80211	1.24672	83120	1.19442	85765	1.17064
45	64322	1.55467	66818	1.49671	69372	1.44137	71990	1.39678	74674	1.33220	77428	1.29151	80258	1.24597	83169	1.19349	85807	1.17016
46	64363	1.55368	66860	1.49576	69415	1.44046	72034	1.39611	74719	1.33128	77475	1.29074	80306	1.24523	83218	1.19256	85849	1.16968
47	64404	1.55269	66902	1.49481	69458	1.43955	72078	1.39544	74764	1.33036	77521	1.28997	80354	1.24449	83268	1.19163	85891	1.16920
48	64445	1.55170	66944	1.49386	69501	1.43864	72122	1.39477	74810	1.32944	77568	1.28919	80402	1.24375	83317	1.19070	85933	1.16872
49	64487	1.55071	66986	1.49291	69544	1.43773	72166	1.39410	74855	1.32852	77615	1.28842	80450	1.24301	83366	1.18977	85975	1.16824
50	64528	1.54972	67028	1.49196	69588	1.43682	72211	1.39343	74900	1.32760	77661	1.28764	80498	1.24227	83415	1.18884	86017	1.16776
51	64569	1.54873	67071	1.49097	69631	1.43591	72255	1.39276	74946	1.32668	77708	1.28687	80546	1.24153	83465	1.18791	86059	1.16728
52	64610	1.54774	67113	1.49003	69675	1.43500	72299	1.39209	74991	1.32576	77754	1.28610	80594	1.24079	83514	1.18698	86101	1.16680
53	64652	1.54675	67155	1.48909	69718	1.43409	72343	1.39142	75037	1.32484	77801	1.28533	80642	1.24005	83564	1.18605	86143	1.16632
54	64693	1.54576	67197	1.48816	69761	1.43318	72388	1.39075	75082	1.32392	77848	1.28456	80690	1.23931	83613	1.18512	86185	1.16584
55	64734	1.54478	67239	1.48722	69804	1.43227	72432	1.39008	75128	1.32300	77895	1.28379	80738	1.23858	83662	1.18419	86227	1.16536
56	64775	1.54379	67282	1.48629	69847	1.43136	72477	1.38941	75173	1.32208	77941	1.28302	80786	1.23784	83712	1.18326</		

°	43°		41°		40°		44°		44°		44°		45°		45°	
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang
0	81910	1.19175	84929	1.15037	90040	1.11061	93252	1.07237	96569	1.03553	99700	1.02355	1.02355	98843	1.01170	20
1	81960	1.19105	84980	1.14969	90093	1.10996	93306	1.07174	96625	1.03493	99756	1.02295	98901	1.01112	19	
2	82009	1.19035	85031	1.14902	90146	1.10931	93350	1.07112	96681	1.03433	99813	1.02236	98958	1.01053	18	
3	82059	1.18964	85082	1.14834	90199	1.10862	93404	1.07049	96738	1.03372	99870	1.02176	99016	1.00994	17	
4	82108	1.18894	85133	1.14767	90251	1.10793	93458	1.06987	96794	1.03312	99927	1.02117	99073	1.00935	16	
5	82158	1.18824	85184	1.14699	90304	1.10724	93512	1.06925	96850	1.03252	99984	1.02057	99131	1.00876	15	
6	82208	1.18754	85236	1.14632	90357	1.10655	93566	1.06862	96907	1.03192	1.00041	1.01998	99189	1.00818	14	
7	82258	1.18684	85287	1.14565	90410	1.10586	93620	1.06800	96963	1.03132	1.00098	1.01939	99247	1.00759	13	
8	82307	1.18614	85338	1.14498	90463	1.10517	93674	1.06738	97020	1.03072	1.00155	1.01879	99304	1.00701	12	
9	82357	1.18544	85389	1.14430	90516	1.10448	93728	1.06676	97076	1.03012	1.00212	1.01820	99362	1.00642	11	
10	82407	1.18474	85441	1.14363	90569	1.10379	93782	1.06613	97133	1.02952	1.00269	1.01761	99420	1.00583	10	
11	82457	1.18404	85492	1.14296	90621	1.10310	93836	1.06551	97189	1.02892	1.00326	1.01702	99478	1.00525	9	
12	82507	1.18334	85543	1.14229	90674	1.10241	93890	1.06489	97246	1.02832	1.00383	1.01642	99536	1.00467	8	
13	82556	1.18264	85595	1.14162	90727	1.10172	93944	1.06427	97302	1.02772	1.00440	1.01583	99594	1.00408	7	
14	82606	1.18194	85646	1.14095	90781	1.10103	94000	1.06365	97359	1.02713	1.00497	1.01524	99652	1.00350	6	
15	82656	1.18125	85698	1.14028	90834	1.10034	94054	1.06303	97416	1.02653	1.00554	1.01465	99710	1.00291	5	
16	82706	1.18055	85749	1.13961	90887	1.10007	94108	1.06241	97472	1.02593	1.00611	1.01406	99768	1.00233	4	
17	82756	1.17986	85801	1.13894	90940	1.09938	94162	1.06179	97529	1.02533	1.00668	1.01347	99826	1.00175	3	
18	82806	1.17916	85852	1.13828	90993	1.09869	94216	1.06117	97586	1.02474	1.00725	1.01288	99884	1.00116	2	
19	82856	1.17846	85904	1.13761	91046	1.09800	94270	1.06056	97643	1.02414	1.00782	1.01229	99942	1.00058	1	
20	82906	1.17777	85955	1.13694	91099	1.09731	94324	1.05994	97700	1.02355	1.00840	1.01170	1.00000	0		

TM 684-168

°	43°		41°		40°		45°		46°		47°		48°		49°	
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang
21	82956	1.17708	86007	1.13627	91153	1.09662	94378	1.05932	94385	1.05924	94392	1.05916	94400	1.05908	94408	1.05900
22	83006	1.17638	86059	1.13561	91206	1.09593	94431	1.05870	94438	1.05862	94445	1.05854	94453	1.05846	94460	1.05838
23	83056	1.17569	86110	1.13494	91259	1.09524	94484	1.05809	94491	1.05799	94498	1.05791	94506	1.05783	94513	1.05775
24	83107	1.17500	86162	1.13427	91312	1.09455	94537	1.05747	94544	1.05737	94551	1.05729	94558	1.05721	94565	1.05713
25	83157	1.17430	86214	1.13360	91365	1.09386	94590	1.05685	94600	1.05675	94607	1.05667	94614	1.05659	94621	1.05651
26	83207	1.17361	86265	1.13293	91418	1.09317	94643	1.05623	94653	1.05613	94660	1.05605	94667	1.05597	94674	1.05589
27	83257	1.17292	86317	1.13226	91471	1.09248	94696	1.05561	94706	1.05551	94713	1.05543	94720	1.05535	94727	1.05527
28	83308	1.17223	86369	1.13159	91524	1.09179	94749	1.05500	94759	1.05490	94766	1.05482	94773	1.05474	94780	1.05466
29	83358	1.17154	86421	1.13092	91577	1.09110	94802	1.05438	94812	1.05428	94819	1.05420	94826	1.05412	94833	1.05404
30	83408	1.17085	86473	1.13025	91630	1.09041	94855	1.05376	94865	1.05366	94872	1.05358	94879	1.05350	94886	1.05342
31	83458	1.17016	86524	1.12958	91683	1.08972	94908	1.05314	94918	1.05304	94925	1.05296	94932	1.05288	94939	1.05280
32	83509	1.16947	86576	1.12891	91736	1.08903	94961	1.05252	94971	1.05242	94978	1.05234	94985	1.05226	94992	1.05218
33	83559	1.16878	86628	1.12823	91789	1.08834	95014	1.05190	95024	1.05180	95031	1.05172	95038	1.05164	95045	1.05156
34	83609	1.16809	86680	1.12755	91842	1.08765	95067	1.05128	95077	1.05118	95084	1.05110	95091	1.05102	95098	1.05094
35	83660	1.16741	86732	1.12687	91895	1.08696	95110	1.05066	95120	1.05056	95127	1.05048	95134	1.05040	95141	1.05032
36	83710	1.16672	86784	1.12619	91948	1.08627	95163	1.05004	95173	1.05004	95180	1.04996	95187	1.04988	95194	1.04980
37	83761	1.16603	86836	1.12551	92001	1.08558	95206	1.04942	95216	1.04942	95223	1.04934	95230	1.04926	95237	1.04918
38	83811	1.16535	86888	1.12483	92054	1.08489	95259	1.04880	95269	1.04880	95276	1.04872	95283	1.04864	95290	1.04856
39	83861	1.16466	86940	1.12415	92107	1.08420	95312	1.04818	95322	1.04818	95329	1.04810	95336	1.04802	95343	1.04794
40	83912	1.16398	86992	1.12347	92160	1.08351	95365	1.04756	95375	1.04756	95382	1.04748	95389	1.04740	95396	1.04732
41	83963	1.16329	87044	1.12279	92213	1.08282	95418	1.04694	95428	1.04694	95435	1.04686	95442	1.04678	95449	1.04670
42	84014	1.16261	87097	1.12211	92266	1.08213	95471	1.04632	95481	1.04632	95488	1.04624	95495	1.04616	95502	1.04608
43	84064	1.16192	87149	1.12143	92319	1.08144	95524	1.04570	95534	1.04570	95541	1.04562	95548	1.04554	95555	1.04546
44	84115	1.16124	87201	1.12075	92372	1.08075	95577	1.04508	95587	1.04508	95594	1.04500	95601	1.04492	95608	1.04484
45	84166	1.16056	87253	1.12007	92425	1.08006	95630	1.04446	95640	1.04446	95647	1.04438	95654	1.04430	95661	1.04422
46	84217	1.15987	87305	1.11939	92478	1.07937	95683	1.04384	95693	1.04384	95700	1.04376	95707	1.04368	95714	1.04360
47	84267	1.15919	87358	1.11871	92531	1.07868	95736	1.04322	95746	1.04322	95753	1.04314	95760	1.04306	95767	1.04298
48	84318	1.15851	87410	1.11803	92584	1.07799	95789	1.04260	95799	1.04260	95806	1.04252	95813	1.04244	95820	1.04236
49	84368	1.15783	87463	1.11735	92637	1.07730	95842	1.04198	95852	1.04198	95859	1.04190	95866	1.04182	95873	1.04174
50	84419	1.15715	87515	1.11667	92690	1.07661	95895	1.04136	95905	1.04136	95912	1.04128	95919	1.04120	95926	1.04112
51	84470	1.15647	87567	1.11599	92743	1.07592	95948	1.04074	95958	1.04074	95965	1.04066	95972	1.04058	95979	1.04050
52	84521	1.15579	87619	1.11531	92796	1.07523	96001	1.04012	96011	1.04012	96018	1.04004	96025	1.04006	96032	1.03998
53	84572	1.15511	87672	1.11463	92849	1.07454	96054	1.03950	96064	1.03950	96071	1.03942	96078	1.03934	96085	1.03926
54	84623	1.15443	87725	1.11395	92902	1.07385	96107	1.03888	96117	1.03888	96124	1.03880	96131	1.03872	96138	1.03864
55	84674	1.15375	87777	1.11327	92955	1.07316	96150	1.03826	96160	1.03826	96167	1.03818	96174	1.03810	96181	1.03802
56	84725	1.15307	87830	1.11259	93008	1.07247	96193	1.03764	96203	1.03764	96210	1.03756	96217	1.03748	96224	1.03740
57	84776	1.15239	87883	1.11191	93061	1.07178	96246	1.03702	96256	1.03702	96263	1.03694	96270	1.03686	96277	1.03678
58	84827	1.15171	87936	1.11123	93114	1.07109	96299	1.03640	96309	1.03640	96316	1.03632	96323	1.03624	96330	1.03616
59	84878	1.15103	87989	1.11055	93167	1.07040	96352	1.03578	96362	1.03578	96369	1.03570	96376	1.03562	96383	1.03554
60	84929	1.15035	88042	1.10987	93220	1.06971	96405	1.03516	96415	1.03516	96422	1.03508	96429	1.03500	96436	1.03492

TM 684-167

0°		1°		2°		3°		4°		5°		6°		7°	
Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.
0	Infinite	1.0001	57.399	1.0006	28.654	1.0014	19.107	1.0024	14.335	1.0038	11.474	1.0055	9.5648	1.0075	8.2055
1	3437.70	1.0001	56.359	1.0006	28.417	1.0014	19.092	1.0025	14.276	1.0038	11.436	1.0055	9.5404	1.0075	8.1861
2	1718.90	1.0002	55.450	1.0006	28.184	1.0014	18.997	1.0025	14.217	1.0039	11.398	1.0056	9.5161	1.0076	8.1674
3	1145.90	1.0002	54.570	1.0006	27.955	1.0014	18.904	1.0025	14.159	1.0039	11.360	1.0056	9.4880	1.0076	8.1476
4	859.44	1.0002	53.718	1.0006	27.730	1.0014	18.820	1.0025	14.101	1.0039	11.323	1.0056	9.4620	1.0076	8.1285
5	687.55	1.0002	52.891	1.0007	27.508	1.0015	18.736	1.0025	14.043	1.0039	11.286	1.0057	9.4362	1.0077	8.1095
6	572.96	1.0002	52.090	1.0007	27.290	1.0015	18.653	1.0025	13.986	1.0040	11.249	1.0057	9.4105	1.0077	8.0904
7	491.11	1.0002	51.313	1.0007	27.075	1.0015	18.570	1.0025	13.930	1.0040	11.213	1.0057	9.3850	1.0078	8.0717
8	429.72	1.0002	50.558	1.0007	26.864	1.0015	18.489	1.0025	13.874	1.0040	11.176	1.0057	9.3596	1.0078	8.0529
9	381.97	1.0002	49.826	1.0007	26.655	1.0015	18.408	1.0025	13.818	1.0040	11.140	1.0058	9.3343	1.0078	8.0342
10	343.77	1.0002	49.114	1.0007	26.450	1.0015	18.326	1.0025	13.763	1.0041	11.104	1.0058	9.3092	1.0079	8.0156
11	312.52	1.0002	48.422	1.0007	26.249	1.0015	18.245	1.0025	13.708	1.0041	11.069	1.0058	9.2842	1.0079	7.9971
12	286.48	1.0002	47.750	1.0007	26.050	1.0016	18.164	1.0026	13.654	1.0041	11.033	1.0059	9.2593	1.0079	7.9787
13	264.44	1.0002	47.096	1.0007	25.854	1.0016	18.083	1.0026	13.600	1.0041	10.998	1.0059	9.2346	1.0080	7.9604
14	245.55	1.0002	46.460	1.0008	25.661	1.0016	18.002	1.0026	13.547	1.0042	10.963	1.0059	9.2100	1.0080	7.9421
15	229.18	1.0002	45.840	1.0008	25.471	1.0016	17.921	1.0026	13.494	1.0042	10.929	1.0059	9.1855	1.0080	7.9240
16	214.86	1.0002	45.237	1.0008	25.284	1.0016	17.840	1.0026	13.441	1.0042	10.894	1.0059	9.1612	1.0081	7.9059
17	202.22	1.0002	44.650	1.0008	25.100	1.0016	17.759	1.0026	13.389	1.0043	10.860	1.0059	9.1370	1.0081	7.8879
18	190.99	1.0002	44.077	1.0008	24.918	1.0017	17.678	1.0026	13.337	1.0043	10.826	1.0059	9.1129	1.0081	7.8700
19	180.73	1.0003	43.520	1.0008	24.739	1.0017	17.597	1.0027	13.286	1.0043	10.792	1.0059	9.0890	1.0082	7.8522
20	171.89	1.0003	42.976	1.0008	24.562	1.0017	17.516	1.0027	13.235	1.0043	10.758	1.0059	9.0651	1.0082	7.8344
21	163.70	1.0003	42.445	1.0008	24.388	1.0017	17.435	1.0027	13.184	1.0044	10.725	1.0059	9.0414	1.0083	7.8168
22	156.26	1.0003	41.928	1.0008	24.216	1.0017	17.354	1.0027	13.134	1.0044	10.692	1.0059	9.0179	1.0083	7.7992
23	149.47	1.0003	41.423	1.0009	24.047	1.0017	17.273	1.0027	13.084	1.0044	10.659	1.0059	8.9944	1.0084	7.7817
24	143.24	1.0003	40.930	1.0009	23.880	1.0018	17.192	1.0027	13.034	1.0044	10.626	1.0059	8.9711	1.0084	7.7642
25	137.51	1.0003	40.448	1.0009	23.716	1.0018	17.111	1.0027	12.985	1.0045	10.593	1.0059	8.9478	1.0084	7.7466
26	132.22	1.0003	39.978	1.0009	23.553	1.0018	17.030	1.0027	12.937	1.0045	10.561	1.0059	8.9248	1.0085	7.7290
27	127.32	1.0003	39.518	1.0009	23.393	1.0018	16.949	1.0027	12.888	1.0046	10.529	1.0059	8.9019	1.0085	7.7114
28	122.78	1.0003	39.069	1.0009	23.235	1.0018	16.868	1.0027	12.840	1.0046	10.497	1.0059	8.8790	1.0085	7.6935
29	118.54	1.0003	38.631	1.0009	23.079	1.0018	16.787	1.0027	12.793	1.0046	10.465	1.0059	8.8563	1.0086	7.6758
30	114.59	1.0003	38.201	1.0009	22.925	1.0019	16.706	1.0027	12.745	1.0046	10.433	1.0059	8.8337	1.0086	7.6583
31	110.90	1.0003	37.782	1.0010	22.774	1.0019	16.625	1.0027	12.698	1.0046	10.402	1.0059	8.8112	1.0087	7.6404
32	107.43	1.0003	37.371	1.0010	22.624	1.0019	16.544	1.0027	12.652	1.0047	10.371	1.0059	8.7888	1.0087	7.6226
33	104.17	1.0004	36.969	1.0010	22.476	1.0019	16.463	1.0027	12.606	1.0047	10.340	1.0059	8.7665	1.0087	7.6048
34	101.11	1.0004	36.576	1.0010	22.330	1.0019	16.382	1.0027	12.560	1.0047	10.309	1.0059	8.7444	1.0088	7.5872
35	98.223	1.0004	36.191	1.0010	22.186	1.0019	16.301	1.0027	12.514	1.0048	10.278	1.0059	8.7223	1.0088	7.5700
36	95.493	1.0004	35.814	1.0010	22.044	1.0020	16.220	1.0027	12.469	1.0048	10.248	1.0067	8.7006	1.0089	7.5511
37	92.914	1.0004	35.445	1.0010	21.904	1.0020	16.139	1.0027	12.424	1.0048	10.217	1.0067	8.6786	1.0089	7.5346
38	90.489	1.0004	35.084	1.0010	21.765	1.0020	16.058	1.0027	12.379	1.0048	10.187	1.0067	8.6569	1.0089	7.5182
39	88.149	1.0004	34.729	1.0011	21.629	1.0020	15.977	1.0027	12.335	1.0049	10.157	1.0068	8.6353	1.0090	7.5019
40	85.946	1.0004	34.382	1.0011	21.494	1.0020	15.896	1.0027	12.291	1.0049	10.127	1.0068	8.6138	1.0090	7.4857
41	83.849	1.0004	34.042	1.0011	21.360	1.0021	15.815	1.0027	12.248	1.0049	10.098	1.0058	8.5924	1.0090	7.4700
42	81.853	1.0004	33.708	1.0011	21.228	1.0021	15.734	1.0027	12.204	1.0050	10.068	1.0058	8.5711	1.0091	7.4543
43	79.950	1.0004	33.381	1.0011	21.098	1.0021	15.653	1.0027	12.161	1.0050	10.039	1.0058	8.5499	1.0091	7.4387
44	78.133	1.0004	33.060	1.0011	20.970	1.0021	15.572	1.0027	12.118	1.0050	10.010	1.0059	8.5289	1.0092	7.4231
45	76.396	1.0005	32.745	1.0011	20.843	1.0021	15.491	1.0027	12.076	1.0050	9.9812	1.0070	8.5079	1.0092	7.4156
46	74.736	1.0005	32.437	1.0012	20.717	1.0022	15.410	1.0027	12.034	1.0051	9.9525	1.0070	8.4871	1.0092	7.3998
47	73.146	1.0005	32.134	1.0012	20.593	1.0022	15.329	1.0027	11.992	1.0051	9.9239	1.0070	8.4663	1.0093	7.3840
48	71.622	1.0005	31.836	1.0012	20.471	1.0022	15.248	1.0027	11.950	1.0051	9.8953	1.0071	8.4457	1.0093	7.3683
49	70.160	1.0005	31.544	1.0012	20.350	1.0022	15.167	1.0027	11.909	1.0052	9.8672	1.0071	8.4251	1.0094	7.3527
50	68.757	1.0005	31.257	1.0012	20.230	1.0022	15.086	1.0027	11.868	1.0052	9.8391	1.0071	8.4046	1.0094	7.3372
51	67.409	1.0005	30.976	1.0012	20.112	1.0023	15.005	1.0027	11.828	1.0052	9.8112	1.0072	8.3843	1.0094	7.3217
52	66.113	1.0005	30.699	1.0012	19.995	1.0023	14.924	1.0027	11.787	1.0052	9.7834	1.0072	8.3640	1.0095	7.3063
53	64.866	1.0005	30.428	1.0013	19.880	1.0023	14.843	1.0027	11.747	1.0053	9.7558	1.0073	8.3439	1.0095	7.2909
54	63.664	1.0005	30.161	1.0013	19.766	1.0023	14.762	1.0027	11.707	1.0053	9.7283	1.0073	8.3238	1.0096	7.2757
55	62.507	1.0005	29.899	1.0013	19.653	1.0023	14.681	1.0027	11.668	1.0053	9.7010	1.0073	8.3039	1.0096	7.2604
56	61.391	1.0006	29.641	1.0013	19.541	1.0024	14.599	1.0027	11.628	1.0054	9.6739	1.0074	8.2840	1.0097	7.2453
57	60.317	1.0006	29.388	1.0013	19.431	1.0024	14.518	1.0027	11.589	1.0054	9.6469	1.0074	8.2642	1.0097	7.2302
58	59.274	1.0006	29.139	1.0013	19.322	1.0024	14.437	1.0027	11.550	1.0054	9.6200	1.0074	8.2446	1.0097	7.2152
59	58.270	1.0006	28.894												

°	8°		9°		10°		11°		12°		13°		14°		15°	
	Sec.	Conc.	Sec.	Conc.	Sec.	Conc.	Sec.	Conc.	Sec.	Conc.	Sec.	Conc.	Sec.	Conc.	Sec.	Conc.
0	1.0098	7.1853	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	1.0223	4.8097	1.0263	4.4454	1.0306	4.1336	1.0353	3.8637
1	1.0099	7.1704	1.0125	6.3807	1.0155	5.7493	1.0188	5.2330	1.0224	4.8032	1.0264	4.4398	1.0307	4.1287	1.0353	3.8595
2	1.0099	7.1557	1.0125	6.3690	1.0155	5.7398	1.0188	5.2252	1.0225	4.7966	1.0264	4.4342	1.0308	4.1239	1.0354	3.8553
3	1.0099	7.1409	1.0126	6.3574	1.0156	5.7304	1.0189	5.2174	1.0225	4.7901	1.0265	4.4287	1.0308	4.1191	1.0355	3.8512
4	1.0100	7.1263	1.0126	6.3458	1.0156	5.7210	1.0189	5.2097	1.0226	4.7835	1.0266	4.4231	1.0309	4.1144	1.0356	3.8470
5	1.0100	7.1117	1.0127	6.3343	1.0157	5.7117	1.0190	5.2021	1.0226	4.7770	1.0266	4.4176	1.0310	4.1096	1.0357	3.8428
6	1.0101	7.0972	1.0127	6.3228	1.0157	5.7033	1.0191	5.1942	1.0227	4.7706	1.0267	4.4121	1.0311	4.1048	1.0357	3.8387
7	1.0101	7.0827	1.0128	6.3113	1.0158	5.6950	1.0191	5.1862	1.0228	4.7641	1.0268	4.4065	1.0311	4.1001	1.0358	3.8346
8	1.0102	7.0683	1.0128	6.2999	1.0158	5.6868	1.0192	5.1788	1.0228	4.7576	1.0268	4.4011	1.0312	4.0953	1.0359	3.8304
9	1.0102	7.0539	1.0129	6.2885	1.0159	5.6785	1.0192	5.1712	1.0229	4.7512	1.0269	4.3956	1.0313	4.0906	1.0360	3.8263
10	1.0102	7.0396	1.0129	6.2772	1.0159	5.6653	1.0193	5.1636	1.0230	4.7448	1.0270	4.3901	1.0314	4.0859	1.0361	3.8222
11	1.0103	7.0254	1.0130	6.2659	1.0160	5.6561	1.0193	5.1560	1.0230	4.7384	1.0271	4.3847	1.0314	4.0812	1.0362	3.8181
12	1.0103	7.0112	1.0130	6.2546	1.0160	5.6470	1.0194	5.1484	1.0231	4.7320	1.0271	4.3792	1.0315	4.0765	1.0362	3.8140
13	1.0104	6.9971	1.0131	6.2434	1.0161	5.6379	1.0195	5.1409	1.0232	4.7257	1.0272	4.3738	1.0316	4.0718	1.0363	3.8100
14	1.0104	6.9830	1.0131	6.2322	1.0162	5.6288	1.0195	5.1333	1.0233	4.7193	1.0273	4.3684	1.0317	4.0672	1.0364	3.8059
15	1.0105	6.9690	1.0132	6.2211	1.0162	5.6197	1.0196	5.1258	1.0234	4.7130	1.0273	4.3630	1.0317	4.0625	1.0365	3.8018
16	1.0105	6.9550	1.0132	6.2100	1.0163	5.6107	1.0196	5.1183	1.0234	4.7067	1.0274	4.3576	1.0318	4.0579	1.0366	3.7978
17	1.0105	6.9411	1.0133	6.1990	1.0163	5.6017	1.0197	5.1109	1.0235	4.7004	1.0275	4.3522	1.0319	4.0532	1.0367	3.7937
18	1.0106	6.9273	1.0133	6.1880	1.0164	5.5928	1.0198	5.1034	1.0235	4.6942	1.0276	4.3469	1.0320	4.0486	1.0367	3.7897
19	1.0106	6.9135	1.0134	6.1770	1.0164	5.5838	1.0198	5.0960	1.0236	4.6879	1.0276	4.3415	1.0320	4.0440	1.0368	3.7857
20	1.0107	6.8998	1.0134	6.1661	1.0165	5.5749	1.0199	5.0886	1.0236	4.6817	1.0277	4.3362	1.0321	4.0394	1.0369	3.7816
21	1.0107	6.8861	1.0135	6.1552	1.0165	5.5660	1.0199	5.0812	1.0237	4.6754	1.0278	4.3309	1.0322	4.0348	1.0370	3.7776
22	1.0107	6.8725	1.0135	6.1443	1.0166	5.5572	1.0200	5.0739	1.0237	4.6692	1.0278	4.3256	1.0323	4.0302	1.0371	3.7736
23	1.0108	6.8589	1.0136	6.1335	1.0166	5.5484	1.0201	5.0666	1.0238	4.6631	1.0279	4.3203	1.0323	4.0256	1.0371	3.7697
24	1.0108	6.8454	1.0136	6.1227	1.0167	5.5396	1.0201	5.0593	1.0239	4.6569	1.0280	4.3150	1.0324	4.0211	1.0372	3.7657
25	1.0109	6.8320	1.0137	6.1120	1.0167	5.5308	1.0202	5.0520	1.0239	4.6507	1.0281	4.3098	1.0325	4.0165	1.0373	3.7617
26	1.0109	6.8185	1.0137	6.1013	1.0168	5.5221	1.0202	5.0447	1.0240	4.6446	1.0281	4.3045	1.0326	4.0120	1.0374	3.7577
27	1.0110	6.8052	1.0138	6.0906	1.0169	5.5134	1.0203	5.0375	1.0241	4.6385	1.0282	4.2993	1.0327	4.0074	1.0375	3.7538
28	1.0110	6.7919	1.0138	6.0800	1.0169	5.5047	1.0204	5.0302	1.0242	4.6324	1.0283	4.2941	1.0327	4.0029	1.0376	3.7498
29	1.0111	6.7787	1.0139	6.0694	1.0170	5.4960	1.0205	5.0230	1.0242	4.6263	1.0284	4.2889	1.0328	3.9984	1.0377	3.7459
30	1.0111	6.7655	1.0139	6.0588	1.0170	5.4874	1.0205	5.0158	1.0243	4.6202	1.0284	4.2836	1.0329	3.9939	1.0377	3.7420
31	1.0111	6.7523	1.0139	6.0483	1.0171	5.4788	1.0205	5.0087	1.0243	4.6142	1.0285	4.2785	1.0330	3.9894	1.0378	3.7380
32	1.0112	6.7392	1.0140	6.0379	1.0171	5.4702	1.0206	5.0015	1.0244	4.6081	1.0285	4.2733	1.0331	3.9850	1.0379	3.7341
33	1.0112	6.7262	1.0140	6.0274	1.0172	5.4617	1.0207	4.9944	1.0245	4.6021	1.0286	4.2681	1.0331	3.9805	1.0380	3.7302
34	1.0113	6.7133	1.0141	6.0170	1.0172	5.4532	1.0208	4.9873	1.0246	4.5961	1.0287	4.2630	1.0332	3.9760	1.0381	3.7263
35	1.0113	6.7003	1.0141	6.0066	1.0173	5.4447	1.0208	4.9802	1.0246	4.5901	1.0288	4.2579	1.0333	3.9716	1.0382	3.7224
36	1.0114	6.6874	1.0142	5.9963	1.0174	5.4362	1.0209	4.9732	1.0247	4.5841	1.0289	4.2527	1.0334	3.9672	1.0383	3.7186
37	1.0114	6.6745	1.0142	5.9860	1.0174	5.4278	1.0209	4.9661	1.0247	4.5782	1.0289	4.2476	1.0334	3.9627	1.0384	3.7147
38	1.0115	6.6617	1.0143	5.9758	1.0175	5.4194	1.0210	4.9591	1.0248	4.5722	1.0290	4.2425	1.0335	3.9583	1.0385	3.7108
39	1.0115	6.6490	1.0143	5.9655	1.0175	5.4110	1.0210	4.9521	1.0249	4.5663	1.0291	4.2375	1.0336	3.9539	1.0386	3.7070
40	1.0115	6.6363	1.0144	5.9554	1.0176	5.4026	1.0211	4.9452	1.0249	4.5604	1.0291	4.2324	1.0337	3.9495	1.0386	3.7031
41	1.0116	6.6237	1.0144	5.9452	1.0176	5.3943	1.0211	4.9382	1.0250	4.5545	1.0292	4.2273	1.0338	3.9451	1.0387	3.6993
42	1.0116	6.6111	1.0145	5.9351	1.0177	5.3860	1.0212	4.9313	1.0251	4.5486	1.0293	4.2223	1.0339	3.9408	1.0387	3.6955
43	1.0117	6.5985	1.0145	5.9250	1.0177	5.3777	1.0213	4.9243	1.0251	4.5428	1.0293	4.2173	1.0340	3.9364	1.0388	3.6917
44	1.0117	6.5860	1.0146	5.9150	1.0178	5.3695	1.0213	4.9175	1.0252	4.5369	1.0294	4.2123	1.0341	3.9320	1.0389	3.6878
45	1.0118	6.5736	1.0146	5.9049	1.0179	5.3612	1.0214	4.9106	1.0253	4.5311	1.0295	4.2072	1.0341	3.9277	1.0390	3.6840
46	1.0118	6.5612	1.0147	5.8950	1.0179	5.3530	1.0215	4.9037	1.0254	4.5253	1.0296	4.2022	1.0342	3.9234	1.0391	3.6802
47	1.0119	6.5488	1.0147	5.8850	1.0180	5.3449	1.0215	4.8969	1.0254	4.5195	1.0296	4.1972	1.0343	3.9191	1.0392	3.6765
48	1.0119	6.5365	1.0148	5.8751	1.0180	5.3367	1.0216	4.8901	1.0255	4.5137	1.0297	4.1923	1.0344	3.9147	1.0393	3.6727
49	1.0119	6.5243	1.0148	5.8652	1.0181	5.3286	1.0216	4.8833	1.0255	4.5079	1.0298	4.1875	1.0345	3.9104	1.0394	3.6689
50	1.0120	6.5121	1.0149	5.8554	1.0181	5.3205	1.0217	4.8765	1.0256	4.5021	1.0299	4.1824	1.0346	3.9061	1.0395	3.6651
51	1.0120	6.4999	1.0150	5.8456	1.0182	5.3124	1.0218	4.8697	1.0257	4.4964	1.0299	4.1774	1.0347	3.9018	1.0395	3.6614
52	1.0121	6.4878	1.0150	5.8358	1.0182	5.3044	1.0218	4.8630	1.0258	4.4907	1.0300	4.1725	1.0348	3.8976	1.0396	3.6576
53	1.0121	6.4757	1.0151	5.8261	1.0183	5.2963	1.0219	4.8563	1.0259	4.4850	1.0301	4.1677	1.0349	3.8933	1.0397	3.6539
54	1.0122	6.4637	1.0151	5.8163	1.0184	5.2883	1.0220	4.8496	1.0260	4.4793	1.0302	4.1627	1.0350	3.8890	1.0398	3.6502
55	1.0122	6.4517	1.0152	5.8067	1.0185	5.2803	1.0221	4.8429	1.0261	4.4736	1.0303	4.1578	1.0351	3.8848	1.0399	3.6464
56	1.0123	6.4398	1.0152	5.7970	1.0185	5.2724	1.0221	4.8362	1.0262	4.4679	1.0304	4.1529	1.0352	3.8805	1.0400	3.6427
57	1.0123	6.4279	1.0153	5.7874	1.0185	5.2645	1.0222	4.8296	1.0263	4.4623	1.0305	4.1481	1.0353	3.8763	1.0401	3.6390
58	1.0124	6.4160	1.0153	5.7778	1.0186	5.2566	1.0223	4.8229	1.0264	4.4566	1.0306	4.1432	1.0354	3.8721	1.0402	3.6353
59	1.0124	6.4042	1.0154	5.7683	1.0186	5.2487	1.0223	4.8163	1.0265	4.4510	1.0307	4.1384	1.0355	3.8679	1.0403	3.6316
60	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	1.0223	4.8097	1.0266	4.4454	1.0308	4.1336	1.0356	3.8637	1.0404	3.6279

TM004-172

TM004-171

°	16°		17°		18°		19°		20°		21°		22°		23°	
	Sec.	Consec.	Sec.	Consec.	Sec.	Consec.	Sec.	Consec.	Sec.	Consec.	Sec.	Consec.	Sec.	Consec.	Sec.	Consec.
0	1.0403	3.6279	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	1.0642	2.9238	1.0711	2.7904	1.0785	2.6695	1.0864	2.5593
1	1.0404	3.6280	1.0458	3.4138	1.0516	3.2362	1.0577	3.0716	1.0643	2.9239	1.0712	2.7883	1.0786	2.6675	1.0865	2.5575
2	1.0405	3.6281	1.0459	3.4106	1.0517	3.2363	1.0578	3.0717	1.0644	2.9240	1.0713	2.7862	1.0787	2.6657	1.0866	2.5558
3	1.0406	3.6282	1.0460	3.4073	1.0518	3.2364	1.0579	3.0718	1.0645	2.9241	1.0714	2.7841	1.0788	2.6638	1.0867	2.5540
4	1.0407	3.6283	1.0461	3.4041	1.0519	3.2365	1.0580	3.0719	1.0646	2.9242	1.0715	2.7820	1.0789	2.6618	1.0868	2.5523
5	1.0408	3.6284	1.0462	3.4009	1.0520	3.2366	1.0581	3.0720	1.0647	2.9243	1.0716	2.7799	1.0790	2.6599	1.0869	2.5506
6	1.0409	3.6285	1.0463	3.3977	1.0521	3.2367	1.0582	3.0721	1.0648	2.9244	1.0717	2.7778	1.0791	2.6580	1.0870	2.5488
7	1.0410	3.6286	1.0464	3.3945	1.0522	3.2368	1.0583	3.0722	1.0649	2.9245	1.0718	2.7757	1.0792	2.6561	1.0871	2.5471
8	1.0411	3.6287	1.0465	3.3913	1.0523	3.2369	1.0584	3.0723	1.0650	2.9246	1.0719	2.7736	1.0793	2.6542	1.0872	2.5453
9	1.0412	3.6288	1.0466	3.3881	1.0524	3.2370	1.0585	3.0724	1.0651	2.9247	1.0720	2.7715	1.0794	2.6523	1.0873	2.5436
10	1.0413	3.6289	1.0467	3.3849	1.0525	3.2371	1.0586	3.0725	1.0652	2.9248	1.0721	2.7694	1.0795	2.6504	1.0874	2.5419
11	1.0414	3.6290	1.0468	3.3817	1.0526	3.2372	1.0587	3.0726	1.0653	2.9249	1.0722	2.7673	1.0796	2.6485	1.0875	2.5402
12	1.0415	3.6291	1.0469	3.3785	1.0527	3.2373	1.0588	3.0727	1.0654	2.9250	1.0723	2.7652	1.0797	2.6466	1.0876	2.5384
13	1.0416	3.6292	1.0470	3.3753	1.0528	3.2374	1.0589	3.0728	1.0655	2.9251	1.0724	2.7631	1.0798	2.6447	1.0877	2.5367
14	1.0417	3.6293	1.0471	3.3722	1.0529	3.2375	1.0590	3.0729	1.0656	2.9252	1.0725	2.7610	1.0799	2.6428	1.0878	2.5350
15	1.0418	3.6294	1.0472	3.3690	1.0530	3.2376	1.0591	3.0730	1.0657	2.9253	1.0726	2.7589	1.0800	2.6409	1.0879	2.5333
16	1.0419	3.6295	1.0473	3.3659	1.0531	3.2377	1.0592	3.0731	1.0658	2.9254	1.0727	2.7568	1.0801	2.6390	1.0880	2.5316
17	1.0420	3.6296	1.0474	3.3627	1.0532	3.2378	1.0593	3.0732	1.0659	2.9255	1.0728	2.7547	1.0802	2.6371	1.0881	2.5299
18	1.0421	3.6297	1.0475	3.3596	1.0533	3.2379	1.0594	3.0733	1.0660	2.9256	1.0729	2.7526	1.0803	2.6352	1.0882	2.5282
19	1.0422	3.6298	1.0476	3.3565	1.0534	3.2380	1.0595	3.0734	1.0661	2.9257	1.0730	2.7505	1.0804	2.6333	1.0883	2.5264
20	1.0423	3.6299	1.0477	3.3534	1.0535	3.2381	1.0596	3.0735	1.0662	2.9258	1.0731	2.7484	1.0805	2.6314	1.0884	2.5247
21	1.0424	3.6300	1.0478	3.3503	1.0536	3.2382	1.0597	3.0736	1.0663	2.9259	1.0732	2.7463	1.0806	2.6295	1.0885	2.5230
22	1.0425	3.6301	1.0479	3.3472	1.0537	3.2383	1.0598	3.0737	1.0664	2.9260	1.0733	2.7442	1.0807	2.6276	1.0886	2.5213
23	1.0426	3.6302	1.0480	3.3441	1.0538	3.2384	1.0599	3.0738	1.0665	2.9261	1.0734	2.7421	1.0808	2.6257	1.0887	2.5196
24	1.0427	3.6303	1.0481	3.3410	1.0539	3.2385	1.0600	3.0739	1.0666	2.9262	1.0735	2.7400	1.0809	2.6238	1.0888	2.5179
25	1.0428	3.6304	1.0482	3.3379	1.0540	3.2386	1.0601	3.0740	1.0667	2.9263	1.0736	2.7379	1.0810	2.6219	1.0889	2.5162
26	1.0429	3.6305	1.0483	3.3348	1.0541	3.2387	1.0602	3.0741	1.0668	2.9264	1.0737	2.7358	1.0811	2.6200	1.0890	2.5145
27	1.0430	3.6306	1.0484	3.3317	1.0542	3.2388	1.0603	3.0742	1.0669	2.9265	1.0738	2.7337	1.0812	2.6181	1.0891	2.5128
28	1.0431	3.6307	1.0485	3.3286	1.0543	3.2389	1.0604	3.0743	1.0670	2.9266	1.0739	2.7316	1.0813	2.6162	1.0892	2.5111
29	1.0432	3.6308	1.0486	3.3255	1.0544	3.2390	1.0605	3.0744	1.0671	2.9267	1.0740	2.7295	1.0814	2.6143	1.0893	2.5094
30	1.0433	3.6309	1.0487	3.3224	1.0545	3.2391	1.0606	3.0745	1.0672	2.9268	1.0741	2.7274	1.0815	2.6124	1.0894	2.5077
31	1.0434	3.6310	1.0488	3.3193	1.0546	3.2392	1.0607	3.0746	1.0673	2.9269	1.0742	2.7253	1.0816	2.6105	1.0895	2.5060
32	1.0435	3.6311	1.0489	3.3162	1.0547	3.2393	1.0608	3.0747	1.0674	2.9270	1.0743	2.7232	1.0817	2.6086	1.0896	2.5043
33	1.0436	3.6312	1.0490	3.3131	1.0548	3.2394	1.0609	3.0748	1.0675	2.9271	1.0744	2.7211	1.0818	2.6067	1.0897	2.5026
34	1.0437	3.6313	1.0491	3.3100	1.0549	3.2395	1.0610	3.0749	1.0676	2.9272	1.0745	2.7190	1.0819	2.6048	1.0898	2.5009
35	1.0438	3.6314	1.0492	3.3069	1.0550	3.2396	1.0611	3.0750	1.0677	2.9273	1.0746	2.7169	1.0820	2.6029	1.0899	2.4992
36	1.0439	3.6315	1.0493	3.3038	1.0551	3.2397	1.0612	3.0751	1.0678	2.9274	1.0747	2.7148	1.0821	2.6010	1.0900	2.4975
37	1.0440	3.6316	1.0494	3.3007	1.0552	3.2398	1.0613	3.0752	1.0679	2.9275	1.0748	2.7127	1.0822	2.5991	1.0901	2.4958
38	1.0441	3.6317	1.0495	3.2976	1.0553	3.2399	1.0614	3.0753	1.0680	2.9276	1.0749	2.7106	1.0823	2.5972	1.0902	2.4941
39	1.0442	3.6318	1.0496	3.2945	1.0554	3.2400	1.0615	3.0754	1.0681	2.9277	1.0750	2.7085	1.0824	2.5953	1.0903	2.4924
40	1.0443	3.6319	1.0497	3.2914	1.0555	3.2401	1.0616	3.0755	1.0682	2.9278	1.0751	2.7064	1.0825	2.5934	1.0904	2.4907
41	1.0444	3.6320	1.0498	3.2883	1.0556	3.2402	1.0617	3.0756	1.0683	2.9279	1.0752	2.7043	1.0826	2.5915	1.0905	2.4890
42	1.0445	3.6321	1.0499	3.2852	1.0557	3.2403	1.0618	3.0757	1.0684	2.9280	1.0753	2.7022	1.0827	2.5896	1.0906	2.4873
43	1.0446	3.6322	1.0500	3.2821	1.0558	3.2404	1.0619	3.0758	1.0685	2.9281	1.0754	2.7001	1.0828	2.5877	1.0907	2.4856
44	1.0447	3.6323	1.0501	3.2790	1.0559	3.2405	1.0620	3.0759	1.0686	2.9282	1.0755	2.6980	1.0829	2.5858	1.0908	2.4839
45	1.0448	3.6324	1.0502	3.2759	1.0560	3.2406	1.0621	3.0760	1.0687	2.9283	1.0756	2.6959	1.0830	2.5839	1.0909	2.4822
46	1.0449	3.6325	1.0503	3.2728	1.0561	3.2407	1.0622	3.0761	1.0688	2.9284	1.0757	2.6938	1.0831	2.5820	1.0910	2.4805
47	1.0450	3.6326	1.0504	3.2697	1.0562	3.2408	1.0623	3.0762	1.0689	2.9285	1.0758	2.6917	1.0832	2.5801	1.0911	2.4788
48	1.0451	3.6327	1.0505	3.2666	1.0563	3.2409	1.0624	3.0763	1.0690	2.9286	1.0759	2.6896	1.0833	2.5782	1.0912	2.4771
49	1.0452	3.6328	1.0506	3.2635	1.0564	3.2410	1.0625	3.0764	1.0691	2.9287	1.0760	2.6875	1.0834	2.5763	1.0913	2.4754
50	1.0453	3.6329	1.0507	3.2604	1.0565	3.2411	1.0626	3.0765	1.0692	2.9288	1.0761	2.6854	1.0835	2.5744	1.0914	2.4737
51	1.0454	3.6330	1.0508	3.2573	1.0566	3.2412	1.0627	3.0766	1.0693	2.9289	1.0762	2.6833	1.0836	2.5725	1.0915	2.4720
52	1.0455	3.6331	1.0509	3.2542	1.0567	3.2413	1.0628	3.0767	1.0694	2.9290	1.0763	2.6812	1.0837	2.5706	1.0916	2.4703
53	1.0456	3.6332	1.0510	3.2511	1.0568	3.2414	1.0629	3.0768	1.0695	2.9291	1.0764	2.6791	1.0838	2.5687	1.0917	2.4686
54	1.0457	3.6333	1.0511	3.2480	1.0569	3.2415	1.0630	3.0769	1.0696	2.9292	1.0765	2.6770	1.0839	2.5668	1.0918	2.4669
55	1.0458	3.6334	1.0512	3.2449	1.0570	3.2416	1.0631	3.0770	1.0697	2.9293	1.0766	2.6749	1.0840	2.5649	1.0919	2.4652
56	1.0459	3.6335	1.0513	3.2418	1.0571	3.2417	1.0632	3.0771	1.0698	2.9294	1.0767	2.6728	1.0841	2.5630	1.0920	2.4635
57	1.0460	3.6336	1.0514	3.2387	1.0572	3.2418	1.0633	3.0772	1.0699	2.9295	1.0768	2.6707	1.0842	2.5611	1.0921	2.4618
58	1.0461	3.6337	1.0515	3.2356	1.0573	3.2419	1.0634	3.0773	1.0700	2.9296	1.0769	2.6686	1.0843	2.5592	1.0922	2.4601
59	1.0462	3.6338	1.0516	3.2325	1.0574	3.2420	1.0635	3.0774	1.0701	2.9297	1.0770	2.6665	1.0844	2.5573	1.0923	2.4584
60	1.0463	3.6339	1.0517	3.2294	1.0575	3.2421	1.0636	3.0775	1.0702	2.9298	1.0771	2.6644	1.0845	2.5554	1.0924	2.4567

TM 664-174

TM 664-173

34°			35°			36°			37°			38°			39°			31°		
Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	
0	1.0946	2.4586	1.1034	2.3662	1.1126	2.2812	1.1223	2.2017	60	0	1.1326	2.1300	1.1433	2.0627	1.1547	2.0000	1.1666	1.9416		
1	1.0948	2.4570	1.1035	2.3647	1.1127	2.2798	1.1225	2.2004	59	1	1.1327	2.1289	1.1435	2.0616	1.1548	1.9990	1.1668	1.9407		
2	1.0949	2.4554	1.1037	2.3632	1.1129	2.2784	1.1226	2.2002	58	2	1.1329	2.1277	1.1437	2.0605	1.1551	1.9980	1.1670	1.9397		
3	1.0951	2.4538	1.1038	2.3618	1.1131	2.2771	1.1228	2.1989	57	3	1.1331	2.1266	1.1439	2.0594	1.1553	1.9970	1.1672	1.9388		
4	1.0952	2.4522	1.1040	2.3603	1.1132	2.2757	1.1230	2.1977	56	4	1.1333	2.1254	1.1441	2.0583	1.1555	1.9960	1.1674	1.9378		
5	1.0953	2.4506	1.1041	2.3588	1.1134	2.2744	1.1231	2.1964	55	5	1.1335	2.1242	1.1443	2.0573	1.1557	1.9950	1.1676	1.9369		
6	1.0955	2.4490	1.1043	2.3574	1.1135	2.2730	1.1232	2.1952	54	6	1.1336	2.1231	1.1445	2.0562	1.1559	1.9940	1.1678	1.9360		
7	1.0956	2.4474	1.1044	2.3559	1.1137	2.2717	1.1233	2.1939	53	7	1.1338	2.1219	1.1446	2.0551	1.1561	1.9930	1.1681	1.9350		
8	1.0958	2.4458	1.1046	2.3544	1.1139	2.2703	1.1235	2.1927	52	8	1.1340	2.1208	1.1448	2.0540	1.1562	1.9920	1.1683	1.9341		
9	1.0959	2.4442	1.1047	2.3530	1.1140	2.2690	1.1236	2.1914	51	9	1.1341	2.1196	1.1448	2.0530	1.1564	1.9910	1.1685	1.9332		
0	1.0961	2.4426	1.1049	2.3515	1.1142	2.2676	1.1238	2.1902	50	10	1.1343	2.1185	1.1452	2.0519	1.1566	1.9900	1.1687	1.9322		
1	1.0962	2.4411	1.1050	2.3501	1.1143	2.2663	1.1242	2.1889	49	11	1.1345	2.1173	1.1454	2.0508	1.1568	1.9890	1.1689	1.9313		
2	1.0963	2.4395	1.1052	2.3486	1.1145	2.2650	1.1243	2.1877	48	12	1.1347	2.1162	1.1456	2.0498	1.1570	1.9880	1.1691	1.9304		
3	1.0965	2.4379	1.1053	2.3472	1.1147	2.2636	1.1245	2.1865	47	13	1.1349	2.1150	1.1458	2.0487	1.1572	1.9870	1.1693	1.9295		
4	1.0966	2.4363	1.1055	2.3457	1.1148	2.2623	1.1247	2.1852	46	14	1.1350	2.1139	1.1459	2.0476	1.1574	1.9860	1.1695	1.9285		
5	1.0968	2.4347	1.1056	2.3443	1.1150	2.2610	1.1248	2.1840	45	15	1.1352	2.1127	1.1461	2.0466	1.1576	1.9850	1.1697	1.9276		
6	1.0969	2.4332	1.1058	2.3428	1.1151	2.2596	1.1250	2.1828	44	16	1.1354	2.1116	1.1463	2.0455	1.1578	1.9840	1.1699	1.9267		
7	1.0971	2.4316	1.1059	2.3414	1.1153	2.2583	1.1252	2.1815	43	17	1.1356	2.1104	1.1465	2.0444	1.1580	1.9830	1.1701	1.9258		
8	1.0972	2.4300	1.1061	2.3399	1.1154	2.2570	1.1253	2.1803	42	18	1.1357	2.1093	1.1467	2.0434	1.1582	1.9820	1.1703	1.9249		
9	1.0973	2.4285	1.1062	2.3385	1.1155	2.2556	1.1255	2.1791	41	19	1.1359	2.1082	1.1469	2.0423	1.1584	1.9811	1.1705	1.9239		
0	1.0975	2.4269	1.1064	2.3371	1.1158	2.2543	1.1257	2.1778	40	20	1.1361	2.1070	1.1471	2.0413	1.1586	1.9801	1.1707	1.9230		
1	1.0976	2.4254	1.1065	2.3356	1.1159	2.2530	1.1258	2.1766	39	21	1.1363	2.1059	1.1473	2.0402	1.1588	1.9791	1.1709	1.9221		
2	1.0978	2.4238	1.1067	2.3342	1.1161	2.2517	1.1260	2.1754	38	22	1.1365	2.1048	1.1474	2.0392	1.1590	1.9781	1.1712	1.9212		
3	1.0979	2.4222	1.1068	2.3328	1.1163	2.2503	1.1262	2.1742	37	23	1.1366	2.1036	1.1476	2.0381	1.1592	1.9771	1.1714	1.9203		
4	1.0981	2.4207	1.1070	2.3313	1.1164	2.2490	1.1264	2.1730	36	24	1.1368	2.1025	1.1478	2.0370	1.1594	1.9761	1.1716	1.9193		
5	1.0982	2.4191	1.1072	2.3299	1.1166	2.2477	1.1265	2.1717	35	25	1.1370	2.1014	1.1480	2.0360	1.1596	1.9752	1.1718	1.9184		
6	1.0984	2.4176	1.1073	2.3285	1.1167	2.2464	1.1267	2.1705	34	26	1.1372	2.1002	1.1482	2.0350	1.1598	1.9742	1.1720	1.9175		
7	1.0985	2.4160	1.1075	2.3271	1.1169	2.2451	1.1269	2.1693	33	27	1.1375	2.0991	1.1484	2.0339	1.1600	1.9732	1.1722	1.9166		
8	1.0986	2.4145	1.1076	2.3256	1.1171	2.2438	1.1270	2.1681	32	28	1.1377	2.0980	1.1486	2.0328	1.1602	1.9722	1.1724	1.9157		
9	1.0988	2.4130	1.1078	2.3242	1.1172	2.2425	1.1272	2.1669	31	29	1.1379	2.0969	1.1488	2.0318	1.1604	1.9713	1.1726	1.9148		
0	1.0989	2.4114	1.1079	2.3228	1.1174	2.2411	1.1274	2.1657	30	30	1.1379	2.0957	1.1489	2.0308	1.1606	1.9703	1.1728	1.9139		
1	1.0991	2.4099	1.1081	2.3214	1.1176	2.2398	1.1275	2.1645	29	31	1.1381	2.0946	1.1491	2.0297	1.1608	1.9693	1.1730	1.9130		
2	1.0992	2.4083	1.1082	2.3200	1.1177	2.2385	1.1277	2.1633	28	32	1.1382	2.0935	1.1493	2.0287	1.1610	1.9683	1.1732	1.9121		
3	1.0994	2.4068	1.1084	2.3186	1.1179	2.2372	1.1279	2.1621	27	33	1.1384	2.0924	1.1495	2.0276	1.1612	1.9674	1.1734	1.9112		
4	1.0995	2.4053	1.1085	2.3172	1.1180	2.2359	1.1281	2.1608	26	34	1.1386	2.0912	1.1497	2.0266	1.1614	1.9664	1.1737	1.9102		
5	1.0996	2.4037	1.1087	2.3158	1.1182	2.2346	1.1282	2.1596	25	35	1.1388	2.0901	1.1499	2.0256	1.1616	1.9654	1.1739	1.9093		
6	1.0998	2.4022	1.1088	2.3143	1.1184	2.2333	1.1284	2.1584	24	36	1.1390	2.0890	1.1501	2.0245	1.1618	1.9645	1.1741	1.9084		
7	1.1000	2.4007	1.1090	2.3129	1.1185	2.2320	1.1286	2.1572	23	37	1.1391	2.0879	1.1503	2.0235	1.1620	1.9635	1.1743	1.9075		
8	1.1001	2.3992	1.1092	2.3115	1.1187	2.2307	1.1287	2.1560	22	38	1.1393	2.0868	1.1505	2.0224	1.1622	1.9625	1.1745	1.9066		
9	1.1003	2.3976	1.1093	2.3101	1.1189	2.2294	1.1289	2.1548	21	39	1.1395	2.0857	1.1507	2.0214	1.1624	1.9616	1.1747	1.9057		
0	1.1004	2.3961	1.1095	2.3087	1.1190	2.2282	1.1291	2.1536	20	40	1.1397	2.0846	1.1508	2.0204	1.1626	1.9606	1.1749	1.9048		
1	1.1005	2.3946	1.1096	2.3073	1.1192	2.2269	1.1293	2.1525	19	41	1.1399	2.0835	1.1510	2.0194	1.1628	1.9596	1.1751	1.9039		
2	1.1007	2.3931	1.1098	2.3059	1.1193	2.2256	1.1294	2.1513	18	42	1.1401	2.0824	1.1512	2.0183	1.1630	1.9587	1.1753	1.9030		
3	1.1008	2.3916	1.1099	2.3046	1.1195	2.2243	1.1296	2.1501	17	43	1.1402	2.0812	1.1514	2.0173	1.1632	1.9577	1.1756	1.9021		
4	1.1010	2.3901	1.1101	2.3032	1.1197	2.2230	1.1298	2.1489	16	44	1.1404	2.0801	1.1516	2.0163	1.1634	1.9568	1.1758	1.9013		
5	1.1011	2.3886	1.1102	2.3018	1.1198	2.2217	1.1299	2.1477	15	45	1.1406	2.0790	1.1518	2.0153	1.1636	1.9558	1.1760	1.9004		
6	1.1013	2.3871	1.1104	2.3004	1.1200	2.2204	1.1301	2.1465	14	46	1.1408	2.0779	1.1520	2.0142	1.1638	1.9548	1.1762	1.8995		
7	1.1014	2.3856	1.1106	2.2990	1.1202	2.2192	1.1303	2.1453	13	47	1.1410	2.0768	1.1522	2.0132	1.1640	1.9539	1.1764	1.8986		
8	1.1016	2.3841	1.1107	2.2976	1.1203	2.2179	1.1305	2.1443	12	48	1.1411	2.0757	1.1524	2.0122	1.1642	1.9530	1.1766	1.8977		
9	1.1017	2.3826	1.1109	2.2962	1.1205	2.2166	1.1306	2.1430	11	49	1.1413	2.0746	1.1526	2.0111	1.1644	1.9520	1.1768	1.8968		
0	1.1019	2.3811	1.1110	2.2949	1.1207	2.2153	1.1308	2.1418	10	50	1.1415	2.0735	1.1528	2.0101	1.1646	1.9510	1.1770	1.8959		
1	1.1020	2.3796	1.1112	2.2935	1.1208	2.2141	1.1310	2.1406	9	51	1.1417	2.0725	1.1530	2.0091	1.1648	1.9501	1.1772	1.8950		
2	1.1022	2.3781	1.1113	2.2921	1.1210	2.2128	1.1312	2.1394	8	52	1.1419	2.0714	1.1532	2.0081	1.1650	1.9492	1.1774	1.8941		
3	1.1023	2.3766	1.1115	2.2907	1.1212	2.2115	1.1313	2.1382	7	53	1.1421	2.0703	1.1533	2.0071	1.1652	1.9483	1.1777	1.8932		
4	1.1025	2.3751	1.1116	2.2894	1.1213	2.2103	1.1315	2.1371	6	54	1.1422	2.0692	1.1535	2.0061	1.1654	1.9474	1.1779	1.8924		
5	1.1026	2.3736	1.1118	2.2880	1.1215	2.2090	1.1317	2.1359	5	55	1.1424	2.0681	1.1537	2.0050	1.1656	1.9465	1.1781	1.8915		
6	1.1028	2.3721	1.1120	2.2866	1.1217	2.2077	1.1319	2.1347	4	56	1.1426	2.0670	1.1539	2.0040	1.1658	1.9456	1.1783	1.8906		
7	1.1029	2.3706	1.1121	2.2853	1.1218	2.2065	1.1320	2.1335	3	57	1.1428	2.0659	1.1541	2.0030	1.1660	1.9447	1.1785	1.8897		
8	1.1031	2.3691	1.1123	2.2839	1.1220	2.2052	1.1322	2.1324	2	58	1.1430	2.0648	1.1543	2.0020	1.1662	1.9438	1.1787	1.8888		
9	1.1032	2.3677	1.1124	2.2825	1.1222	2.2039	1.1324	2.1312	1	59	1.1432	2.0637	1.1545	2.0010	1.1664	1.9429	1.1789	1.8879		
0	1.1034	2.3662	1.1126	2.2812	1.1223	2.2027	1.1326	2.1300	0	60	1.1433	2.0627	1.1547	2.0000	1.1666	1.9416	1.1792	1.8871		

°	32°		33°		34°		35°		36°		37°		38°		39°	
	Sec.	Coast.	Sec.	Coast.	Sec.	Coast.	Sec.	Coast.	Sec.	Coast.	Sec.	Coast.	Sec.	Coast.	Sec.	Coast.
0	1.1792	1.8871	1.1924	1.8361	1.2062	1.7883	1.2208	1.7434	1.2361	1.7013	1.2521	1.6616	1.2689	1.2857	1.2407	1.2400
1	1.1794	1.8862	1.1926	1.8352	1.2064	1.7875	1.2210	1.7427	1.2363	1.7006	1.2523	1.6618	1.2691	1.2859	1.2409	1.2402
2	1.1796	1.8853	1.1928	1.8344	1.2067	1.7867	1.2213	1.7420	1.2365	1.6999	1.2526	1.6620	1.2703	1.2861	1.2411	1.2404
3	1.1798	1.8844	1.1930	1.8336	1.2069	1.7859	1.2215	1.7413	1.2367	1.6991	1.2528	1.6622	1.2705	1.2863	1.2413	1.2406
4	1.1800	1.8836	1.1933	1.8328	1.2072	1.7852	1.2218	1.7405	1.2369	1.6983	1.2531	1.6624	1.2707	1.2865	1.2415	1.2408
5	1.1802	1.8827	1.1935	1.8320	1.2074	1.7844	1.2220	1.7398	1.2371	1.6975	1.2533	1.6626	1.2709	1.2867	1.2417	1.2410
6	1.1805	1.8818	1.1938	1.8311	1.2077	1.7837	1.2223	1.7391	1.2373	1.6967	1.2536	1.6628	1.2711	1.2869	1.2419	1.2412
7	1.1807	1.8809	1.1941	1.8303	1.2079	1.7829	1.2225	1.7384	1.2375	1.6959	1.2538	1.6630	1.2713	1.2871	1.2421	1.2414
8	1.1809	1.8801	1.1944	1.8295	1.2081	1.7821	1.2228	1.7377	1.2377	1.6951	1.2541	1.6632	1.2715	1.2873	1.2423	1.2416
9	1.1811	1.8792	1.1947	1.8287	1.2083	1.7814	1.2230	1.7370	1.2379	1.6943	1.2543	1.6634	1.2717	1.2875	1.2425	1.2418
10	1.1813	1.8783	1.1949	1.8279	1.2086	1.7806	1.2233	1.7362	1.2381	1.6935	1.2546	1.6636	1.2719	1.2877	1.2427	1.2420
11	1.1815	1.8775	1.1952	1.8271	1.2088	1.7798	1.2235	1.7355	1.2383	1.6928	1.2548	1.6638	1.2721	1.2879	1.2429	1.2422
12	1.1818	1.8766	1.1955	1.8263	1.2091	1.7791	1.2238	1.7348	1.2385	1.6920	1.2551	1.6640	1.2723	1.2881	1.2431	1.2424
13	1.1820	1.8757	1.1958	1.8255	1.2093	1.7783	1.2240	1.7341	1.2387	1.6912	1.2553	1.6642	1.2725	1.2883	1.2433	1.2426
14	1.1822	1.8749	1.1961	1.8247	1.2096	1.7776	1.2243	1.7334	1.2389	1.6904	1.2556	1.6644	1.2727	1.2885	1.2435	1.2428
15	1.1824	1.8740	1.1964	1.8239	1.2098	1.7768	1.2245	1.7327	1.2391	1.6896	1.2558	1.6646	1.2729	1.2887	1.2437	1.2430
16	1.1826	1.8731	1.1967	1.8231	1.2100	1.7760	1.2248	1.7320	1.2393	1.6888	1.2561	1.6648	1.2731	1.2889	1.2439	1.2432
17	1.1828	1.8723	1.1970	1.8223	1.2103	1.7753	1.2250	1.7313	1.2395	1.6880	1.2563	1.6650	1.2733	1.2891	1.2441	1.2434
18	1.1831	1.8714	1.1973	1.8215	1.2105	1.7745	1.2253	1.7305	1.2397	1.6872	1.2566	1.6652	1.2735	1.2893	1.2443	1.2436
19	1.1833	1.8706	1.1976	1.8207	1.2107	1.7738	1.2255	1.7298	1.2399	1.6864	1.2568	1.6654	1.2737	1.2895	1.2445	1.2438
20	1.1835	1.8697	1.1979	1.8199	1.2110	1.7730	1.2258	1.7291	1.2401	1.6856	1.2571	1.6656	1.2739	1.2897	1.2447	1.2440
21	1.1837	1.8688	1.1982	1.8191	1.2112	1.7723	1.2260	1.7284	1.2403	1.6848	1.2573	1.6658	1.2741	1.2899	1.2449	1.2442
22	1.1839	1.8680	1.1985	1.8183	1.2115	1.7715	1.2263	1.7277	1.2405	1.6840	1.2576	1.6660	1.2743	1.2901	1.2451	1.2444
23	1.1841	1.8671	1.1988	1.8175	1.2117	1.7708	1.2265	1.7270	1.2407	1.6832	1.2578	1.6662	1.2745	1.2903	1.2453	1.2446
24	1.1844	1.8663	1.1991	1.8167	1.2119	1.7700	1.2268	1.7263	1.2409	1.6824	1.2581	1.6664	1.2747	1.2905	1.2455	1.2448
25	1.1846	1.8654	1.1994	1.8159	1.2122	1.7693	1.2270	1.7256	1.2411	1.6816	1.2583	1.6666	1.2749	1.2907	1.2457	1.2450
26	1.1848	1.8646	1.1997	1.8151	1.2124	1.7685	1.2273	1.7249	1.2413	1.6808	1.2586	1.6668	1.2751	1.2909	1.2459	1.2452
27	1.1850	1.8637	1.1999	1.8143	1.2127	1.7678	1.2275	1.7242	1.2415	1.6800	1.2588	1.6670	1.2753	1.2911	1.2461	1.2454
28	1.1852	1.8629	1.1999	1.8135	1.2129	1.7670	1.2278	1.7235	1.2417	1.6792	1.2591	1.6672	1.2755	1.2913	1.2463	1.2456
29	1.1855	1.8620	1.1999	1.8127	1.2132	1.7663	1.2281	1.7227	1.2419	1.6784	1.2593	1.6674	1.2757	1.2915	1.2465	1.2458
30	1.1857	1.8611	1.1999	1.8119	1.2134	1.7655	1.2283	1.7220	1.2421	1.6776	1.2595	1.6676	1.2759	1.2917	1.2467	1.2460
31	1.1859	1.8603	1.1999	1.8111	1.2136	1.7648	1.2286	1.7213	1.2423	1.6768	1.2597	1.6678	1.2761	1.2919	1.2469	1.2462
32	1.1861	1.8595	1.1999	1.8103	1.2139	1.7640	1.2288	1.7206	1.2425	1.6760	1.2599	1.6680	1.2763	1.2921	1.2471	1.2464
33	1.1863	1.8586	1.1999	1.8095	1.2141	1.7633	1.2291	1.7199	1.2427	1.6752	1.2601	1.6682	1.2765	1.2923	1.2473	1.2466
34	1.1866	1.8578	1.2001	1.8087	1.2144	1.7625	1.2293	1.7192	1.2429	1.6744	1.2603	1.6684	1.2767	1.2925	1.2475	1.2468
35	1.1868	1.8569	1.2004	1.8079	1.2146	1.7618	1.2296	1.7185	1.2431	1.6736	1.2605	1.6686	1.2769	1.2927	1.2477	1.2470
36	1.1870	1.8561	1.2006	1.8071	1.2149	1.7610	1.2298	1.7178	1.2433	1.6728	1.2607	1.6688	1.2771	1.2929	1.2479	1.2472
37	1.1872	1.8552	1.2008	1.8063	1.2151	1.7603	1.2301	1.7171	1.2435	1.6720	1.2609	1.6690	1.2773	1.2931	1.2481	1.2474
38	1.1874	1.8544	1.2010	1.8055	1.2153	1.7596	1.2304	1.7164	1.2437	1.6712	1.2611	1.6692	1.2775	1.2933	1.2483	1.2476
39	1.1877	1.8535	1.2013	1.8047	1.2156	1.7588	1.2306	1.7157	1.2439	1.6704	1.2613	1.6694	1.2777	1.2935	1.2485	1.2478
40	1.1879	1.8527	1.2015	1.8039	1.2158	1.7581	1.2309	1.7151	1.2441	1.6696	1.2615	1.6696	1.2779	1.2937	1.2487	1.2480
41	1.1881	1.8519	1.2017	1.8031	1.2161	1.7573	1.2311	1.7144	1.2443	1.6688	1.2617	1.6698	1.2781	1.2939	1.2489	1.2482
42	1.1883	1.8510	1.2020	1.8023	1.2163	1.7566	1.2314	1.7137	1.2445	1.6680	1.2619	1.6700	1.2783	1.2941	1.2491	1.2484
43	1.1886	1.8502	1.2022	1.8015	1.2166	1.7559	1.2316	1.7130	1.2447	1.6672	1.2621	1.6702	1.2785	1.2943	1.2493	1.2486
44	1.1888	1.8493	1.2024	1.8007	1.2168	1.7551	1.2319	1.7123	1.2449	1.6664	1.2623	1.6704	1.2787	1.2945	1.2495	1.2488
45	1.1890	1.8485	1.2027	1.7999	1.2171	1.7544	1.2322	1.7116	1.2451	1.6656	1.2625	1.6706	1.2789	1.2947	1.2497	1.2490
46	1.1892	1.8477	1.2029	1.7992	1.2173	1.7537	1.2324	1.7109	1.2453	1.6648	1.2627	1.6708	1.2791	1.2949	1.2499	1.2492
47	1.1894	1.8468	1.2031	1.7984	1.2175	1.7530	1.2327	1.7102	1.2455	1.6640	1.2629	1.6710	1.2793	1.2951	1.2501	1.2494
48	1.1897	1.8460	1.2034	1.7976	1.2178	1.7523	1.2329	1.7095	1.2457	1.6632	1.2631	1.6712	1.2795	1.2953	1.2503	1.2496
49	1.1899	1.8452	1.2036	1.7968	1.2180	1.7516	1.2332	1.7088	1.2459	1.6624	1.2633	1.6714	1.2797	1.2955	1.2505	1.2498
50	1.1901	1.8443	1.2039	1.7960	1.2183	1.7509	1.2335	1.7081	1.2461	1.6616	1.2635	1.6716	1.2799	1.2957	1.2507	1.2500
51	1.1903	1.8435	1.2041	1.7953	1.2185	1.7502	1.2337	1.7075	1.2463	1.6608	1.2637	1.6718	1.2801	1.2959	1.2509	1.2502
52	1.1906	1.8427	1.2043	1.7945	1.2188	1.7495	1.2340	1.7068	1.2465	1.6600	1.2639	1.6720	1.2803	1.2961	1.2511	1.2504
53	1.1908	1.8418	1.2046	1.7937	1.2190	1.7488	1.2342	1.7061	1.2467	1.6592	1.2641	1.6722	1.2805	1.2963	1.2513	1.2506
54	1.1910	1.8410	1.2048	1.7929	1.2193	1.7481	1.2345	1.7054	1.2469	1.6584	1.2643	1.6724	1.2807	1.2965	1.2515	1.2508
55	1.1912	1.8402	1.2050	1.7921	1.2195	1.7474	1.2348	1.7047	1.2471	1.6576	1.2645	1.6726	1.2809	1.2967	1.2517	1.2510
56	1.1915	1.8394	1.2053	1.7913	1.2198	1.7467	1.2350	1.7040	1.2473	1.6568	1.2647	1.6728	1.2811	1.2969	1.2519	1.2512
57	1.1917	1.8386	1.2055	1.7905	1.2200	1.7460	1.2353	1.7033	1.2475	1.6560	1.2649	1.6730	1.2813	1.2971	1.2521	1.2514
58	1.1919	1.8378	1.2057	1.7897	1.2203	1.7453	1.2355	1.7026	1.2477	1.6552	1.2651	1.6732	1.2815	1.2973	1.2523	1.2516
59	1.1921	1.8369	1.2060	1.7889	1.2205	1.7446	1.2358	1.7019	1.2479	1.6544	1.2653	1.6734	1.2817	1.2975	1.2525	1.2518
60	1.1924	1.8361	1.2062	1.7883	1.2208	1.7439	1.2361	1.7013	1.2481	1.6536	1.2655	1.6736	1.2819	1.2977	1.2527	1.2520

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°	40°		41°		42°		43°		44°		45°		°
	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	Sec.	Cons.	
0	1.3054	1.5557	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	1.3902	1.4395	1.4108	1.4221	19
1	1.3057	1.5552	1.3253	1.5237	1.3460	1.4940	1.3677	1.4658	1.3905	1.4391	1.4105	1.4217	18
2	1.3060	1.5546	1.3256	1.5232	1.3463	1.4935	1.3681	1.4654	1.3908	1.4387	1.4102	1.4212	17
3	1.3064	1.5541	1.3260	1.5227	1.3467	1.4930	1.3684	1.4649	1.3911	1.4382	1.4099	1.4208	16
4	1.3067	1.5536	1.3263	1.5222	1.3470	1.4925	1.3688	1.4644	1.3914	1.4378	1.4096	1.4204	15
5	1.3070	1.5530	1.3267	1.5217	1.3474	1.4921	1.3692	1.4640	1.3917	1.4374	1.4093	1.4200	14
6	1.3073	1.5525	1.3270	1.5212	1.3477	1.4916	1.3695	1.4635	1.3920	1.4370	1.4090	1.4196	13
7	1.3076	1.5520	1.3274	1.5207	1.3481	1.4911	1.3699	1.4631	1.3923	1.4366	1.4087	1.4192	12
8	1.3080	1.5514	1.3277	1.5202	1.3485	1.4906	1.3703	1.4628	1.3926	1.4362	1.4084	1.4188	11
9	1.3083	1.5509	1.3280	1.5197	1.3488	1.4901	1.3707	1.4624	1.3929	1.4358	1.4081	1.4184	10
10	1.3086	1.5503	1.3284	1.5192	1.3492	1.4897	1.3710	1.4619	1.3932	1.4354	1.4078	1.4180	9
11	1.3089	1.5498	1.3287	1.5187	1.3495	1.4892	1.3714	1.4613	1.3935	1.4350	1.4075	1.4176	8
12	1.3092	1.5493	1.3290	1.5182	1.3499	1.4887	1.3718	1.4608	1.3938	1.4346	1.4072	1.4172	7
13	1.3096	1.5487	1.3294	1.5177	1.3502	1.4882	1.3722	1.4604	1.3941	1.4342	1.4069	1.4168	6
14	1.3099	1.5482	1.3297	1.5171	1.3506	1.4877	1.3725	1.4599	1.3944	1.4338	1.4066	1.4164	5
15	1.3102	1.5477	1.3301	1.5166	1.3509	1.4873	1.3729	1.4595	1.3947	1.4334	1.4063	1.4160	4
16	1.3105	1.5471	1.3304	1.5161	1.3513	1.4868	1.3733	1.4590	1.3950	1.4330	1.4060	1.4156	3
17	1.3109	1.5466	1.3307	1.5156	1.3517	1.4863	1.3737	1.4586	1.3953	1.4326	1.4057	1.4152	2
18	1.3112	1.5461	1.3311	1.5151	1.3520	1.4858	1.3740	1.4581	1.3956	1.4322	1.4054	1.4148	1
19	1.3115	1.5456	1.3314	1.5146	1.3524	1.4854	1.3744	1.4577	1.3959	1.4318	1.4051	1.4144	0
20	1.3118	1.5450	1.3318	1.5141	1.3527	1.4849	1.3748	1.4572	1.3962	1.4314	1.4048	1.4140	
21	1.3121	1.5445	1.3321	1.5136	1.3531	1.4844	1.3752	1.4568	1.3965	1.4310	1.4045	1.4136	
22	1.3125	1.5440	1.3324	1.5131	1.3534	1.4839	1.3756	1.4563	1.3968	1.4306	1.4042	1.4132	
23	1.3128	1.5434	1.3328	1.5126	1.3538	1.4835	1.3759	1.4559	1.3971	1.4302	1.4039	1.4128	
24	1.3131	1.5429	1.3331	1.5121	1.3542	1.4830	1.3763	1.4554	1.3974	1.4298	1.4036	1.4124	
25	1.3134	1.5424	1.3335	1.5116	1.3545	1.4825	1.3767	1.4550	1.3977	1.4294	1.4033	1.4120	
26	1.3138	1.5419	1.3338	1.5111	1.3549	1.4821	1.3771	1.4545	1.3980	1.4290	1.4030	1.4116	
27	1.3141	1.5414	1.3342	1.5106	1.3552	1.4816	1.3774	1.4541	1.3983	1.4286	1.4027	1.4112	
28	1.3144	1.5408	1.3345	1.5101	1.3556	1.4811	1.3778	1.4536	1.3986	1.4282	1.4024	1.4108	
29	1.3148	1.5403	1.3348	1.5096	1.3560	1.4806	1.3782	1.4532	1.3989	1.4278	1.4021	1.4104	
30	1.3151	1.5398	1.3352	1.5092	1.3563	1.4802	1.3786	1.4527	1.3992	1.4274	1.4018	1.4100	
31	1.3154	1.5392	1.3355	1.5087	1.3567	1.4797	1.3790	1.4523	1.3995	1.4270	1.4015	1.4096	
32	1.3157	1.5387	1.3359	1.5082	1.3571	1.4792	1.3794	1.4518	1.3998	1.4266	1.4012	1.4092	
33	1.3161	1.5382	1.3363	1.5077	1.3574	1.4788	1.3797	1.4514	1.4001	1.4262	1.4009	1.4088	
34	1.3164	1.5377	1.3366	1.5072	1.3578	1.4783	1.3801	1.4510	1.4004	1.4258	1.4006	1.4084	
35	1.3167	1.5371	1.3369	1.5067	1.3581	1.4778	1.3805	1.4505	1.4007	1.4254	1.4003	1.4080	
36	1.3170	1.5366	1.3372	1.5062	1.3585	1.4774	1.3809	1.4501	1.4010	1.4250	1.4000	1.4076	
37	1.3174	1.5361	1.3376	1.5057	1.3589	1.4769	1.3813	1.4496	1.4013	1.4246	1.3997	1.4072	
38	1.3177	1.5356	1.3379	1.5052	1.3592	1.4764	1.3816	1.4492	1.4016	1.4242	1.3994	1.4068	
39	1.3180	1.5351	1.3383	1.5047	1.3596	1.4760	1.3820	1.4487	1.4019	1.4238	1.3991	1.4064	
40	1.3184	1.5345	1.3386	1.5042	1.3600	1.4755	1.3824	1.4483	1.4022	1.4234	1.3988	1.4060	
41	1.3187	1.5340	1.3390	1.5037	1.3603	1.4750	1.3828	1.4479	1.4025	1.4230	1.3985	1.4056	
42	1.3190	1.5335	1.3393	1.5032	1.3607	1.4746	1.3832	1.4474	1.4028	1.4226	1.3982	1.4052	
43	1.3193	1.5330	1.3397	1.5027	1.3611	1.4741	1.3836	1.4470	1.4031	1.4222	1.3979	1.4048	
44	1.3197	1.5325	1.3400	1.5022	1.3614	1.4736	1.3839	1.4465	1.4034	1.4218	1.3976	1.4044	
45	1.3200	1.5320	1.3404	1.5018	1.3618	1.4732	1.3843	1.4461	1.4037	1.4214	1.3973	1.4040	
46	1.3203	1.5314	1.3407	1.5013	1.3622	1.4727	1.3847	1.4457	1.4040	1.4210	1.3970	1.4036	
47	1.3207	1.5309	1.3411	1.5008	1.3625	1.4723	1.3851	1.4452	1.4043	1.4206	1.3967	1.4032	
48	1.3210	1.5304	1.3414	1.5003	1.3629	1.4718	1.3855	1.4448	1.4046	1.4202	1.3964	1.4028	
49	1.3213	1.5299	1.3418	1.4998	1.3633	1.4713	1.3859	1.4443	1.4049	1.4198	1.3961	1.4024	
50	1.3217	1.5294	1.3421	1.4993	1.3636	1.4709	1.3863	1.4439	1.4052	1.4194	1.3958	1.4020	
51	1.3220	1.5289	1.3425	1.4988	1.3640	1.4704	1.3867	1.4435	1.4055	1.4190	1.3955	1.4016	
52	1.3223	1.5283	1.3428	1.4983	1.3644	1.4699	1.3870	1.4430	1.4058	1.4186	1.3952	1.4012	
53	1.3227	1.5278	1.3432	1.4979	1.3647	1.4695	1.3874	1.4426	1.4061	1.4182	1.3949	1.4008	
54	1.3230	1.5273	1.3435	1.4974	1.3651	1.4690	1.3878	1.4422	1.4064	1.4178	1.3946	1.4004	
55	1.3233	1.5268	1.3439	1.4969	1.3655	1.4686	1.3882	1.4417	1.4067	1.4174	1.3943	1.4000	
56	1.3237	1.5263	1.3442	1.4964	1.3658	1.4681	1.3886	1.4413	1.4070	1.4170	1.3940	1.3996	
57	1.3240	1.5258	1.3446	1.4959	1.3662	1.4676	1.3890	1.4408	1.4073	1.4166	1.3937	1.3992	
58	1.3243	1.5253	1.3449	1.4954	1.3665	1.4671	1.3894	1.4404	1.4076	1.4162	1.3934	1.3988	
59	1.3247	1.5248	1.3453	1.4949	1.3669	1.4667	1.3898	1.4400	1.4079	1.4158	1.3931	1.3984	
60	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	1.3902	1.4395	1.4082	1.4154	1.3928	1.3980	

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4. Frequently Used Angles and Their Functions

Angle	sin A	cos A	tan A	cot A	sec A	csc A
0°	0	1	0	∞	1	∞
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
90°	1	0	∞	0	∞	1
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$
180°	0	-1	0	∞	-1	∞
270°	-1	0	∞	0	∞	-1
360°	0	1	0	∞	1	∞

5. All Functions of an Angle Expressed in Terms of Any One Function

Function	sine	cosine	tangent	cotangent	secant	cosecant
Sin A	sin A	$\pm\sqrt{1-\cos^2 A}$	$\frac{\tan A}{\pm\sqrt{1+\tan^2 A}}$	$\frac{1}{\pm\sqrt{1+\cot^2 A}}$	$\frac{\pm\sqrt{\sec^2 A-1}}{\sec A}$	$\frac{1}{\csc A}$
Cos A	$\pm\sqrt{1-\sin^2 A}$	cos A	$\frac{1}{\pm\sqrt{1+\tan^2 A}}$	$\frac{\cot A}{\pm\sqrt{1+\cot^2 A}}$	$\frac{1}{\sec A}$	$\frac{\pm\sqrt{\csc^2 A-1}}{\csc A}$
Tan A	$\frac{\sin A}{\pm\sqrt{1-\sin^2 A}}$	$\frac{\pm\sqrt{1-\cos^2 A}}{\cos A}$	tan A	$\frac{1}{\cot A}$	$\pm\sqrt{\sec^2 A-1}$	$\frac{1}{\pm\sqrt{\csc^2 A-1}}$
Cot A	$\frac{\pm\sqrt{1-\sin^2 A}}{\sin A}$	$\frac{\cos A}{\pm\sqrt{1-\cos^2 A}}$	$\frac{1}{\tan A}$	cot A	$\frac{1}{\pm\sqrt{\sec^2 A-1}}$	$\pm\sqrt{\csc^2 A-1}$
Sec A	$\frac{1}{\pm\sqrt{1-\sin^2 A}}$	$\frac{1}{\cos A}$	$\pm\sqrt{1+\tan^2 A}$	$\frac{\pm\sqrt{1+\cot^2 A}}{\cot A}$	sec A	$\frac{\csc A}{\pm\sqrt{\csc^2 A-1}}$
Csc A	$\frac{1}{\sin A}$	$\frac{1}{\pm\sqrt{1-\cos^2 A}}$	$\frac{\pm\sqrt{1+\tan^2 A}}{\tan A}$	$\pm\sqrt{1+\cot^2 A}$	$\frac{\sec A}{\pm\sqrt{\sec^2 A-1}}$	csc A

ANSWERS TO PROBLEMS

Paragraph 12.

- $a(1) \frac{3}{5}; .6; 60\%.$ $(2) \frac{1}{2}; .5; 50\%.$ $(3) \frac{3}{8}; .375; 37\frac{1}{2}\%.$ $(4) \frac{1}{4}; .25; 25\%.$
 $(5) \frac{5}{8}; .625; 62\frac{1}{2}\%.$ $(6) \frac{3}{5}; .6; 60\%.$
 $(7) \frac{3}{10}; .3; 30\%.$ $(8) \frac{7}{10}; .7; 70\%.$ $(9) 2\frac{1}{4}; 2.25; 225\%.$ $(10) \frac{7}{8}; .875; 87\frac{1}{2}\%.$
 $(11) \frac{2}{25}; .08; 8\%.$ $(12) \frac{3}{50}; .06; 6\%.$
 $(13) \frac{9}{50}; .18; 18\%.$ $(14) \frac{1}{400}; .0025; 25\%.$ $(15) \frac{1}{40}; .025; 2\frac{1}{2}\%.$ $(16) \frac{1}{20}; .05; 5\%.$ $(17) \frac{1}{12}; .08\frac{1}{3}$ (See note below); $8\frac{1}{3}\%.$ $(18) \frac{3}{8}; .375; 37\frac{1}{2}\%.$ $(19) 1\frac{1}{20}; 1.05; 105\%.$ $(20) \frac{1}{25}; .04; 4\%.$

Note. This mixed decimal and fractional form is often used when an unending decimal would result.

- $b(1) 150; (2) 50; (3) 4; (4) 900.$
 $c(1) 150\%; (2) 275\%; (3) 150\%; (4) 550\%.$
 $d(1) 1.64; (2) 2,496; (3) .34; (4) 4.42.$
 $e(1) .207\%; (2) .028\%.$
 $f(1) 433\frac{1}{8}; (2) 2,500; (3) 520; (4) 200; (5) 200.$

Paragraph 21.

- a 336.6 pounds. b $3\frac{8}{7}$ days. c \$5.00. d \$1400.00. e .372 ohm. .298 ohm; .459 ohm; .898 ohm. f 2.820 pounds; 3.776 pounds; 4.119 pounds; 2,567 pounds. g 300 rpm. h 157.5 rpm.

Paragraph 25.

- $a(1) 21; (2) 33; (3) 50; (4) 2.90; (5) 50.1; (6) 70.01; (7) 86.5; (8) 75.89.$
 $b(1) 2.236; (2) 2.646; (3) 3.317; (4) 3.606; (5) 3.873; (6) 4.123.$
 $c(1) .158$ ampere; $(2) .085$ ampere; $(3) .233$ ampere; $(4) 1.118$ amperes.

Paragraph 42.

- $a(1) 17; (2) 58; (3) -21; (4) -139^\circ; (5) -252$ volts.
 $b(1) 251$ amperes; $(2) -8$ volts; $(3) -.6375cy$; $(4) -81.99ax^2$; $(5) 1.810x^2y$.
 $c(1) -17.92; (2) -72; (3) \frac{8}{85}; (4) .075852; (5) .0028125; (6) 120.$
 $d(1) 9; (2) -\frac{20}{21}; (3) 700; (4) 250; (5) -\frac{2}{8}$ ampere; $(6) -.0025.$
 $e(1) -4; (2) 14; (3) -25; (4) 19; (5) 11; (6) 16; (7) 44; (8) 66; (9) -46; (10) 18.$

Paragraph 50.

- $a(1) 4a^4 - 4a^2b^2 + 4b^4.$ $(2) E + 3RI + 20ZI.$ $(3) w + x + 9y + 8z.$
 $b(1) 19ax + 17by - 9cz.$ $(2) -25w - 3x + 8y + 2z.$ $(3) 4a^2 - 34ab + 6b^2.$
 $c(1) 7.$ $(2) 1.$ $(3) 1.$
 $d(1) f^{10}.$ $(2) y^{a+b}.$ $(3) v^{2a}.$ $(4) r^2.$ $(5) R^{2m}.$ $(6) r^{m+1}.$
 $e(1) \frac{4}{x^4}.$ $(2) \frac{1}{r^2s^4}.$ $(3) \frac{1}{36^3a^{21}}.$ $(4) \frac{1}{r^2R}.$ $(5) \frac{a^2}{8b^2}.$ $(6) \frac{3E}{4r^2R}.$
 $f(1) 10a^2b - 15a^2b^2 + 35ab^3.$ $(2) 4a^3 + 12a^2 + 4a.$ $(3) i^3 - 27.$ $(4) 2x^4 + 5x^2y + 4x^2y^2 + 2xy^3 - y^4.$ $(5) 9x^4 - 4x^2y^2 +$

$$20xy^2 - 25y^4. (6) \frac{a-e}{ca}. (7) \frac{3L-R^2}{R}$$

$$(8) 1 - 2a^2b + 3a^4b^3. (9) 2x^2 + z - 1 + \frac{3z+4}{x^2-z+8}. (10) 4b^2 - b.$$

Paragraph 61.

$$a(1) 5(5+1-6); (2) 4(2+1-8);$$

$$(3) 3(3-6+7); (4) 7r(1-3+5);$$

$$(5) 2(5x+4y+3z).$$

$$b(1) 49x^2y^3; (2) 4w^{10}; (3) 64a^4b^3; (4)$$

$$729a^3x^3; (5) -27b^3x^{12}.$$

$$c(1) 5; (2) -8; (3) \pm ab^3; (4) \pm 6yz^2;$$

$$(5) -10ab^3; (6) \pm 20a; (7) -3; (8)$$

$$-x^3; (9) 4; (10) 5x^4y^3z^2.$$

$$d(1) 3(x+2); (2) 5a(a+3); (3) 2x(5x^2$$

$$-7x-1); (4) 3z(2ay+3bx-4c); (5)$$

$$m(m^2+m-5x); (6) 3a^3(a^2-2ab-b^2);$$

$$(7) 7ry^3(1-2+3) \text{ or } 14^3ry; (8)$$

$$2xam(6x+7a+8m); (9) \pi(r_1^2+r_2^2);$$

$$(10) \frac{1}{16}cd(4c^2-2cd+d^2).$$

Paragraph 69.

$$a(1) x = 5\frac{2}{5}; (2) x = 4; (3) r = 2; (4)$$

$$x = -1\frac{1}{12}; (5) t = 1; (6) x = 7\frac{3}{4}; (7)$$

$$r = 4; (8) x = 1.$$

$$b(1) 8; (2) x; (3) 3(r+s); (4) 3(a-s);$$

$$(5) (I-6)(I-9); (6) \frac{8E^2I^2}{2I^2R}; (7)$$

$$\frac{2f}{6\pi f^2c}.$$

$$c(1) \frac{rR}{rR^2}, \frac{r}{rR^2}, \frac{R^2}{rR^2}; (2) \frac{a-1}{a^2-1},$$

$$\frac{x(a+1)}{a^2-1}; (3) \frac{3b}{6x}, \frac{2c}{6x}; (4) \frac{y(y+3)}{2(y+3)},$$

$$\frac{y}{2(y+3)}; (5) \frac{2(c+1)}{c(c+1)}, \frac{3c}{c(c+1)};$$

$$(6) \frac{2i}{2e-10}, \frac{i}{2e-10}; (7) \frac{y}{C^2-d^2}$$

$$\frac{z(c+d)}{C^2-d^2}.$$

$$d(1) \frac{12}{a}; (2) \frac{7s+11}{4t};$$

$$(3) \frac{9y^2a+10xb}{12x^2y^3}; (4) \frac{6(x^2-2)}{x^4-5x^2+14};$$

$$(5) \frac{9c+2cd-12d}{12c^2d^2}; (6) \frac{2r^2+r-13}{r^2+2r-15};$$

$$(7) \frac{12y-1}{4}; (8) \frac{4ab}{a^2-b^2};$$

$$(9) \frac{16(2-5q)}{25q^2}; (10) \frac{3t+4y}{12tv^2}$$

$$e(1) \frac{3y^2}{8}; (2) \frac{a^3}{b^3}; (3) \frac{xz}{21my};$$

$$(4) \frac{(s-r)^2}{s^2}; (5) \frac{3}{5x}; (6) \frac{1}{a^3};$$

$$(7) 15z; (8) \frac{a^3}{6cd}; (9) \frac{4su}{5};$$

$$(10) \frac{e+3}{e+2}.$$

Paragraph 76.

$$a(1) 2; (2) 16; (3) 5\sqrt{2}; (4) \frac{\sqrt[3]{4}}{4};$$

$$(5) 3\sqrt{2x-1}; (6) \frac{x^2\sqrt[3]{6}}{y}; (7) x^2y;$$

$$(8) d^{\frac{2}{3}}e^3; (9) \frac{4r^2}{s}; (10) a^3b.$$

$$b(1) \sqrt[3]{4}; (2) \sqrt[3]{a^3b^4}; (3) \sqrt[3]{6^2}; (4) \sqrt[3]{27};$$

$$(5) \sqrt[3]{x}; (6) \sqrt[3]{a^3b^3}; (7) 6\sqrt[3]{r}; (8) 26\sqrt[3]{a^3};$$

$$(9) \sqrt{2r_1+3r_2}; (10) 3y\sqrt[3]{x}.$$

$$c(1) a^4; (2) (5x)^4; (3) 6xd^4; (4) z^4;$$

$$(5) (3a^3b^5)^4; (6) y^3a^4; (7) 8(3e)^4;$$

$$(8) 9g^4; (9) 3bcd^4; (10) (x-y)^4.$$

$$d(1) 2\sqrt{3}; (2) 3\sqrt{7}; (3) 3x\sqrt{7};$$

$$(4) 12ab^2\sqrt{2}; (5) 2bd\sqrt{15}; (6) 2l\sqrt{2R};$$

$$(7) 9pz\sqrt{7p}; (8) 12dr^4s\sqrt{3ds}; (9)$$

$$45a^2\sqrt{b}; (10) 112w^4x^2y\sqrt{2xz}.$$

$$e(1) \frac{\sqrt{2}}{10}; (2) \frac{\sqrt{x}}{2x}; (3) \frac{\sqrt[3]{3a}}{3}; (4) \frac{\sqrt[3]{x^3}}{x};$$

$$(5) \frac{\sqrt[3]{27a^3x^3}}{3ax}; (6) \frac{\sqrt[3]{(3-2x)^3}}{3-2x};$$

$$(7) \frac{\sqrt[3]{a(a+b)}}{a}; (8) \frac{\sqrt[3]{ab^3c^2}}{bc};$$

$$(9) \frac{\sqrt[3]{s+1}}{s+1}; (10) \sqrt[3]{(i+3)^3}.$$

$$f(1) 10; (2) 14\sqrt{5}; (3) x - \frac{x\sqrt{3}}{2}; (4)$$

$$\frac{3a\sqrt{2} + a}{2}; (5) (r + 1)\sqrt{rst}; (6)$$

$$\frac{2y\sqrt{x^2 - y^2}}{x^2 - y^2}; (7) \sqrt[3]{5} + 8\sqrt{x}; (8) 7\sqrt{a} -$$

$$6\sqrt{b}; (9) 3\sqrt{x + y} - 4\sqrt{x - y}; (10) 7ab\sqrt{5a}.$$

$$g(1) 12\sqrt{10}; (2) 18; (3) 8ab^2; (4)$$

$$2x^3\sqrt{3z}; (5) 2xy\sqrt[3]{xy}; (6) 24pq^2r\sqrt[3]{qr^2};$$

$$(7) a + b + c + 2(\sqrt{ab} + \sqrt{ac} + \sqrt{bc});$$

$$(8) ax\sqrt{a} (a + x + 1); (9) 8; (10)$$

$$2axy^2\sqrt[3]{2a}.$$

$$h(1) 2; (2) 5; (3) 2\sqrt[3]{x}; (4) 3\sqrt{xy}; (5)$$

$$\sqrt{6} + 2; (6) 12a\sqrt[12]{2^3 \cdot 3^5 \cdot 5^4 \cdot a^2}; (7)$$

$$\frac{c - \sqrt{2c} - 4}{c - 8} \quad (8) \sqrt{15}; (9)$$

$$\frac{e^2 + f^2 + 2f\sqrt{e^2}}{e^2} \sqrt{f^2}; (10) \frac{4b\sqrt{1 - 4b^2} + 1}{8b^2 - 1}$$

Paragraph 79.

$$a(1) j5\sqrt{3}; (2) j\sqrt{\quad}; (3) -j8x^2\sqrt{ax};$$

$$(4) -j10x^2y^2\sqrt{x}; (5) \frac{\div}{3};$$

$$(6) -4xy\sqrt[3]{2x^2y^2}.$$

$$b(1) 16 + j109; (2) 41 - j22; (3) 61 -$$

$$j251; (4) 4 + j10; (5) 6 + j11; (6) -2 - j47.$$

$$c(1) 779 - j371; (2) 59 + j114; (3)$$

$$-22 + j15; (4) 155 - j61; (5) 169 + j23; (6) 9 - j8.$$

$$d(1) -55 + j46; (2) 6 - 6\sqrt{6} + j(6\sqrt{2}$$

$$+ 6\sqrt{3}); (3) 13; (4) -5 - j12; (5)$$

$$-j8; (6) 46 - j48; (7) f^2 + jfg - g^2;$$

$$(8) I^2 + E^2; (9) -68 - j239; (10) 71 - j17.$$

$$e(1) \frac{3}{13} - j\frac{2}{13}; (2) 1 - j6; (3) -\frac{6}{25}$$

$$+ j\frac{17}{25}; (4) 1 + j2; (5) \frac{x^2 + j^2xy - y^2}{x^2y^2};$$

$$(6) 2(1 - j2); (7) \frac{3(1 + j)}{2}; (8) \frac{1 + j13}{10};$$

$$(9) \frac{38 + j34}{65}; (10) \frac{I^2 + j2IE - E^2}{I^2 E^2}$$

Paragraph 86.

$$a(1) 3; (2) 2; (3) 85; (4) 3; (5) 1;$$

$$(6) x = -5, y = 8; (7) a = 3, b = 1;$$

$$(8) x = 3, y = 4; (9) m = 3, n = 5;$$

$$(10) r = 8, s = 1$$

$$b(1) d = \frac{Wh}{F} \quad (2) g = \frac{v^2 - v_0^2}{2h}.$$

$$(3) a = \frac{Fg}{w}. (4) N = \frac{2.534H}{D^2}. (5) l =$$

$$\frac{10^8 F}{22.5BI}.$$

$$c(1) 15; (2) 0; (3) \frac{10}{8}; (4) 4; (5) \frac{28}{9};$$

$$(6) \frac{12}{119}; (7) -2\frac{12}{25}; (8) 8; (9) \frac{40}{109};$$

$$(10) -\frac{1}{19}.$$

$$d(1) x = 4, y = 5; (2) a = 4.95, b = 2.61;$$

$$(3) x = 4, y = 7; (4) x = -2, y = -4;$$

$$(5) x = -3, y = 1; (6) I = 13, Z = 17;$$

$$(7) x = 4, y = \frac{1}{2}; (8) a = 6, b = -4;$$

$$(9) x = 5, y = -1; (10) r = \frac{(a + b)}{2},$$

$$s = \frac{(a - b)}{2}$$

$$e(1) 1 \text{ volt}; (2) R = 20 \text{ ohms}; (3) 110$$

$$\text{volts}; (4) 75 \text{ ohms}; (5) 100 \text{ milliamperes},$$

$$80 \text{ milliamperes}, 60 \text{ milliamperes}; (6) 5.5$$

$$\text{amperes}.$$

Paragraph 94.

$$a(1) 0, -\frac{3}{2}; (2) 0, 4; (3) 0, -3; (4) 0,$$

$$-2; (5) \pm 8; (6) \pm 3; (7) \pm 3; (8) \pm 4;$$

$$(9) 7, -6; (10) 1, 12.$$

$$b(1) \frac{-3 \pm \sqrt{13}}{2}; (2) -3 \pm \sqrt{19}; (3)$$

$$2 \pm \sqrt{3}; (4) -2 \pm \frac{\sqrt{22}}{2}; (5) \frac{1}{2} \pm$$

$$\frac{\sqrt{14}}{4}; (6) -\frac{5}{3} \pm \frac{2\sqrt{10}}{3}; (7) -1, 3;$$

$$(8) -1 \pm \frac{\sqrt{6}}{2}; (9) 1 \pm \sqrt{6}; (10) \frac{1}{2} \pm$$

$$\frac{\sqrt{5}}{2}$$

$$c(1) -1; (2) -\frac{3}{4}, \frac{2}{3}; (3) \frac{-5 \pm \sqrt{13}}{6};$$

$$(4) \frac{3}{2}, \frac{4}{3}; (5) -3, 1; (6) -\frac{1}{5}, \frac{5}{8};$$

$$(7) \pm \sqrt{2}; (8) \pm \sqrt{19}; (9) -1, 2;$$

$$(10) \frac{-5 \pm \sqrt{7}}{8}$$

Paragraph 111.

$$a(1) 1,613 \times 10^3; (2) 500 \times 10^3, \text{ or } 5 \times 10^5; (3) 6,166 \times 10^{-3}.$$

$$b(1) 3,109 \times 10^3; (2) 19 \times 10^{-4}; (3) 4,492 \times 10^{-4}.$$

$$c(1) 892 \times 10^3; (2) 2,464 \times 10^{-3}, \text{ or } 24.64; (3) 3,168 \times 10^{-11}; (4) 14,640.$$

$$d(1) 167; (2) 1,608 \times 10^7; (3) 107; (4) 33 \times 10^{-3}.$$

$$e(1) 4 \times 10^3, \text{ or } 400; (2) 13 \times 10^{-4}; (3) 27 \times 10^{-9}; (4) 9 \times 10^2, \text{ or } 900.$$

Paragraph 127.

$$a(1) 2.8949; (2) 0.5527; (3) 8.5378-10; (4) 6.6776-10; (5) 1.6955; (6) 2.4370; (7) 2.8809; (8) 0.8593; (9) 7.9946-10; (10) 5.7205-10.$$

$$b(1) 70,100; (2) 271; (3) .351; (4) .000676; (5) 3.99; (6) 370.67; (7) .00002718; (8) 500,500; (9) 1.5915; (10) .000003445.$$

$$c(1) 164.2; (2) 39,982; (3) 1,376; (4) .006764; (5) 5,710.$$

$$d(1) .4983; (2) .3874; (3) .3984; (4) .7487; (5) .2437.$$

$$e(1) .0000007372; (2) 51.46; (3) 3.47; (4) 19.43; (5) 783; (6) .2367; (7) 5.343; (8) 87.74; (9) 1.55; (10) .09456.$$

$$f(1) 2.298; (2) 11.77; (3) 24.43; (4) 83.28; (5) .4509; (6) .4725; (7) .04088; (8) .6153; (9) .0576; (10) .35367.$$

Paragraph 142.

a 96 square inches. b 36 square inches.
 c 25 square inches. d 15 square inches.
 e 14.422 square inches. f 5.657 square inches.
 g (1) Parallelogram, $A = bh$, 120 square inches; (2) Triangle, $A = \frac{bh}{2}$ 4.025 square inches; (3) Circle, $A = \pi r^2$, 814 square centimeters, $C = \pi D$, 62.8 centimeters; (4) Trapezoid, $A = \frac{B+b}{2}h$, $A = 60$ square inches. h (1) 3 inches; (2) $4\frac{1}{2}$ inches; (3) 8.8 inches; (4) 5 inches. i 78.5 square inches. j 100 feet. k 82.5 square feet. l 48.496 inches.

Paragraph 153.

$$a(1) c = 8.603. (2) a = 6.08. (3) b = 39.5. (4) c = b\sqrt{10}. (5) b = m^2 - 1.$$

$$b(1) \sin A = \frac{4}{7}, \cos A = \frac{\sqrt{33}}{7}, \tan A = \frac{4}{\sqrt{33}}, \cot A = \frac{\sqrt{33}}{4}, \sec A = \frac{7}{33}\sqrt{33}, \csc A = \frac{7}{4}$$

$$(2) \sin A = \frac{2}{13}\sqrt{13}, \cos A = \frac{3}{13}\sqrt{13}, \tan A = \frac{2}{3}, \cot A = \frac{3}{2}, \sec A = \frac{\sqrt{13}}{3}, \csc A = \frac{\sqrt{13}}{2}$$

$$(3) \sin A = \frac{1}{2}, \cos A = \frac{\sqrt{3}}{2}, \tan A = \frac{\sqrt{3}}{3}, \cot A = \sqrt{3}, \sec A = \frac{2}{3}\sqrt{3}, \csc A = 2.$$

$$(4) \sin A = \frac{1}{2.4}, \cos A = \frac{1.09}{1.2}, \tan A = \frac{1}{2.18}, \cot A = 2.18, \sec A = \frac{1.2}{1.09}, \csc A = 2.4.$$

$$(5) \sin A = y \frac{\sqrt{y^2+1}}{y^2+1}, \cos A = \frac{\sqrt{y^2+1}}{y^2+1}, \tan A = y, \cot A = \frac{1}{y}, \sec A = \sqrt{y^2+1}, \csc A = \frac{\sqrt{y^2+1}}{y}.$$

$$(6) \sin A = \frac{\sqrt{55}}{8}, \cos A = \frac{3}{8}, \tan A = \frac{\sqrt{55}}{3}, \cot A = \frac{3\sqrt{55}}{55}, \sec A = 2\frac{2}{3}, \csc A = \frac{8\sqrt{55}}{55}$$

$c(1) a = 17, b = 29.4, c = 34.$ (2) $a = 9, b = 12, c = 15.$ (3) $a = 12, b = 16, c = 20.$ (4) $a = 17.5, b = 10\sqrt{11}, c = 37.5.$ (5) $a = 10, b = 6, c = 2\sqrt{34}.$ (6) $a = 37.08, b = 18.4, c = 41.4.$

$d(1) b = 10\sqrt{3}, c = 20.$ (2) $a = 7, c = 7\sqrt{2}.$ (3) $a = 4\sqrt{3}, b = 4.$ (4) $b = 3\sqrt{3}, c = 6\sqrt{3}.$ (5) $a = 12.5, b = 12.5\sqrt{3}.$

Paragraph 164.

$a(1) .02618, .99966, .02619, 38.1885.$ (2) $.26584, .96402, .27576, 3.62636.$ (3) $.53238, .84650, .62892, 1.59002.$ (4) $.59693, .80230, .74402, 1.34405.$ (5) $.70690, .70731, .99942, 1.00058.$ (6) $.70706, .70716, .99986, 1.00014.$ (7) $.57649, .81710, .70553, 1.41737.$ (8) $.81370, .58129, 1.39982, .71438.$ (9) $.74811, .66357, 1.12740, .88700.$ (10) $.92429, .38169, 2.42158, .41295.$

$b(1) 14^\circ 54' 51''; (2) 66^\circ 35' 51''; (3) 19^\circ 56' 54''; (4) 25^\circ 17' 5''; (5) 40^\circ 23' 35''; (6) 68^\circ 45' 2''; (7) 22^\circ 11' 47''; (8) 34^\circ 5' 19''; (9) 52^\circ 13' 2''; (10) 51^\circ 29' 49''$

$c(1) 44^\circ 43' 29''; (2) 10.29; (3) 32.9; (4) 19.76; (5) 12.4; (6) 54^\circ 18' 52.5''; (7) 33.69; (8) 16.5; (9) 36^\circ 28' 9''; (10) 128.3; (11) 32.9; (12) 29^\circ 3' 15''$

$d(1) 43.845 \text{ feet}; (2) 80.027 \text{ feet}; (3) 12.226 \text{ feet}, 8.69 \text{ feet high}; (4) 3,149 \text{ feet}; (5) 11.734 \text{ feet}; (6) 91.77 \text{ feet}; (7) 206 \text{ feet}; (8) 3,578 \text{ feet}; (9) 16,647 \text{ feet} (3.153 \text{ miles}); (10) 82.12 \text{ feet}; (11) 1.414 \text{ inches each}; (12) \text{side opposite } 60^\circ \angle 5.196 \text{ inches, side opposite } 30^\circ \angle 3 \text{ inches.}$

Paragraph 173.

$a C = 62^\circ 16' 38'', a = 14.59. b B = 69^\circ 58'. c A = 23^\circ 33' 22'', B = 45^\circ 16' 31'', C = 111^\circ 10' 7''. d A = 81^\circ 31' 41' B = 41^\circ 7' 29''. e 240 \text{ square inches. } 97.880 \text{ square feet. } g 55.424 \text{ square inches. } h A = 32^\circ 33' 45'', B = 84^\circ 36' 15'', c = 15.95 \text{ inches.}$

Paragraph 176.

$a(1) .4 \text{ radian}; (2) 4 \text{ radians}; (3) 1 \text{ radians}; (4) 2.78 \text{ radians.}$

$b(1) 35 \text{ inches}; (2) 17.6 \text{ feet}; (3) 18.1 \text{ miles}; (4) .00198 \text{ inch.}$

$c(1) .52 \text{ radian}; (2) 4.6 \text{ radians}; (3) 2.77 \text{ radians}; (4) 5.89 \text{ radians.}$

$d(1) 45^\circ 50' 11.8''; (2) 1432^\circ 23' 40.2'' (3) 197^\circ 40' 13.44''; (4) 540^\circ.$

$e(1) \pi/6; (2) \pi/3; (3) 5\pi/4; (4) 4\pi.$

Paragraph 192.

$a(1) 5.5 \text{ amperes}; (2) 1.30 \text{ amperes.}$

$b(1) 28 \text{ to } 100 \text{ ohms}; (2) 7 \text{ to } 25 \text{ ohms.}$

$c 221 \text{ volts}; 7.514 \text{ watts.}$

$d(1) .8 \text{ ampere}; (2) 24 \text{ volts}; (3) 80 \text{ volts.}$

$e(1) G_T = .35 \text{ inch}; (2) 2.857 \text{ ohms}; (3) I_2 = 10 \text{ amperes, } I_3 = 5 \text{ amperes}; (4) I_T = 35 \text{ amperes.}$

$f(1) G_1 = 1 \text{ mho, } G_2 = .333 \text{ mho, } G_3 = .1 \text{ mho, } G_4 = .05 \text{ mho, } G_5 = .02 \text{ mho}; (2) G_T = 1.503 \text{ mhos}; (3) R_T = .665 \text{ mho.}$

$g(1) 17.08 \text{ ohms}; (2) 86.4 \text{ volts}; (3) 100.8 \text{ volts}; (4) 228.192 \text{ volts.}$

$h(1) 4.62 \text{ ohms}; (2) 5.359 \text{ ohms}; (3) 4.783 \text{ ohms}; (4) 15.246 \text{ ohms.}$

$i(1) 125 \text{ volts}; (2) E_1 = 50 \text{ volts, } E_2 = 75 \text{ volts.}$

$j(1) 10.754 \text{ volts}; (2) I_1 = .7028 \text{ ampere, } I_2 = 8.269 \text{ amperes, } I_3 = 1.0237 \text{ amperes;}$

(8) $I_1 R_1 = 10.753$ volts, $I_2 R_2 = 10.750$ volts, $I_3 R_3 = 10.749$ volts.

k(1) 5,500 amperes; (2) .22729 ampere.

l A, 5 ohms; B, $6\frac{2}{5}$ ohms; C, 8 ohms.

m A, 3 ohms; B, 12 ohms; C, 7 ohms; D, 25 ohms.

n A, $8\frac{8}{9}$ ohms; B, $8\frac{1}{8}$ ohms; C, 10 ohms.

o A, 3 ohms; B, 4 ohms; C, 2 ohms.

p $I_{R1} = 7.519$ amperes; $I_{R2} = 3.214$ amperes; $I_{R3} = 1.176$ amperes; $I_{R4} = I_{R7} = I_{R8} = 0.392$ ampere; $I_{R6} = I_{R10} = 4.782$ amperes; $I_{R5} = I_{R9} = 1.568$ amperes, $I_{R11} = 12.301$ amperes.

q $R = 239.1688$ ohms.

r $I = .457$ ampere.

Paragraph 204.

a(1) 94 ohms; (2) 134 ohms; (3) current leads voltage because capacitive reactance exceeds inductive reactance; (4) 104 ohms; (5) 5 amperes.

b. 9.425 ohms.

c. 63,662 ohms (approx).

d. 80 ohms.

e. 455 kc.

f(1) 30 amperes; (2) 180 volts; (3) 480 volts; (4) 240 volts.

g(1) 3 amperes; (2) 4 amperes; (3) 6 amperes; (4) 83 ohms; (5) 34° ; (6) 896.4 watts; (7) 1080 watts, or 1.08 kw

Paragraph 208.

a. 39.8 to 1.

b. $-.744$ db per mile.

c. $+10.8$ db.

d. 25.1 to 1.

Paragraph 217.

a. 5%.

b. 77.7%

c. 15%.

d. 12,600 ohms.

e. 86.7%.

f. 2,200 watts.

g. 1.58 amperes.

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